Membrane processes

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Lecture 13

28.05.2025

Reference book: Membrane Technology and Applications, R. W. Baker, Wiley 2012 (3rd ed.)

Agenda

- How to model a membrane stage?
- How to design multi-stage processes?

already presented in the Part 3 of the previous lecture

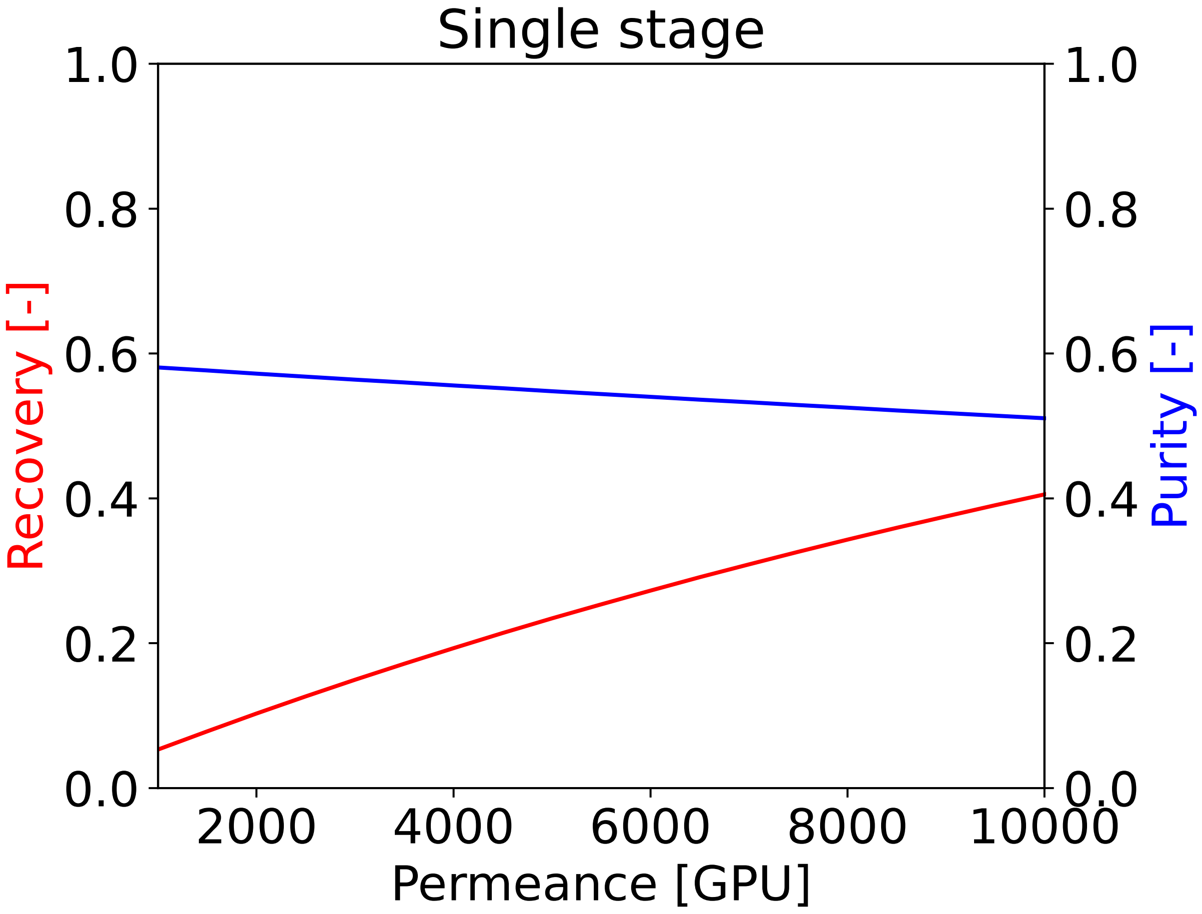
- Results at variable membrane **permeance and selectivity**
- Results at variable **feed concentration and flow rate**
- Results at variable **feed and permeate pressure**
- Results at variable **membrane areas**
- Modelling the non-ideal phenomena

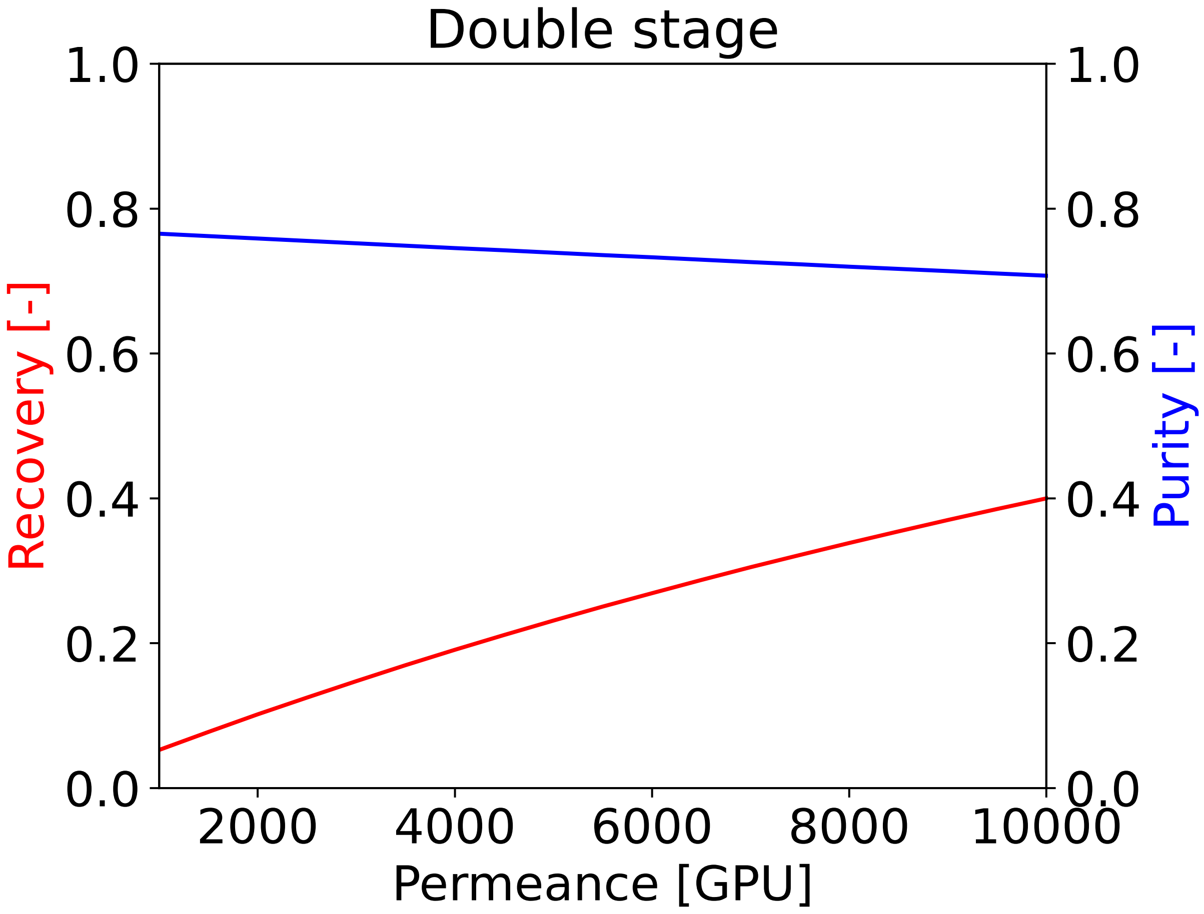
Part 1

Variable CO_2 permeance

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\, X_{CO2,feed}(x) - P_{perm}\, X_{CO2,perm}(x)\right)$$

