Membrane processes

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Lecture 12

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Reference book: Membrane Technology and Applications, R. W. Baker, Wiley 2012 (3rd ed.)

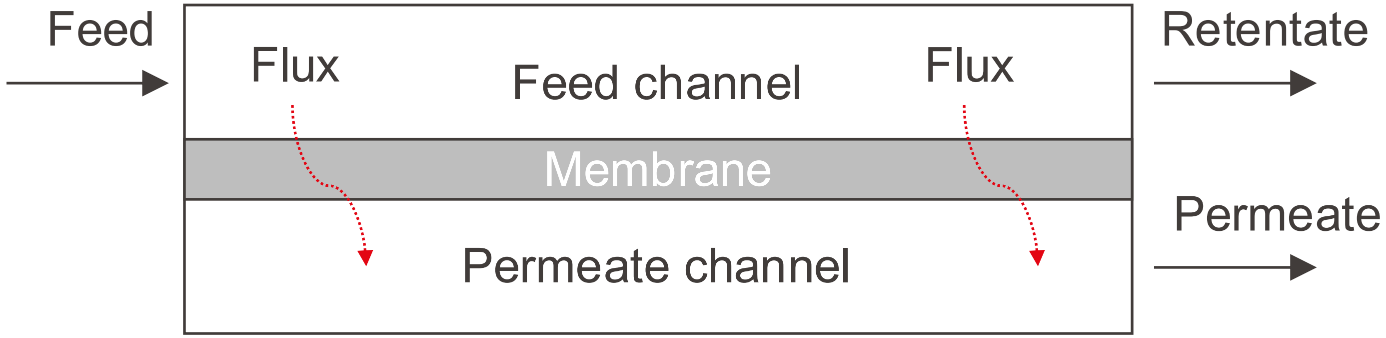
Agenda

THEORY

- What is a membrane stage?
- How to calculate transmembrane flux?
- How to model a membrane stage?
- How to design multi-stage processes?
- How to implement the model for membrane process?

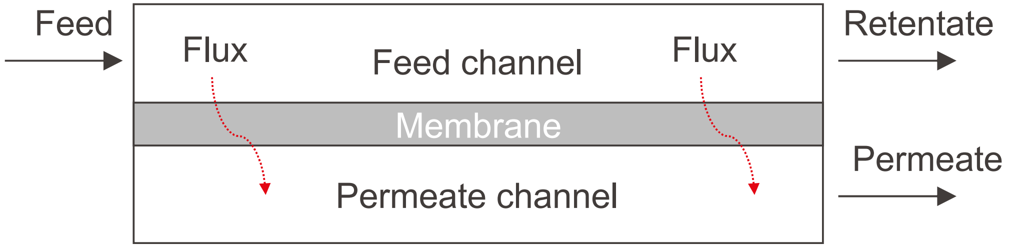
Part 1

Membrane module, definitions



- ✓ **Driving force**: gradient of pressure, concentration or chemical potential that drives the trans-membrane flux from the feed to the permeate channel;
- ✓ **Membrane permeance:** measure of the flux of a component through the membrane for a given driving force;
- ✓ **Membrane selectivity:** ratio between the permeance of two components;
- ✓ **Product recovery:** ratio between the content of the product in the permeate stream and the content in the feed stream;
- ✓ **Product purity:** concentration of the product in the outlet permeate stream.

Overall balances on the membrane stage



- Total mass balance

$$Q_f = Q_r + Q_p$$

- Mass balance for component *i*

$$X_{i,f} Q_f = X_{i,r} Q_r + X_{i,p} Q_p \qquad \sum X_{i,f} = \sum X_{i,r} = \sum X_{i,p} = 1$$

- Total mass balance on the feed channel

$$Q_f = Q_r + \int_0^A J \, dA$$

- Mass balance for component *i* on the feed channel

$$X_{i,f} Q_f = X_{i,r} Q_r + \int_0^A J_i \, dA \qquad \sum J_i = J$$

How do we calculate J?

