ChE-312 Numerical methods

Part 2

PDE tool introduction.

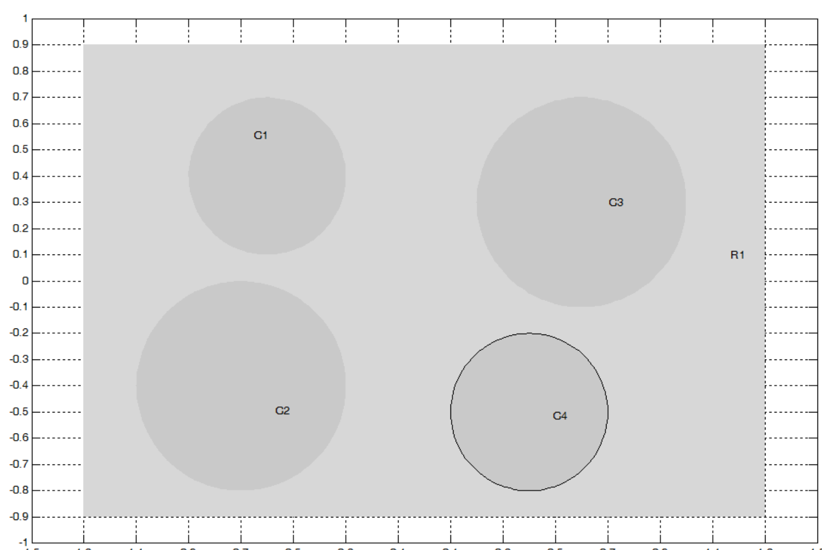
Matlab PDE tooolkit

June 2022, after a difficult term of courses you begin your revisions. At the end of a long day of work, you decide to cook some pasta but you're so tired that you fall asleep and forget to turn off the stove, what happens then? (you will model this process with the Matlab PDE toolkit)

Your stove consists of 4 circular metal heaters (material properties: ρ = 10500 kg m^{-3}, c_p = 235 J $kg^{-1}K^{-1}$, k = 429 W $m^{-1}K^{-1}$, where k is the thermal conductivity) which are isolated by a rectangular glass plate (material: ceran ρ = 2500 kg m^{-3}, c_p = 840 J $kg^{-1}K^{-1}$, k = 1.7 W $m^{-1}K^{-1}$). The plate is totally isolated from the rest of the counter top and allows no heat flux: i.e. $\nabla T = 0$ as boundary conditions. Over the entire 2D-domain of the stove (metal + glass) the temperature will be modeled by the following differential equation:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q_{prod}}{\rho c_p}$$

However, our total domain will have separate regions with different physical properties. In the glass ceran region $q_{prod} = 0$ but not in the all of the metal heater regions, since the one(s) you left on with have a non-zero q_{prod}. The stovetop looks something like this:



At time t = 0 (T_0 = 20°C) you turn all 4 (or some, you choose) metal heater plates on with a power of 50000 W/m^3. Heat is conducted from the metal heaters to the glass plate in the x-y-plane, but we will also consider that convective heat transfer occurs to the air of the room in the z-direction (assume the convective heat transfer coefficient h=25 W/m^2K) and the external temperature of the room is 20°C.

Use Matlab to solve this problem with pdetool:

1. In Matlab, type "pdetool" into the command window and press enter. This command opens a new window where you will define and solve the problem.
2. Choose the options menu and then its submenus as follows:
    a. *Grid*- places grid lines in the drawing space.
    b. *Snap*- makes any object drawn snap to the nearest grid line.
    c. *Grid Spacing*- lets you change grid-line spacing in x and y by unchecking "auto" boxes and assigning new values (you can choose a gird spacing of 0.1 for both x and y)
3. Also in the options menu choose the *application* as *heat transfer*
4. Draw one rectangle and four circles (approximately as shown above) using the buttons on the toolbar to set the geometry. You can assign some geometry values with a double-click on them.
5. In the "set formula line" (below the toolbar), edit the formula to read: (R1-(C1+C2+C3+C4))+C1+C2+C3+C4
6. Go in the *boundary mode* with the *boundary menu*
7. Set the boundary conditions with a double-click on arrows defining the boundary of the glass plate (choose the appropriate type, look at the equation and set the appropriate values).
8. Change to the *PDE mode* with the *PDE* menu. In this mode you can set the corresponding values to the equation. Two equations are possible:
    a. Elliptic for the steady state problem
    b. Parabolic for the transient problem

    Frist solve the transient problem with a double click in one region to get a menu that allows you to set the PDE specifications for that region. Enter the values for all regions.
9. Choose Mesh from the Mesh menu. Next choose Refine Mesh and Jiggle Mesh to improve the first rough mesh.
10. Change the time scale of your solution to solve the PDE from time=0 to 100000 seconds (about 1 day) with 1000 time steps using the *solve parameter* from the *solve menu* by linspace(0,100000,1000). Also here you can define the initial temperature to be 20°C
11. Solve the problem by clicking in the "=" button (the final time step is shown as the solution)
12. You can change the plot parameter from the plot menu and add animation. Try choosing "color", "contour" and "animation" and clicking "plot".
13. Try to obtain the steady state results