$$\alpha_{CO2-N2} = {}^{\boldsymbol{N}_{CO2}}/_{\boldsymbol{N}_{N2}}$$



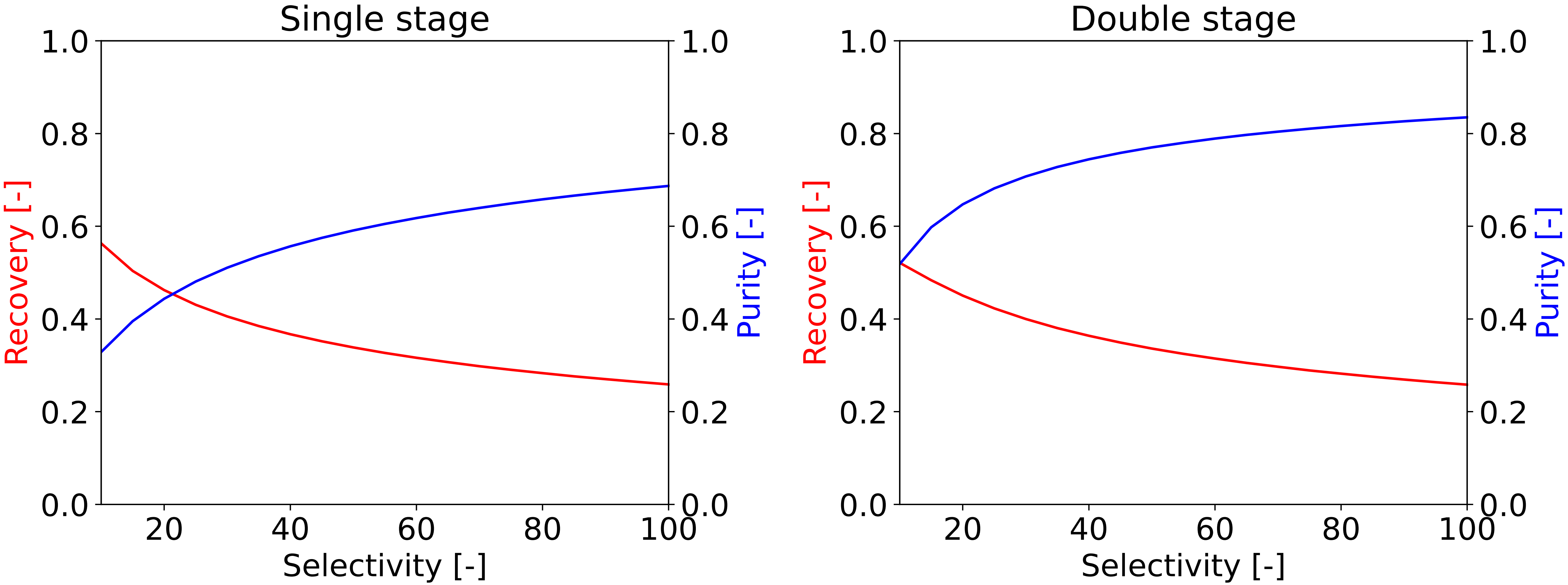


Speaker

Variable membrane selectivity

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\,X_{CO2,feed}(x) - P_{perm}\,X_{CO2,perm}(x)\right)$$

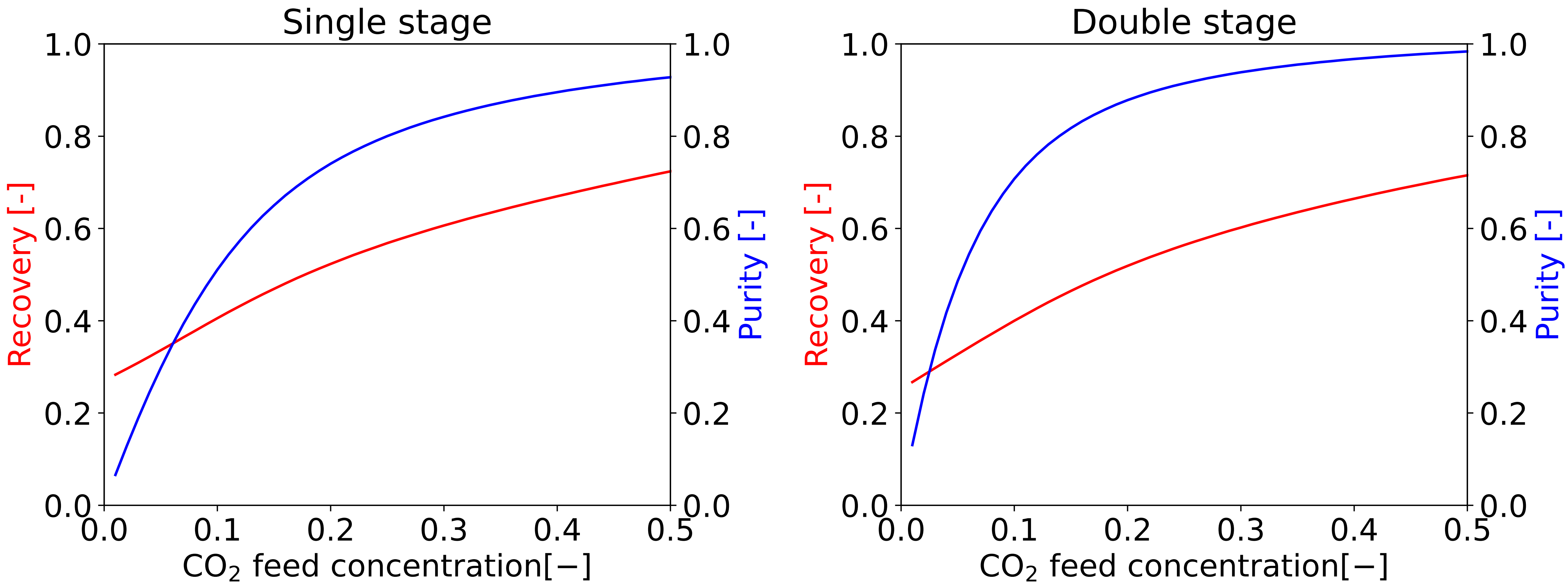
$$\alpha_{CO2-N2} = {}^{\boldsymbol{N}_{CO2}}/_{\boldsymbol{N}_{N2}}$$