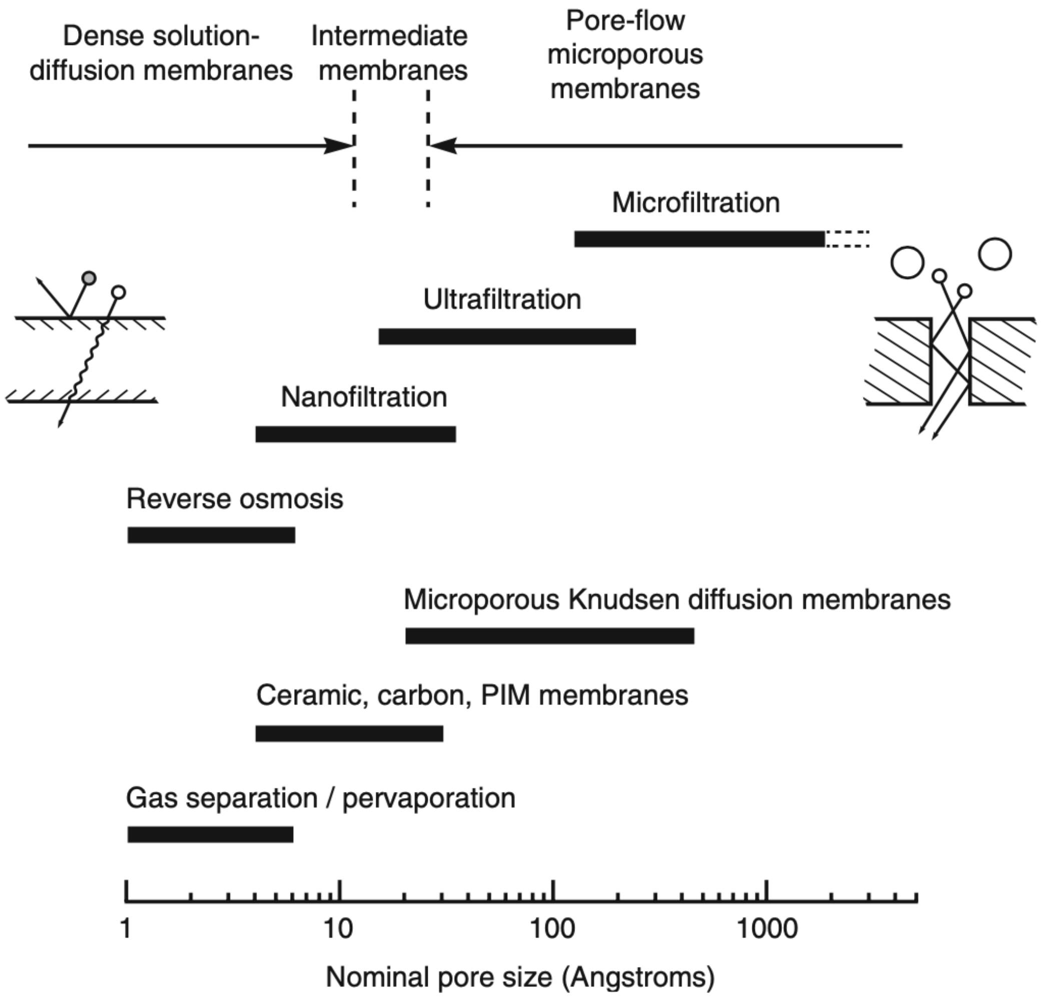
Transport mechanism through the membrane

Dense membranes

Flux driven by the concentration gradient

Transport based on solution-diffusion

Selectivity based on solubility and diffusivity



Porous membranes

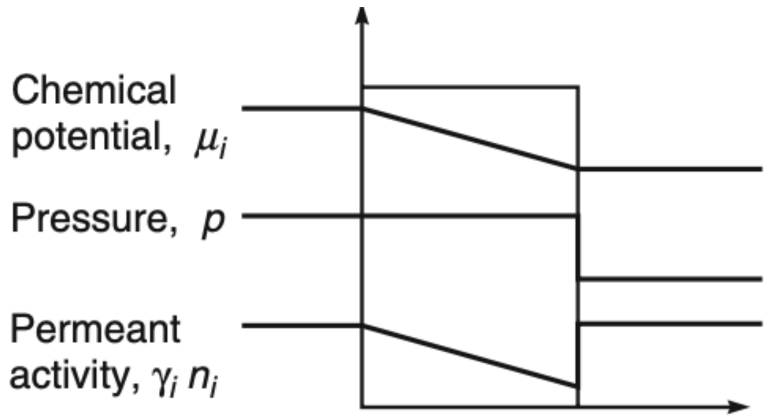
Flux driven by the total pressure gradient

Transport based on viscous flow and diffusion

Selectivity based on pore size

- Baker, R.W. , Membrane Technology and Applications, 2012

Flux from solution-diffusion model in gas separation



$$J_i = \frac{D_i K_{i,G}}{\delta}\left(p_{i,feed} - p_{i,perm}\right) = \frac{\bar{N}_i}{\delta}\left(p_{i,feed} - p_{i,perm}\right)$$

Permeability coefficient given by the product of diffusivity $\boldsymbol{D_i}$ and sorption coefficient $\boldsymbol{K_{i,G}}$

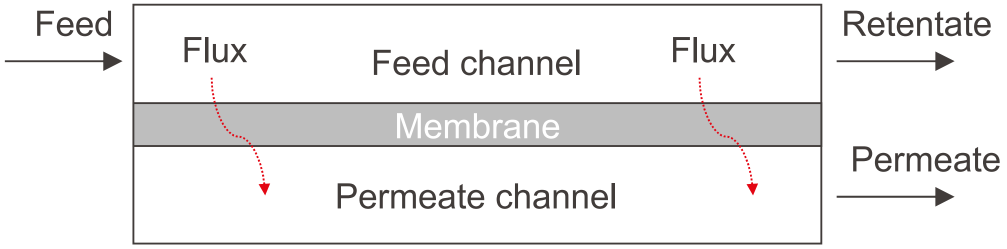
Flux from solution-diffusion model in gas separation

