# Exercises

Tuesday, 25 March 2025

# Exercise 5 - Coding

1) Consider the initial value problem,

$$dy/dt = y\cos^2(t) - y \text{ with } y(t=0) = y_0 = 2$$

A) Write a function *EulF(f, tspan, h, y0)* that solves an initial value problem where f is the Right-Hand Side of an ODE, *tspan* is the interval on which we want to solve the ODE and h is the step size, using the Euler Forward method. Solve the above ODE using this function in the *t-interval [0, 5]* for step sizes, *h = [0.1, 0.5, 1]*. Plot the solutions and comment on it.

B) Solve the same ODE using *scipy.integrate.solve_ivp* with the default method 'RK45'. Compare the performance of *scipy.integrate.solve_ivp* with the Euler forward method in the previous section.

# Exercise 5 - Coding

2) Assuming an initial value problem *dy(t)/dt=f(t, y(t))*, create a function that calculates the function *y(t)* at discrete points of the time interval *tspan* with step size *h* using (i) 2nd order Runge-Kutta method (Heun's Method) (ii) 4th order Runge-Kutta method. The functions can be called in the same format as Question 1. Name these functions as RK2.py and RK4.py

A) Now using these functions, approximate the solution in the t-interval *tspan = [0, 2]* when $f(t, y(t)) = -20y + 20t^2 + 2t$ and *y(t = 0)= y0 = 1*. Plot the solutions for the step sizes , *h = [0.01, 0.1, 0.2]*. Explain your observations.

B) Solve the same ODE using *scipy.integrate.solve_ivp* and compare the performance with the Runge-Kutta methods by plotting the solutions with appropriate legends.

# Exercise 5 - Handwritten

Consider the following system of ODEs

$$\frac{dx_1}{dt} = 998x_1 + 1998x_2$$

$$\frac{dx_2}{dt} = -999x_1 - 1999x_2$$

The general solution of this system can be written in the form:

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 2c_1 + 2c_2 \\ -c_1 - c_2 \end{bmatrix} e^{-t} + \begin{bmatrix} -c_1 - 2c_2 \\ c_1 + 2c_2 \end{bmatrix} e^{-1000t}$$

A) Which method you would apply to solve numerically the above-mentioned system of ODEs? Discuss what would be the appropriate step size.

B) For $x_1(0) = 1$ and $x_2(0) = 1$, apply the proposed method with the proposed step size to solve the system in the interval $t \in (0,100)$. Provide details of calculating $x(t_1)$ using $x(t_0) = [x_1(0) \quad x_2(0)]^T$.