Variable feed concentration

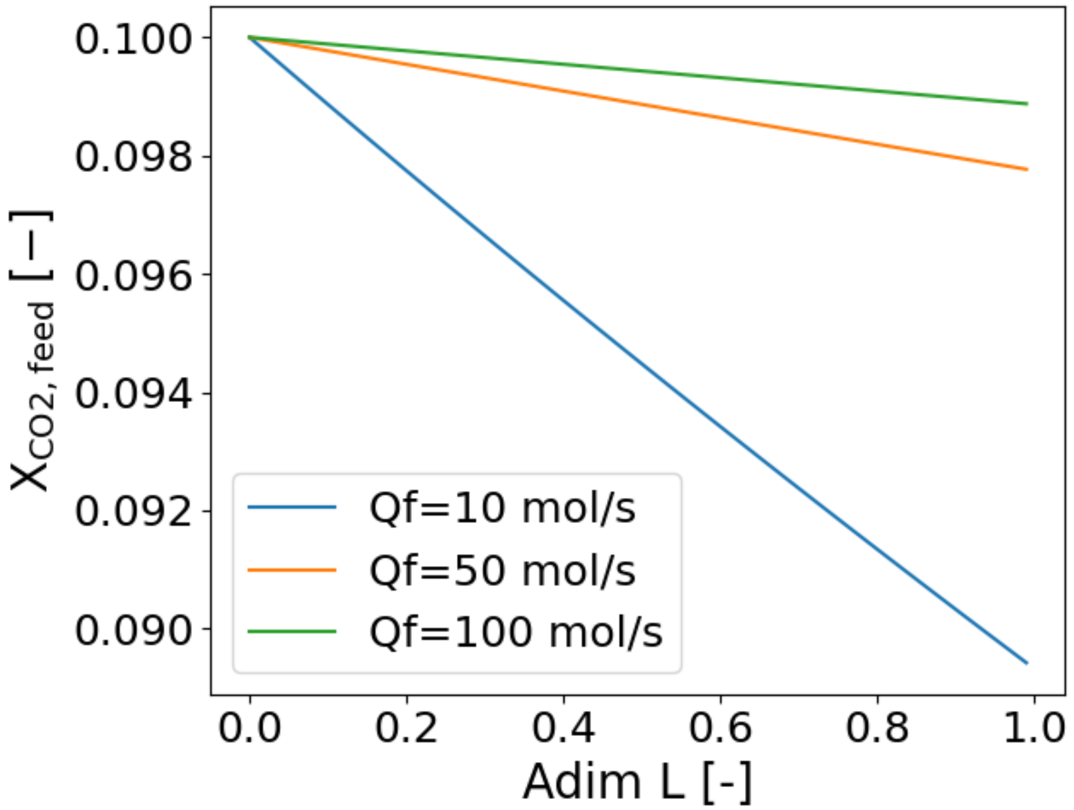
$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\, X_{CO2,feed}(x) - P_{perm}\, X_{CO2,perm}(x)\right)$$

$$Recovery = \frac{Q_{P,out} X_{CO2,p,out}}{Q_{feed,in} X_{CO2,feed,in}}$$



Variable feed flow rate

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\, X_{CO2,feed}(x) - P_{perm}\, X_{CO2,perm}(x)\right)$$



Variable feed flow rate

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\,X_{CO2,feed}(x) - P_{perm}\,X_{CO2,perm}(x)\right)$$