Well-mixed system

$$J_i = \frac{\bar{N}_i}{\delta}\left(p_{i,ret} - p_{i,perm}\right)$$

$$= N_i\left(P_{ret}\, X_{i,ret} - P_{perm}\, X_{i,perm}\right)$$

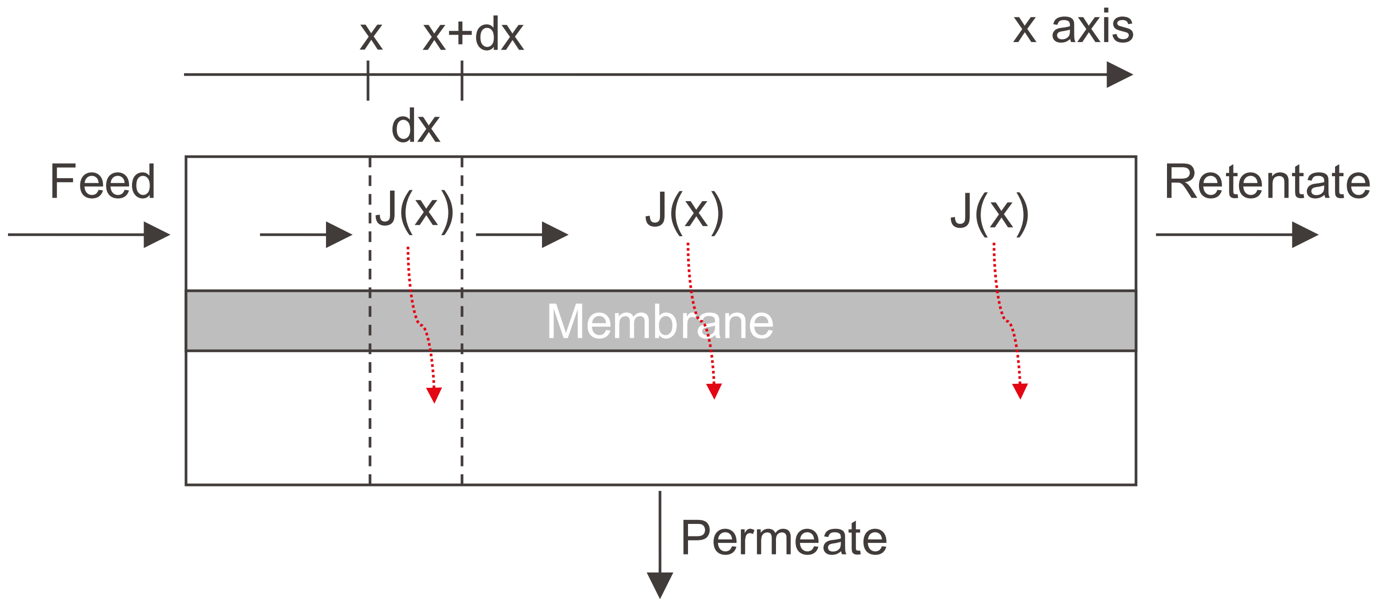
$$J = \sum_{components} J_i$$



Modelling a membrane module

1-dimensional model: discretization along the direction of the feed stream (x axis)

Cross-current flow arrangement



Assumptions

- No variation along the width
- Permeance independent of X
- Negligible pressure drops
- Isothermal process
- No mixing in permeate channel

