Exercises

Tuesday, 18 March 2025

Exercise 4 - Coding

a) A space shuttle reentering the earth atmosphere decelerates in one hour from its orbital speed (7360.56 m/s) to its touchdown speed (96.11 m/s). The following table contains data of its measured velocity. Use the *scipy.integrate.cumtrapz()* and *scipy.integrate.trapz()* to measure the distance covered at each measuring point and over the whole 3600 seconds.

Time (s)	-3600	-3200	-2800	-2400	-2000	-1600	-1200	-800	-400	0
Velocity (m/s)	7360.56	7314.81	7274.32	7245.61	7134.59	6974.67	6722.22	5618.97	3236.74	96.11

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b) Create a function that computes numerically the integral of a single argument function over the given *interval (a,b)* using the composite Simpson 1/3 rule. The function should be called as follows: [intF]=*compSimp13(f, interval, n)* where *f* is the function handle and *n* represents the number of subintervals of the integration interval.

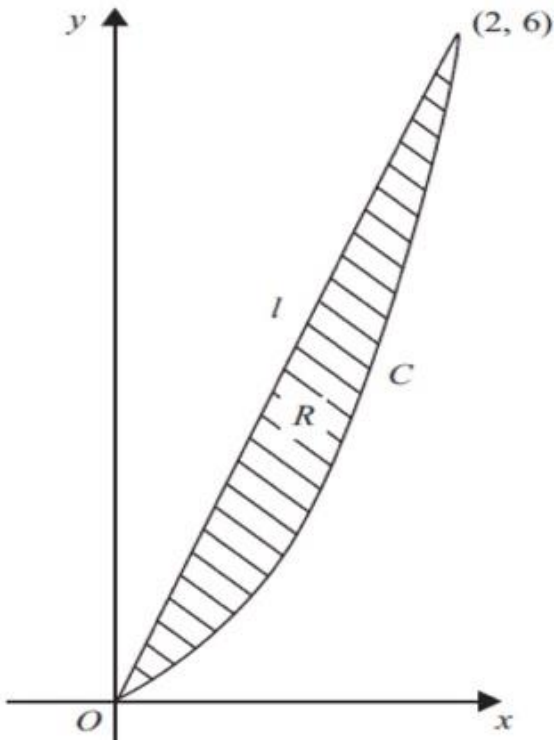
c) Using the function created in a), compute numerically the value of the integral of $f(x)=0.25+4x+x^2+6x^3+x^4$ from $a = 0$ to $b=0.75$ using the values of n=[4,8,…,100]. Compute the error of integration* and see if the error convergence matches the theoretical estimates.

*- integration error we define as the difference between the values that are calculated by your *compSimp13()* function and Python's *scipy.integrate.quad()* function.

Exercise 4 - Handwritten

A boat builder needs to compute the surface of the sail, R, to estimate the aerodynamic forces acting on the boat. The contour of the sail is defined by the linear segment l, which joins the origin (0,0) and the point (2,6), and the curve C, which is described by the equation

$$y = x\sqrt{x^3 + 1}, \qquad 0 \leq x \leq 2$$



Exercise 4 - Handwritten

a) Complete the table below that provides values of y on the curve C.

x	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
y	0				1.414				6

b) Compute the integral $\int_0^2 x\sqrt{x^3+1}\,dx$ using the most appropriate composite rule with the values from the table in a).

c) Use your answer calculated in b) to approximate the surface of the sail, R. Provide your solution with three significant digits.

d) Compute the surface of the sail, R, using the two points Gauss-Legendre formula and compare the result with c).