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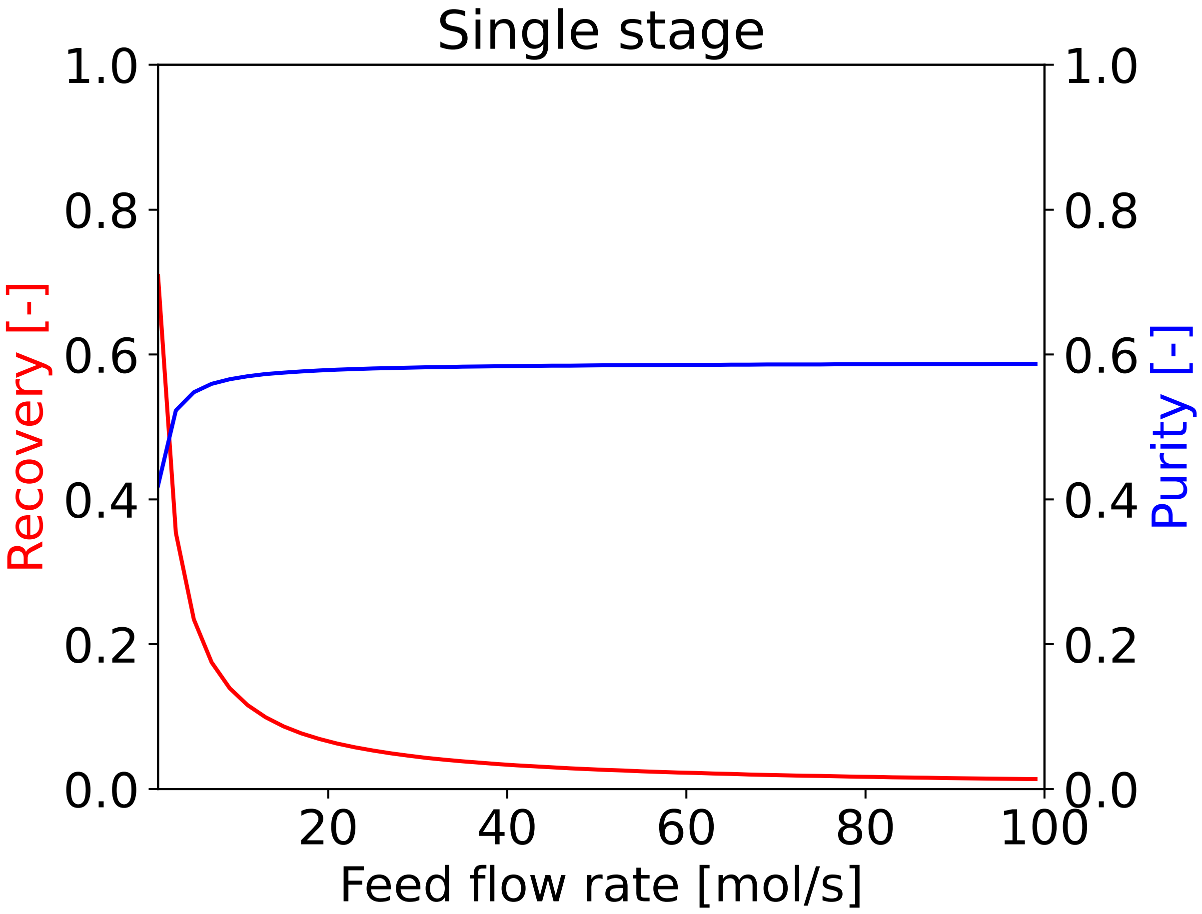
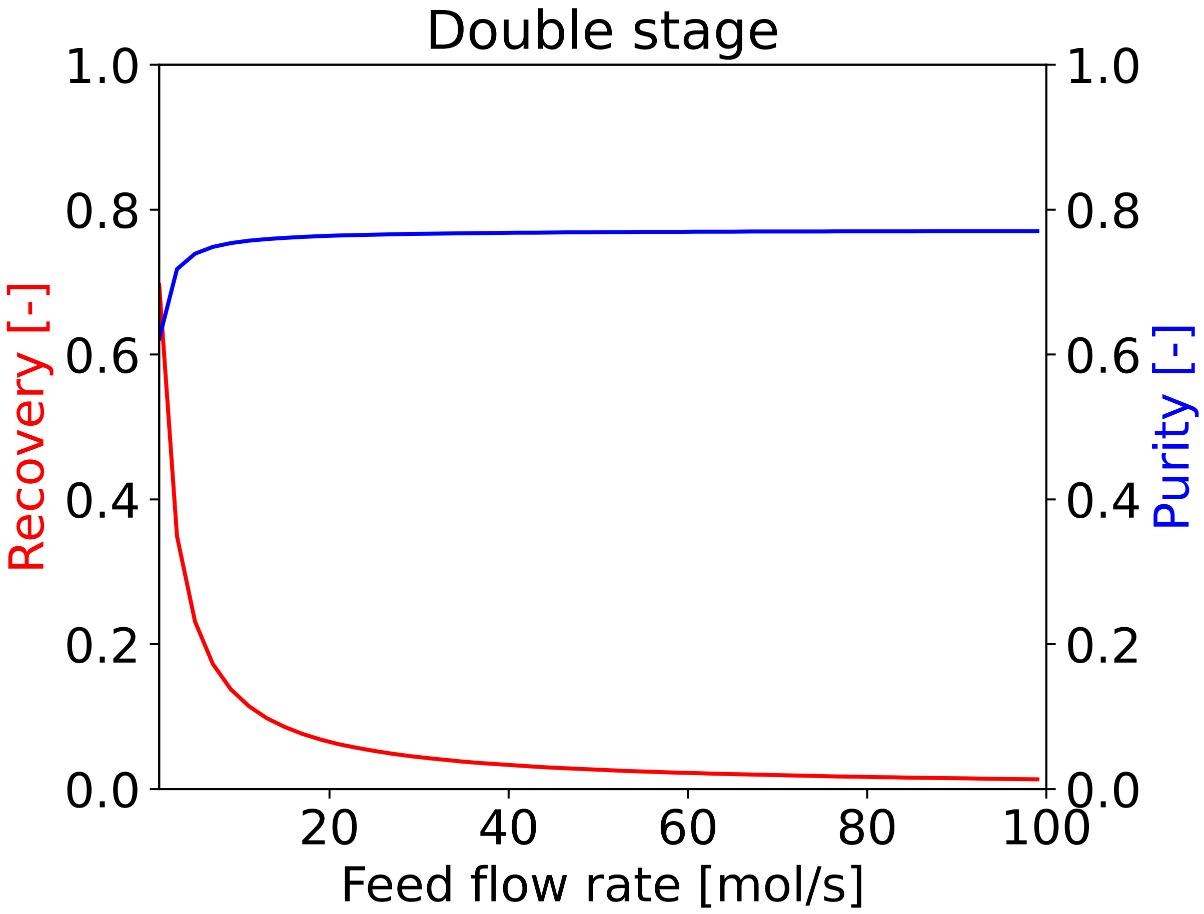


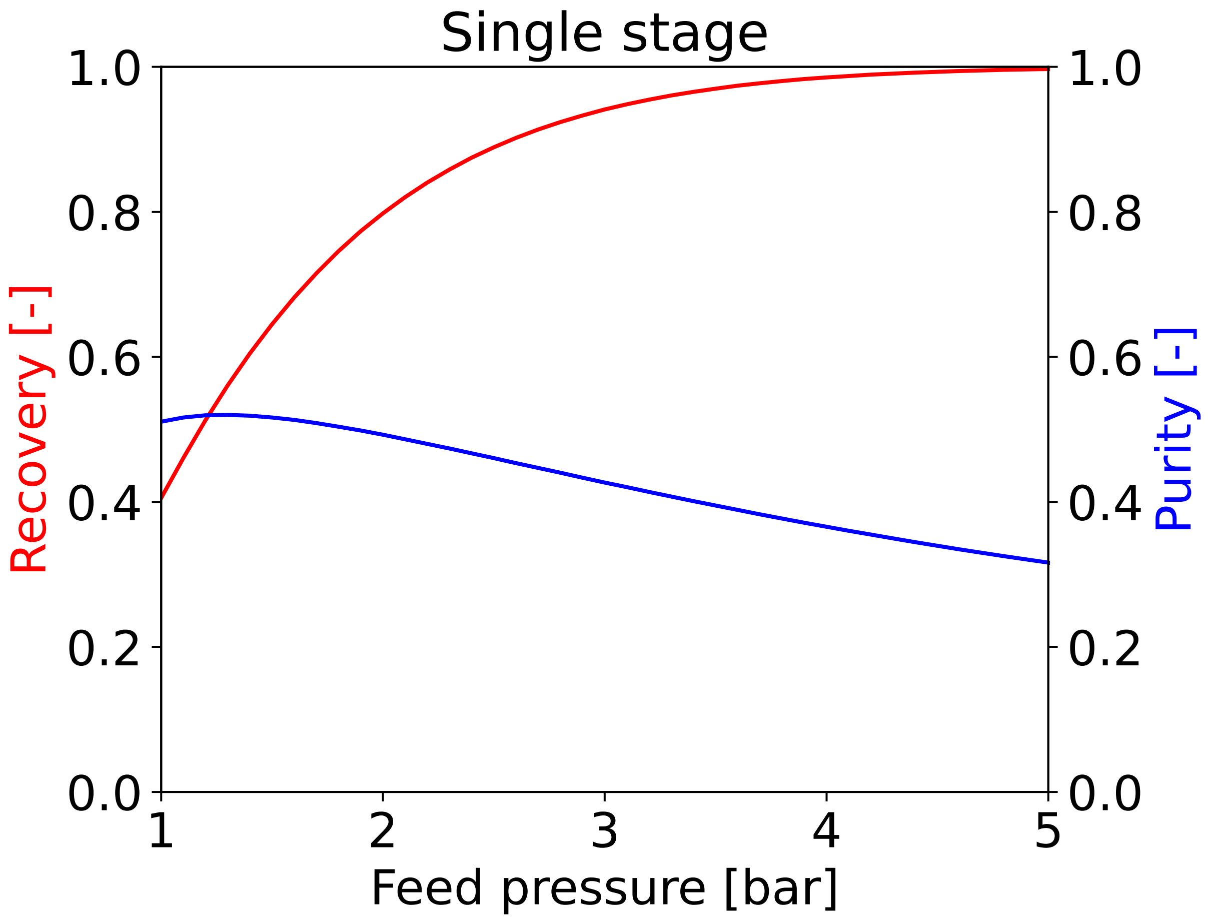


Speaker

Variable feed pressure (1st stage)