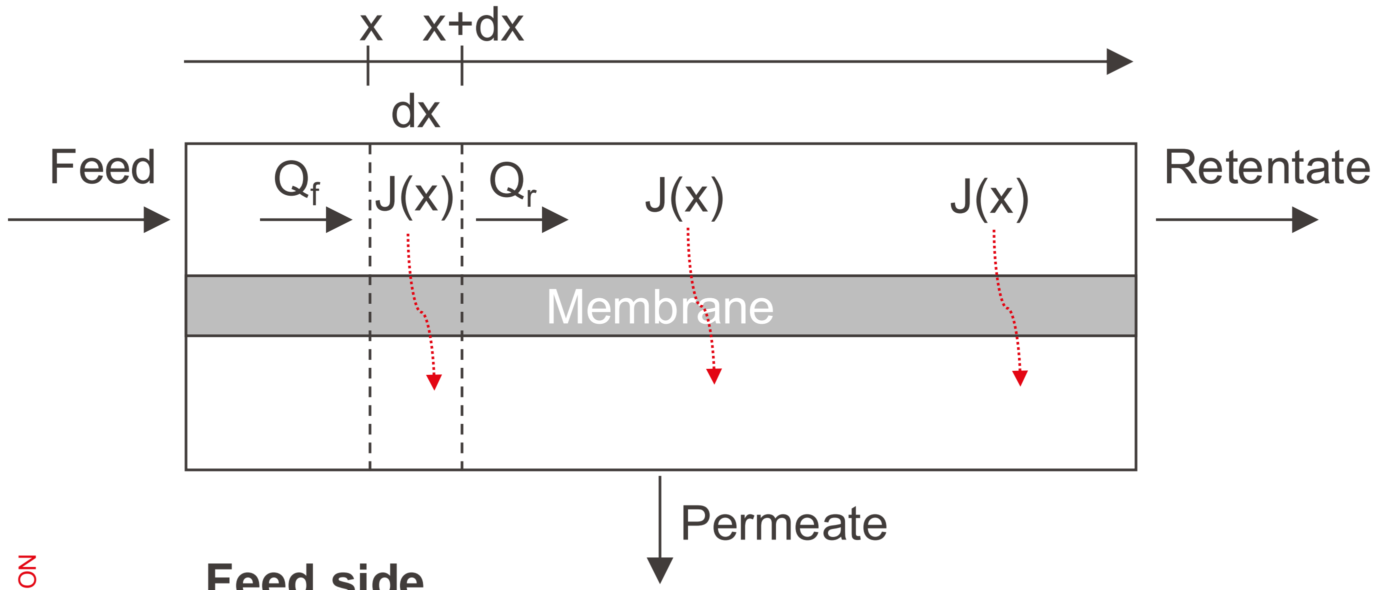
$$J_i(x) = \frac{\bar{N}_i \left(p_{i,feed}(x) - p_{i,perm}(x)\right)}{\delta} = \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,perm}(x)\right)$$

$$J(x) = \sum_{components} J_i(x)$$

Modelling a membrane module

1-dimensional model: discretization along the direction of the feed stream (x-axis)

Cross-current flow arrangement



$$J_i(x) =$$
$$= \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,perm}(x) \right)$$

$$J(x) = \sum_{components} J_i(x)$$

Feed side

$$Q_r(x) = Q_f(x) - J(x)\, dA$$

$$X_{i,r}(x)\, Q_r(x) = X_{i,f}(x)\, Q_f(x) - J_i(x)\, dA$$

$$Q_f(x + dx) = Q_r(x)$$

$$X_{i,f}(x + dx) = X_{i,r}(x)$$

Permeate side

$$Q_{p,out} = Q_{feed} - Q_{r,out}$$

$$X_{i,p,out}\; Q_{p,out} = X_{i,feed}\, Q_{feed} - X_{i,r,out}\, Q_{r,out}$$

Bi-component system: CO2/N2

1 $$J_{CO2}(x) = \boldsymbol{N}_{CO2}\left(P_{feed}\,X_{CO2,feed}(x) - P_{perm}\,X_{CO2,perm}(x)\right)$$

$$X_{N2,perm}(x) = 1 - X_{CO2,perm}(x) \qquad X_{N2,feed}(x) = 1 - X_{CO2,feed}(x)$$

2 $$J_{N2}(x) = \boldsymbol{N}_{N2}\left(P_{feed}\left(1 - X_{CO2,feed}(x)\right) - P_{perm}\left(1 - X_{CO2,perm}(x)\right)\right)$$

3 $$J_{TOT}(x) = J_{CO2}(x) + J_{N2}(x)$$

4 $$J_{TOT}(x) = J_{CO2}(x)/X_{CO2,perm}(x)$$

α selectivity

β pressure ratio: P_{perm}/P_{feed}

$$X_{CO2,perm}(x) = \frac{1 + (\alpha-1)\left(\beta + X_{CO2,feed}(x)\right) - \sqrt{\left[1 + (\alpha-1)\left(\beta + X_{CO2,feed}(x)\right)\right]^2 - 4\alpha\beta(\alpha-1)\,X_{CO2,feed}(x)}}{2\beta(\alpha-1)}$$

Modelling a membrane module

Equation	N equation
$J_i(x) = \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,perm}(x)\right)$	$N_{components} \times N_{elem}$
$J(x) = \sum_{components} J_i(x)$	N_{elem}
$J_i(x) = J(x)\, X_{i,perm}(x)$	$N_{components} \times N_{elem}$
$Q_r(x) = Q_f(x) - J(x)\, dA$	N_{elem}
$X_{i,r}(x)\, Q_r(x) = X_{i,f}(x)\, Q_f(x) - J_i(x)\, dA$	$N_{components} \times N_{elem}$
$Q_{p,out} = Q_{feed} - Q_{r,out}$	1
$X_{i,p,out}\ Q_{p,out} = X_{i,feed}\, Q_{feed} - X_{i,r,out}\, Q_{r,out}$	$N_{components}$
$Q_f(x + dx) = Q_r(x)$	N_{elem} - 1
$X_{i,f}(x + dx) = X_{i,r}(x)$	$N_{components} \times (N_{elem} - 1)$
$Q_f(0) = Q_{feed}$	1
$X_{i,f}(0) = X_{i,feed}$	$N_{components}$
	total: 4 $N_{components} \times N_{elem}$ + 3 N_{elem} + $N_{components}$ + 1

