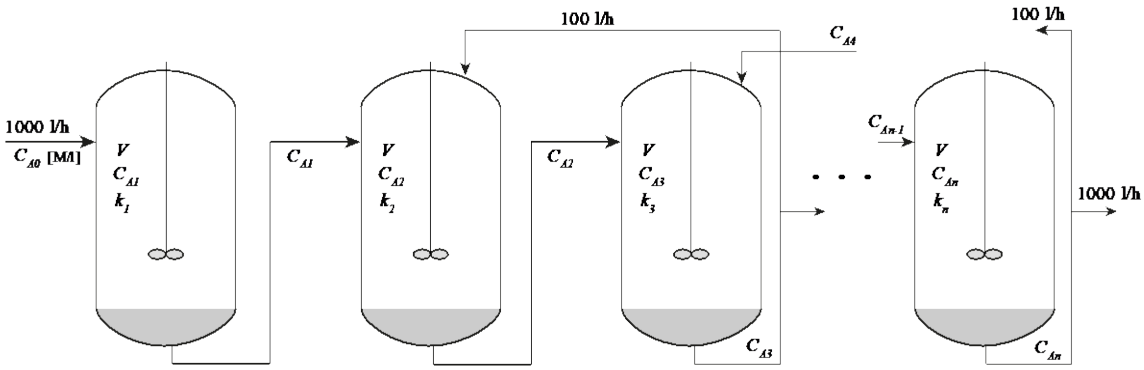
Examples of potential exam problems
13.04.2021

1. **(Linear)** A chemical reaction takes place in a series of *n* continuous stirred tank reactors as shown below.



The material balance equations of these reactors are as follows

$$\begin{array}{ccccccccc}
1100C_{A1} & 0 & 0 & 0 & 0 & 0 & \cdots & = 1000 \\
1000C_{A1} & -1400C_{A2} & 100C_{A3} & 0 & 0 & 0 & \cdots & = 0 \\
0 & 1100C_{A2} & -1240C_{A3} & 100C_{A4} & 0 & 0 & \cdots & = 0 \\
\vdots & 0 & 1100C_{A3} & -1250C_{A4} & 100C_{A5} & 0 & \cdots & \vdots \\
\vdots & \vdots & 0 & \ddots & \ddots & \ddots & \cdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{array}$$

Which method would you use to solve this system? Why? Assuming that *n=3*, show in three steps how you would use the proposed method.

2. **(Nonlinear)** Consider the following nonlinear equation on the interval (2.5,pi):

$$2e^{x}\sin x = 2x - 1$$

   a) Apply the Regula-Falsi method to find numerically the root of this equation. Perform 3 iterations. What is the attained accuracy if you know that the exact solution is *x*= 3.01816421*.
   b) Repeat a. with the Newton-Raphson method and 2 iterations. Take as initial point $x_0=2.5$.
   c) Estimate the number of Newton-Raphson iterations needed to reach this root with a precision of 10^{-9}.

Detailed calculus and utilized expressions should be provided as well.

3. **(Differentiation and Integration)** A space shuttle reentering the earth atmosphere decelerates in one hour from its orbital speed (7360.56 m/s) to its touchdown speed (96.11 m/s). Table 1 contains data of its measured velocity at equally spaced time-points.

Time (s)	-3600	-2400	-1200	0
Velocity (m/s)	7360.56	7245.61	6722.22	96.11

Table 1. Space shuttle descent velocity data

a) Based on the provided data, and using forward differences determine the acceleration values of the space shuttle. Detailed calculations and utilized expressions should be provided.
b) Compute at which time point the astronauts experienced their acceleration dropping below -5 m/s^2.
c) Using the composite trapezoid rule determine the total distance travelled by the shuttle from the first observation until the touchdown. Detailed calculations and utilized expressions should be provided.

4. **(ODEs)** The predator-prey model depicts the interaction of two species in an ecosystem. Consider the interaction between gnus and lions in a closed system. The system of ODEs describing the evolution of the gnu population, G, and the lion population, L, is given by:

$$\frac{dG}{dt} = 0.7G - 0.007GL$$

$$\frac{dL}{dt} = 0.0021GL - 0.5L$$

and

$$x(0) = \begin{bmatrix} G(0) \\ L(0) \end{bmatrix} = \begin{bmatrix} 400 \\ 50 \end{bmatrix}$$

a. Formulate the equations for the Euler Backward scheme to solve this system and explain conceptually the procedure of solving the system. In the equations, the known quantities should be replaced by their actual values in all utilized expressions.
b. For the given initial condition compute *x(1)*, if the time step, *h*, is equal to 1/104. For the calculations, if an iterative method is used, propose and use the fastest converging one in three iterations.
c. Which explicit method would you recommend and for which type of the ODEs it would be applicable? When would you apply an implicit method and which one? Explain why.