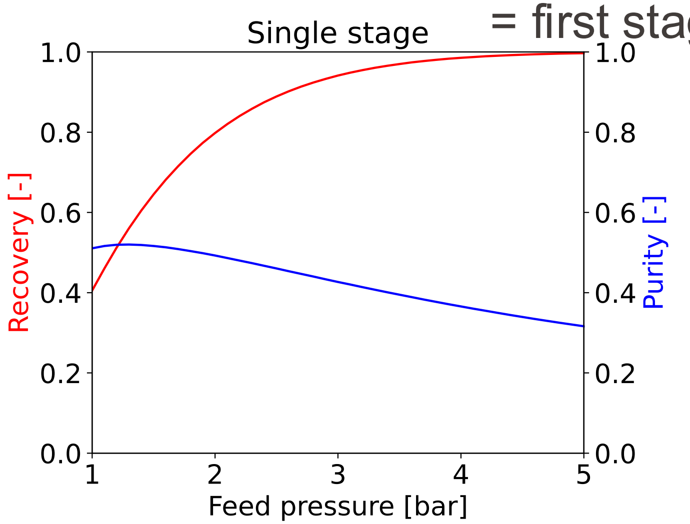
$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\, X_{CO2,feed}(x) - P_{perm}\, X_{CO2,perm}(x)\right)$$

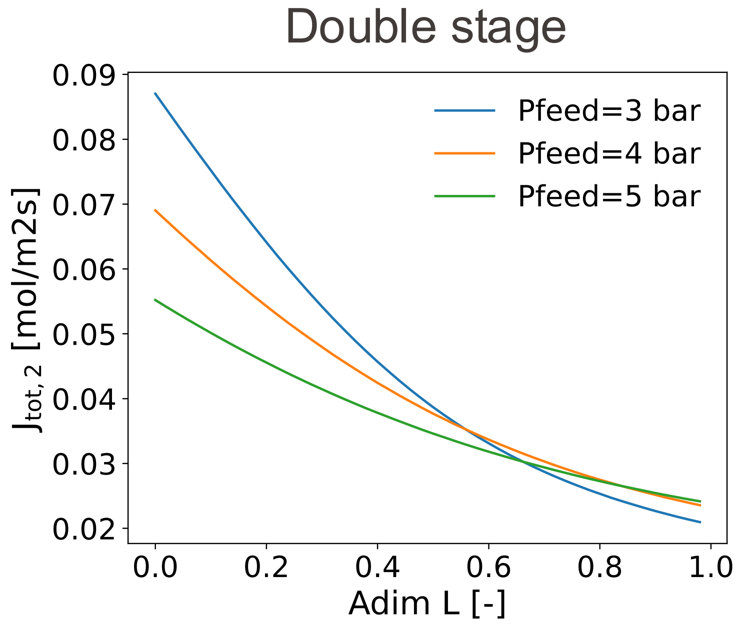
$$J_{N2}(x) = \boldsymbol{N}_{N2}\left(P_{feed}\, X_{N2,feed}(x) - P_{perm}\, X_{N2,perm}(x)\right)$$

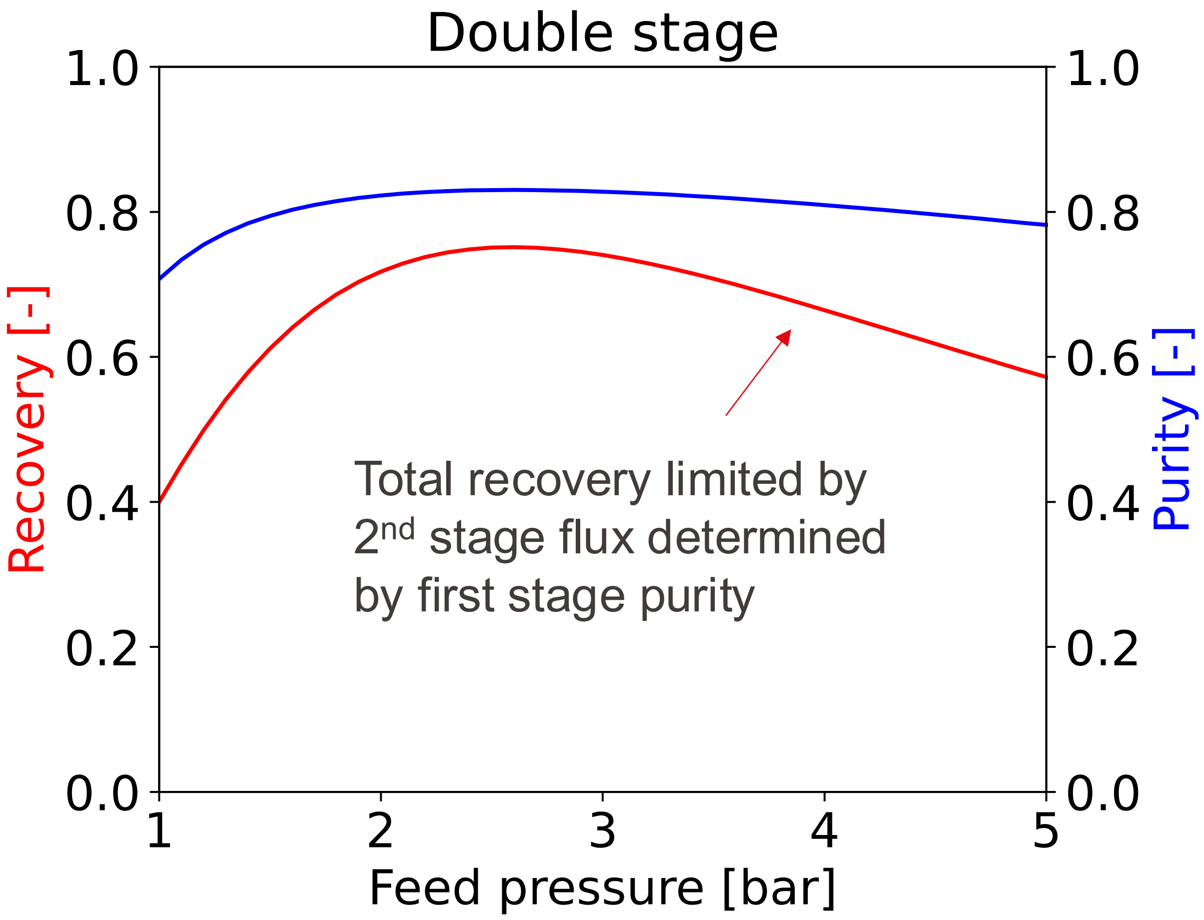


Variable feed pressure (1^{st} stage)