Variable	N variable
$J_i(x)$	$N_{components} \times N_{elem}$
$J(x)$	N_{elem}
$X_{i,perm}(x)$	$N_{components} \times N_{elem}$
$Q_r(x)$	N_{elem}
$X_{i,r}(x)$	$N_{components} \times N_{elem}$
$Q_{p,out}$	1
$X_{i,p,out}$	$N_{components}$
$Q_f(x)$	N_{elem}
$X_{i,f}(x)$	$N_{components} \times N_{elem}$
	total: 4 $N_{components} \times N_{elem}$ + 3 N_{elem} + $N_{components}$ + 1

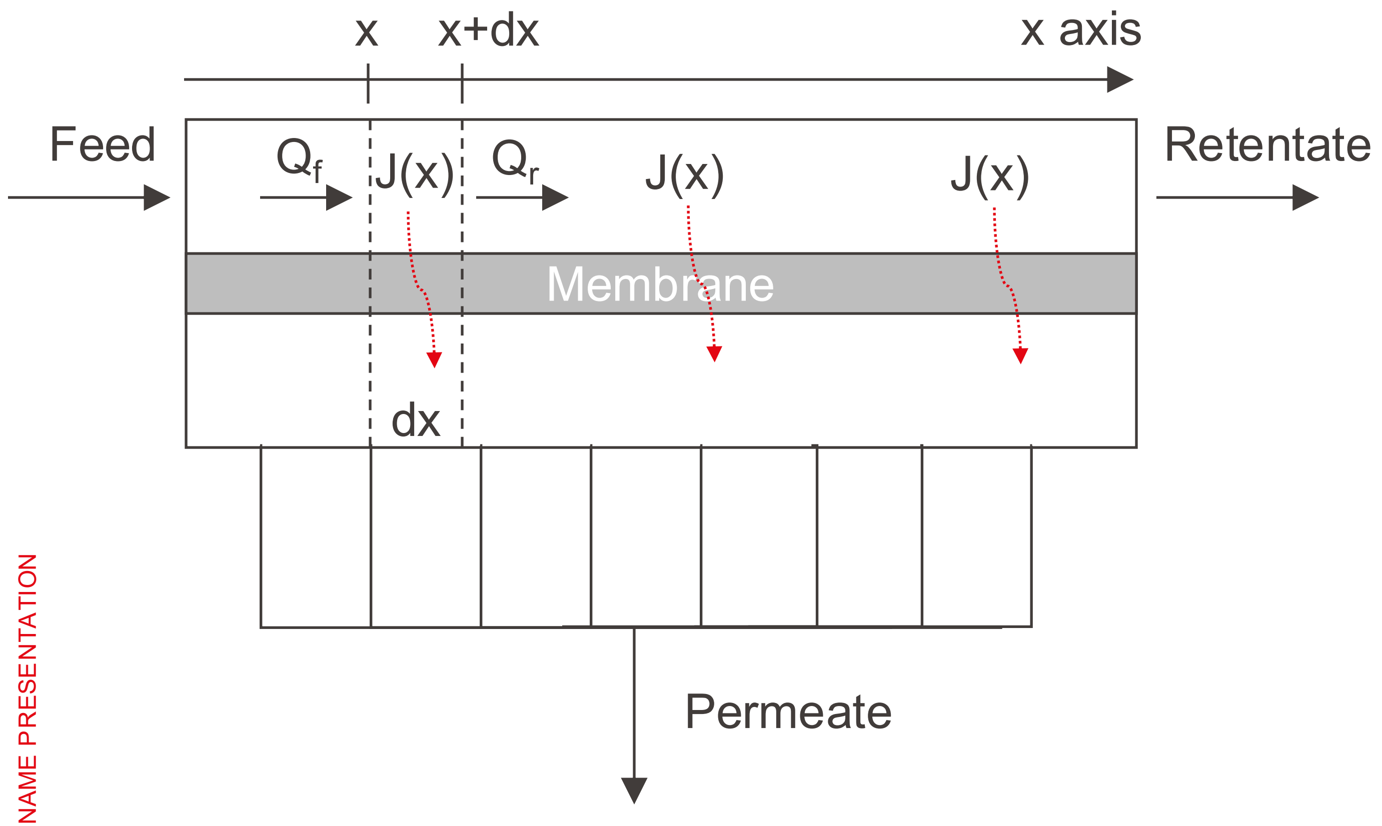
For given $\mathbf{P_{feed}}$, $\mathbf{P_{perm}}$ **and A**

