Exercises

Tuesday, 11 March 2025

Exercise 4

a) Create a function *forwardDiff*(*f*, x_0 ,*h*) that computes numerically the first derivative of a single argument function at the given argument value x_0 using the forward differences approximation. *f* is the function whose first derivative we wish to calculate, and *h* represents the vector of grid sizes

Test *forwardDiff* for the function $f(x) = e^{x^2} - 6x + 5$, $x_0 = 1$.

b) Modify the *forwardDiff* function to create a new function *centralDiff* that computes numerically the first derivative of a single variable function at a given argument x_0. Test this on the same function $f(x)$ as (a).

Exercise 4 (Handwritten)

- A car laps a race track in 54 seconds. The speed of the car at each 6-second interval is determined using a radar gun and is given, from the beginning of the lap, in meters per second, by the entries in the following table:

T (s)	0	6	12	18	24	30	36	42	48	54
v (m/s)	37.81	40.83	45.11	47.25	44.81	40.53	36.89	33.22	30.17	35.36

A) Determine the time points of the maximal acceleration and deceleration using the appropriate finite difference method

B) Taylor series analysis indicates that the error to approximate the derivative of a smooth function $f(x) = e^x$ by the forward finite difference should follow:

$$\left| f'(x) - \frac{f(x+h) - f(x)}{h} \right| = O(h)$$

however, after calculating the derivative of $f(x) = e^x$ at $x = 1$ in MATLAB with 16-decimal-digit accuracy for $h = 10^{-7}$ and $h = 10^{-14}$ the following was obtained:

$$\left| e^1 - \frac{e^{1+10^{-7}} - e^1}{10^{-7}} \right| \approx 1.3 \times 10^{-7}$$

and

$$\left| e^1 - \frac{e^{1+10^{-14}} - e^1}{10^{-14}} \right| \approx 0.053$$

Explain the results above