= first stage of the double stage



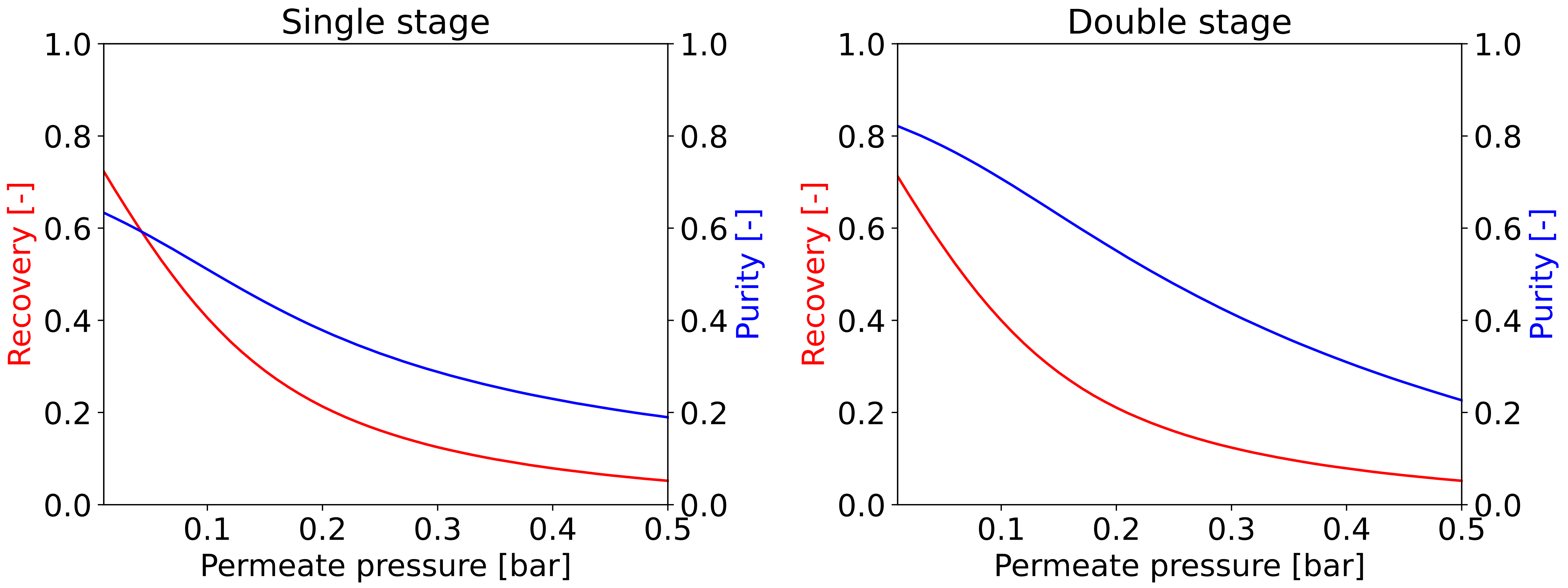




Variable permeate pressure (1st stage)

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\,X_{CO2,feed}(x) - P_{perm}\,X_{CO2,perm}(x)\right)$$

$$J_{N2}(x) = \boldsymbol{N}_{N2}\left(P_{feed}\,X_{N2,feed}(x) - P_{perm}\,X_{N2,perm}(x)\right)$$



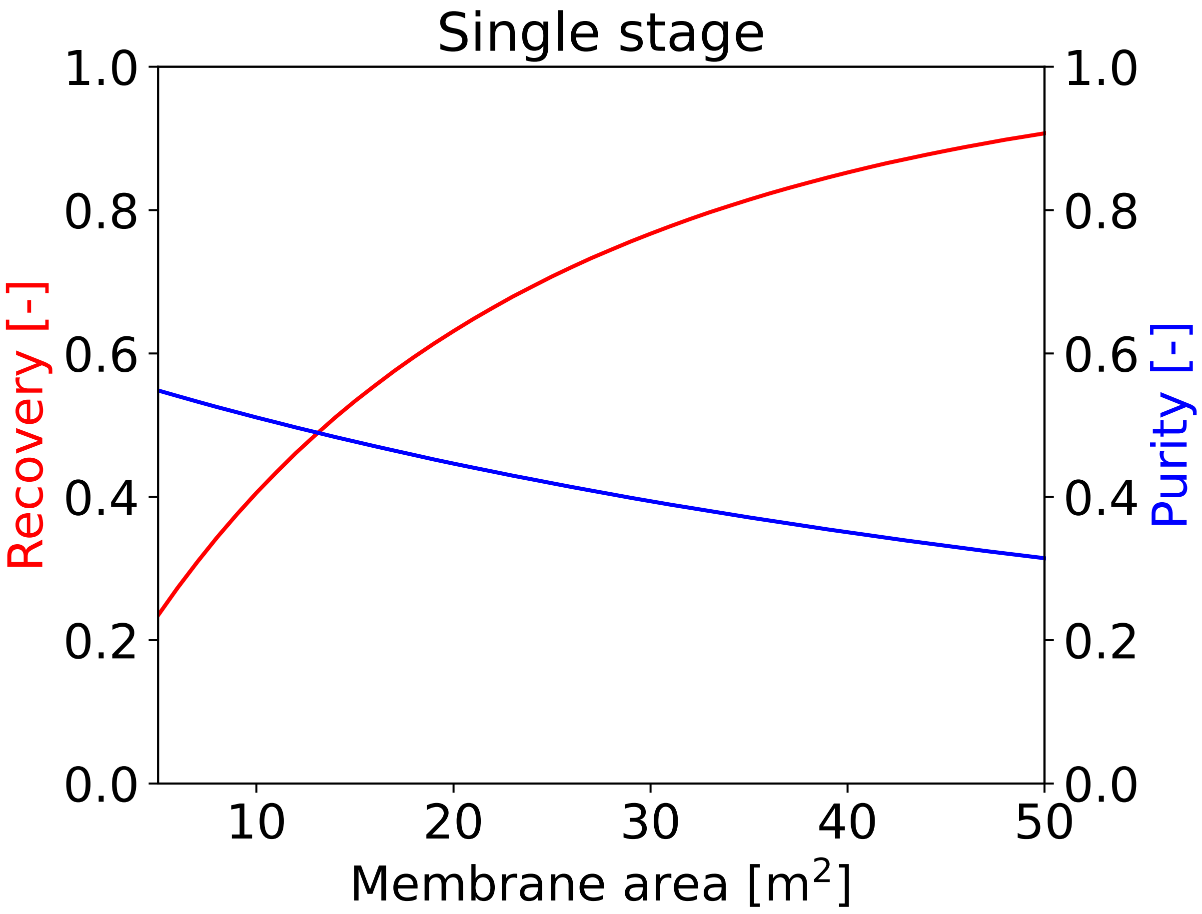
Variable membrane area (1st stage)