$N_{equations} = N_{variables}$

Part 2

Membrane module: flow arrangements

Cross-current flow arrangement



No mixing in the permeate channel

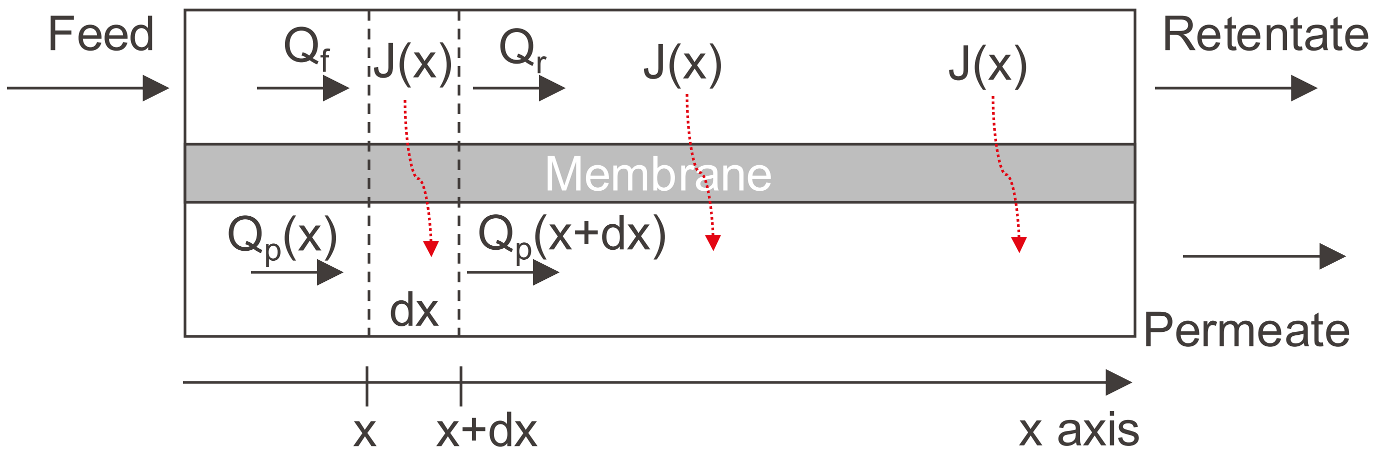
$$J_i(x) = \boldsymbol{N}_i\left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, \boldsymbol{X}_{\boldsymbol{i,perm}}(\boldsymbol{x})\right)$$

$$Q_{p,out} = \sum_{x=0}^{L} J(x)dA$$

$$X_{i,p,out} = \frac{\sum_{x=0}^{L} J_i(x)dA}{Q_{p,out}}$$

Membrane module: flow arrangements

Co-current flow arrangement



$$J_i(x) = \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,p}(x) \right)$$

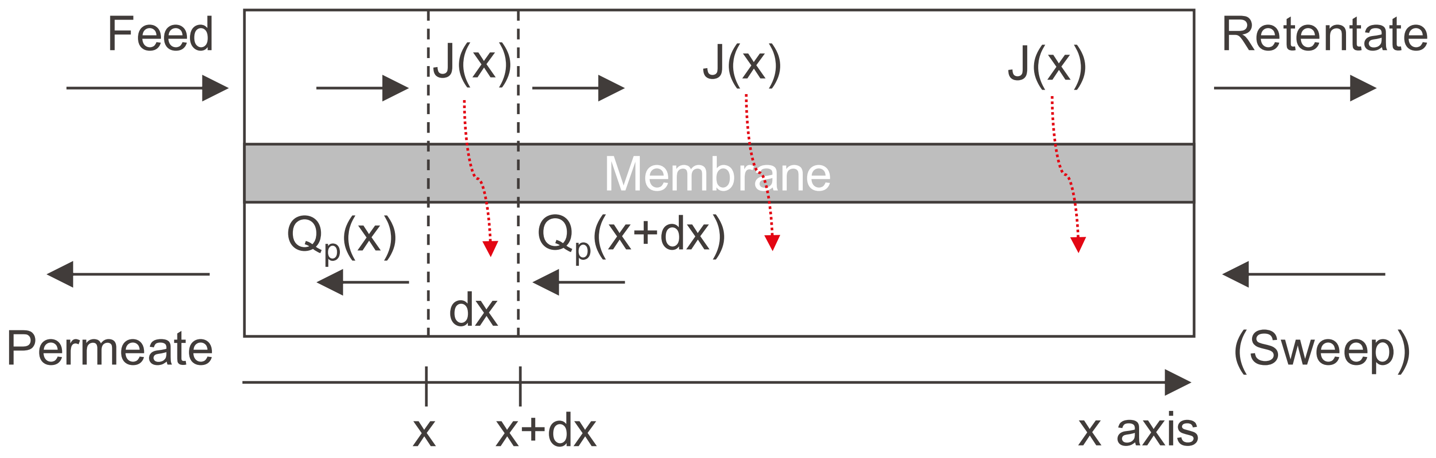
Permeate side

$$Q_p(x+dx) = Q_p(x) + J(x)\, dA$$

$$X_{i,p}(x+dx)\, Q_p(x+dx) = X_{i,p}(x)\, Q_p(x) + J_i(x)\, dA$$

Membrane module: flow arrangements

Counter-current flow arrangement



$$J_i(x) = \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,p}(x) \right)$$