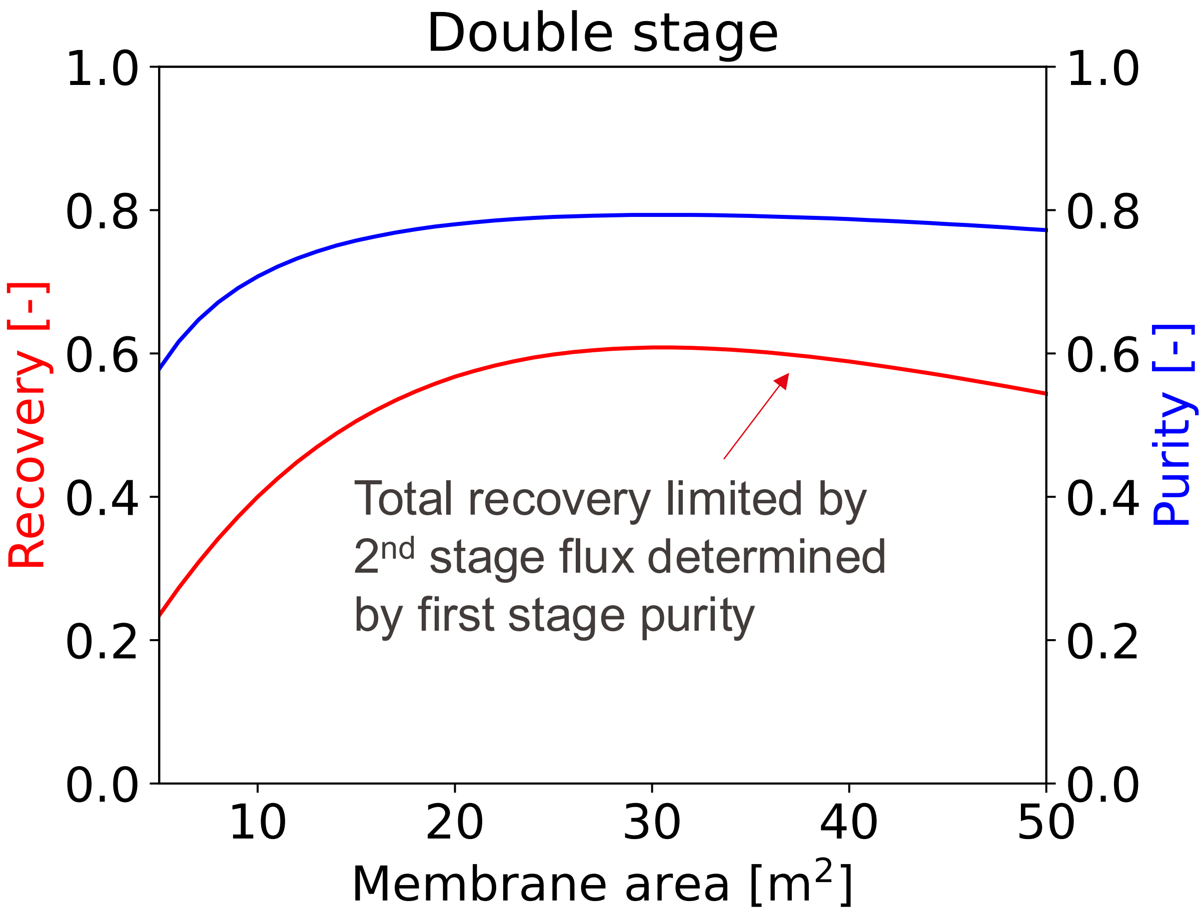
$$Q_{p,out} = \sum_{x=0}^{L} J(x)dA$$

$$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\,X_{CO2,feed}(x) - P_{perm}\,X_{CO2,perm}(x)\right)$$ ↓ along the length

$$J_{N2}(x) = \boldsymbol{N}_{N2}\left(P_{feed}\,X_{N2,feed}(x) - P_{perm}\,X_{N2,perm}(x)\right)$$ ↑ along the length

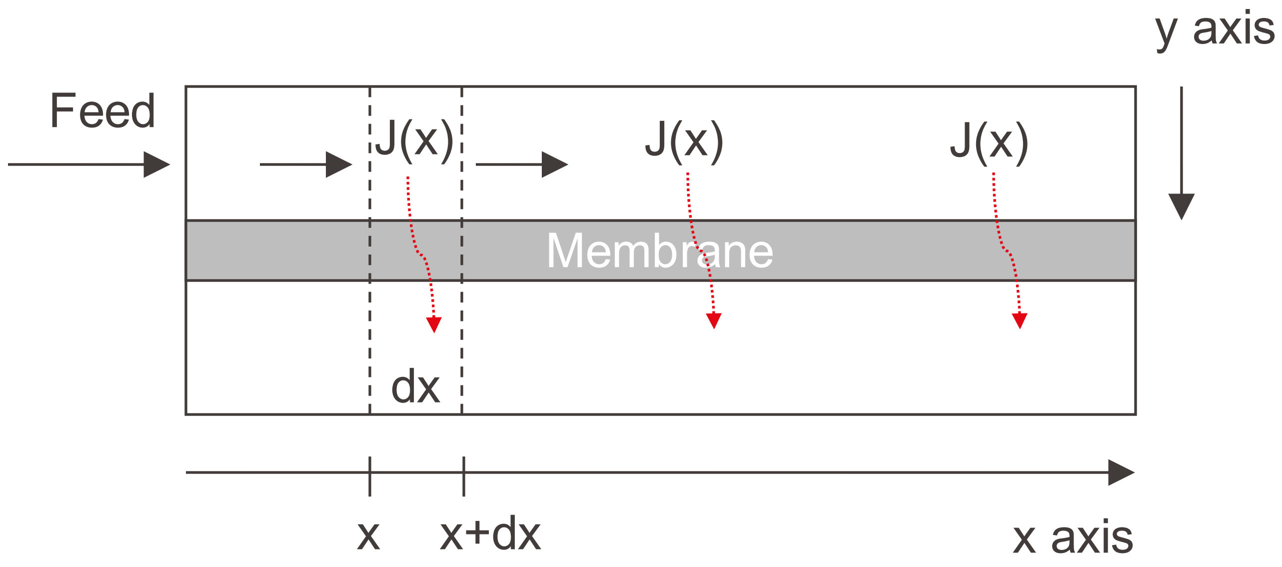
$$X_{CO2,p,out} = \frac{\sum_{x=0}^{L} J_{CO2}(x)dA}{Q_{p,out}}$$





Part 2

Non-ideal phenomena



Pressure drops → decrease of P_{fee}d along the x axis

Concentration polarization → decrease of $X_{i,feed}$ along the y axis

$$J_i(x) = \frac{\bar{N}_i\left(p_{i,feed}(x) - p_{i,perm}(x)\right)}{\delta} = \boldsymbol{N}_i\left(P_{feed}\,X_{i,feed}(x) - P_{perm}\,X_{i,perm}(x)\right)$$