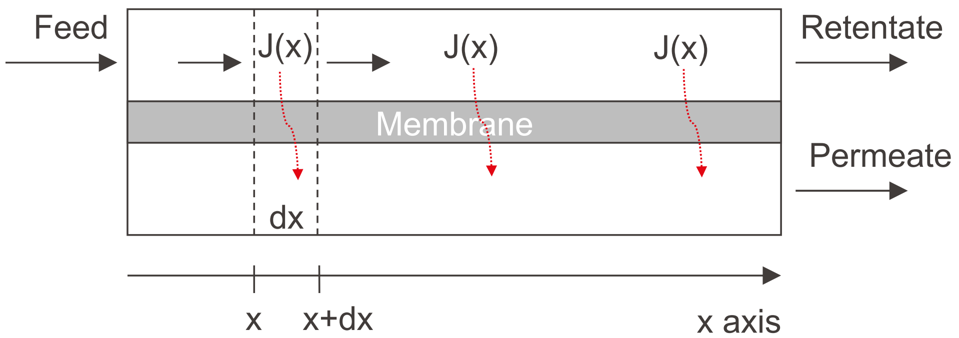
Permeate side

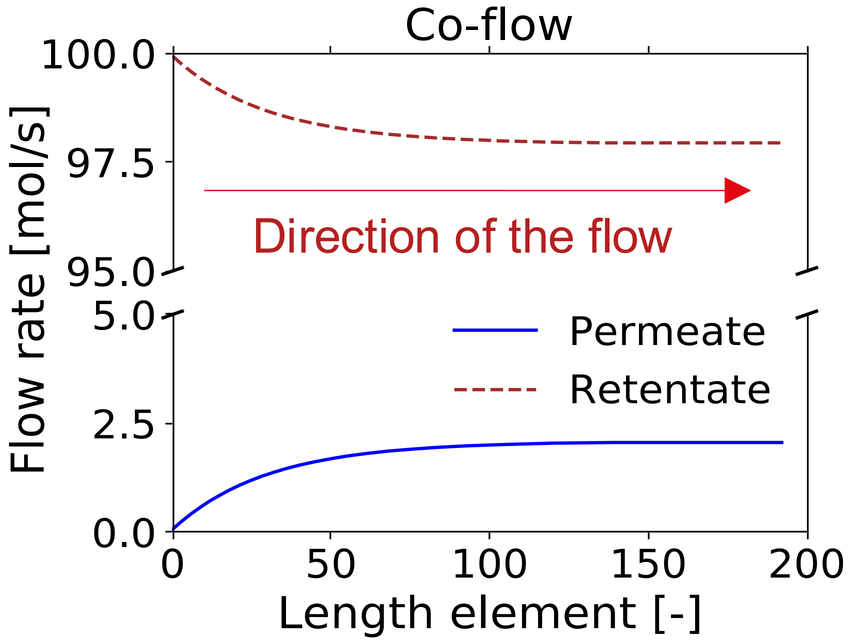
$$Q_p(x) = Q_p\,(x + dx) + J(x)\, dA$$

$$X_{i,p}(x)\, Q_p(x) = X_{i,p}(x + dx)\, Q_p(x + dx) + J_i(x)\, dA$$

Profiles along the membrane module

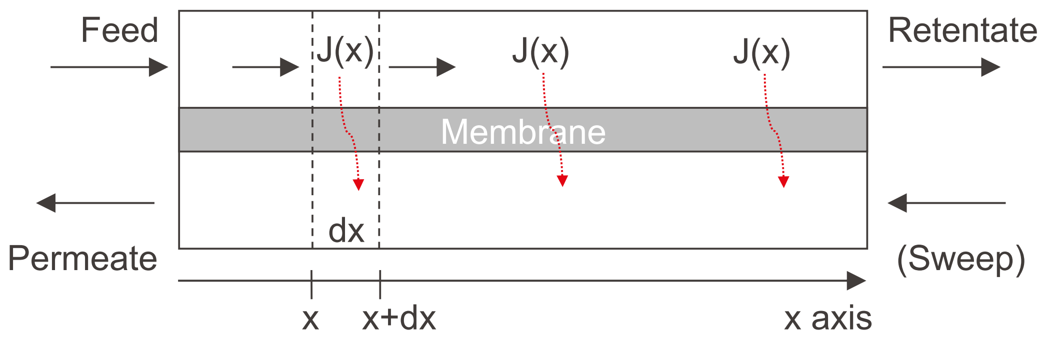
Co-current flow arrangement

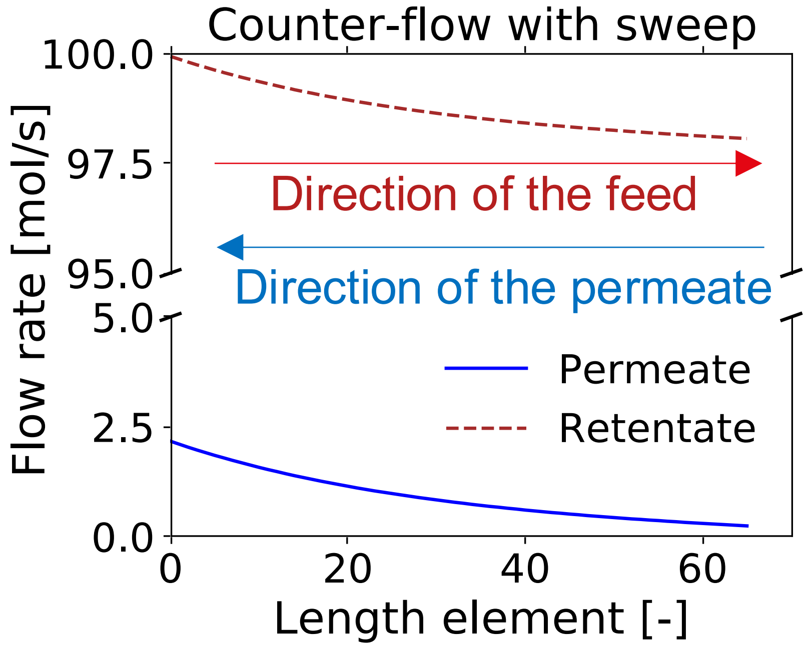




- Reduced driving force along the length

Counter-current flow arrangement





Almost constant driving force along the length