Non-ideal phenomena
Pressure drops in the feed channel

Pressure drops depend on the velocity v and on the friction factor f

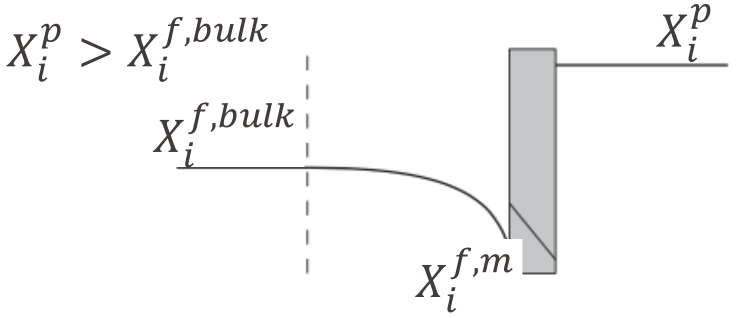
$$\frac{dP_{feed}}{dx} = -f \, \frac{\rho}{2\, d_{hydr}} \, v^2$$

$$f = \frac{64}{Re}$$ friction factor for laminar flow ($Re < 2300$)

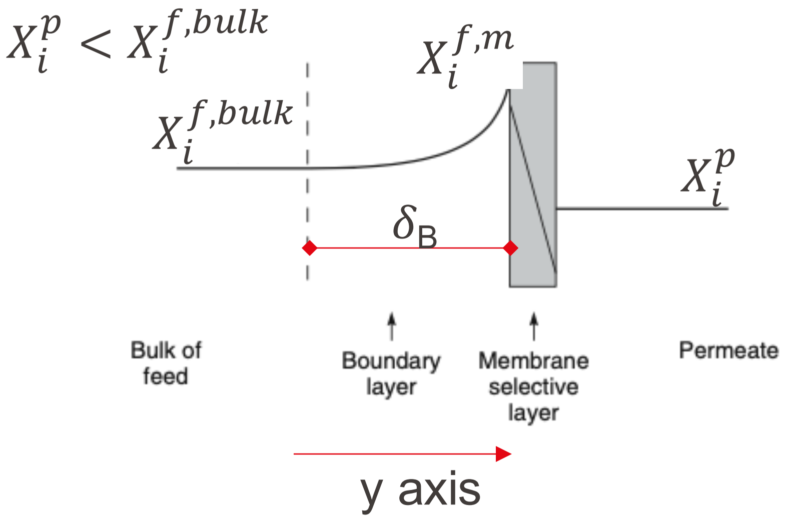
Common limit on pressure drops for channel design: 1.5 psi/m (0.1 bar/m)

Non-ideal phenomena
Concentration polarization

More permeable component



Less permeable component



$$J_{i,bulk \rightarrow interface} = J_{i,transmembrane}$$

$$J_{i,transmembrane} = \boldsymbol{N}_i\left(P_{feed} X_i^{f,m} - P_{perm}\, X_{i,perm}\right)$$

For a binary mixture:

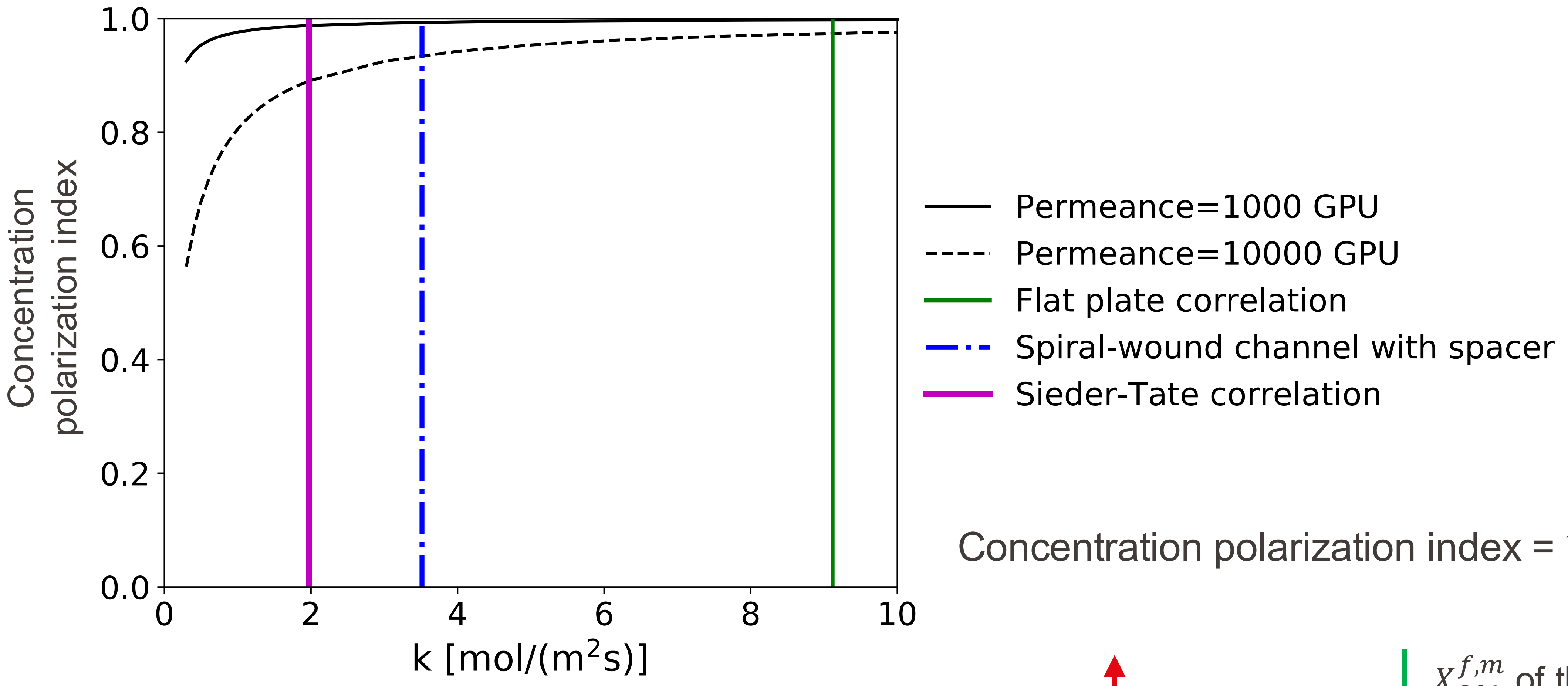
$$\frac{X_1^p - X_1^{f,m}}{X_1^p - X_1^{f,b}} = exp\left(\frac{J_1 + J_2}{k}\right)$$

mass transfer coefficient k

$$k = \frac{D}{\delta_B}$$

velocity δ_B

Concentration polarization
Mass transfer coefficient



Concentration polarization index = $X_{CO2}^{f,m} / X_{CO2}^{f,bulk}$

↑ permeance ↓ $X_{CO2}^{f,m}$ of the permeable component