Multi-stage configurations

To achieve purity and recovery > 90% with one module, one would need very high permeance and selectivity & very high driving force (high P_{feed}, low P_{perm})

$$J_i(x) = \boldsymbol{N}_i\left(P_{feed}\,X_{i,feed}(x) - P_{perm}\,X_{i,perm}(x)\right) > 0 \text{ if } \frac{P_{feed}}{P_{perm}} > \frac{X_{i,perm}}{X_{i,feed}}$$

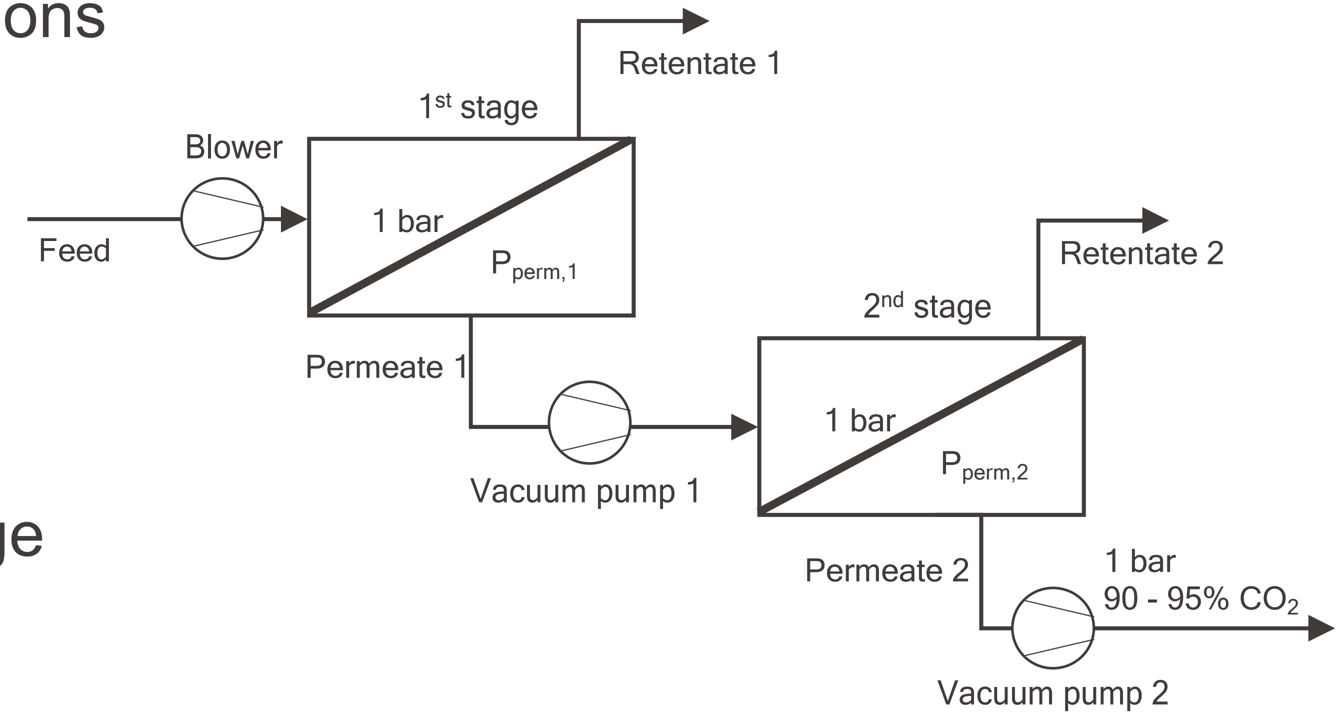
Max separation limited by pressure ratio

Alternative: multi-stage configurations

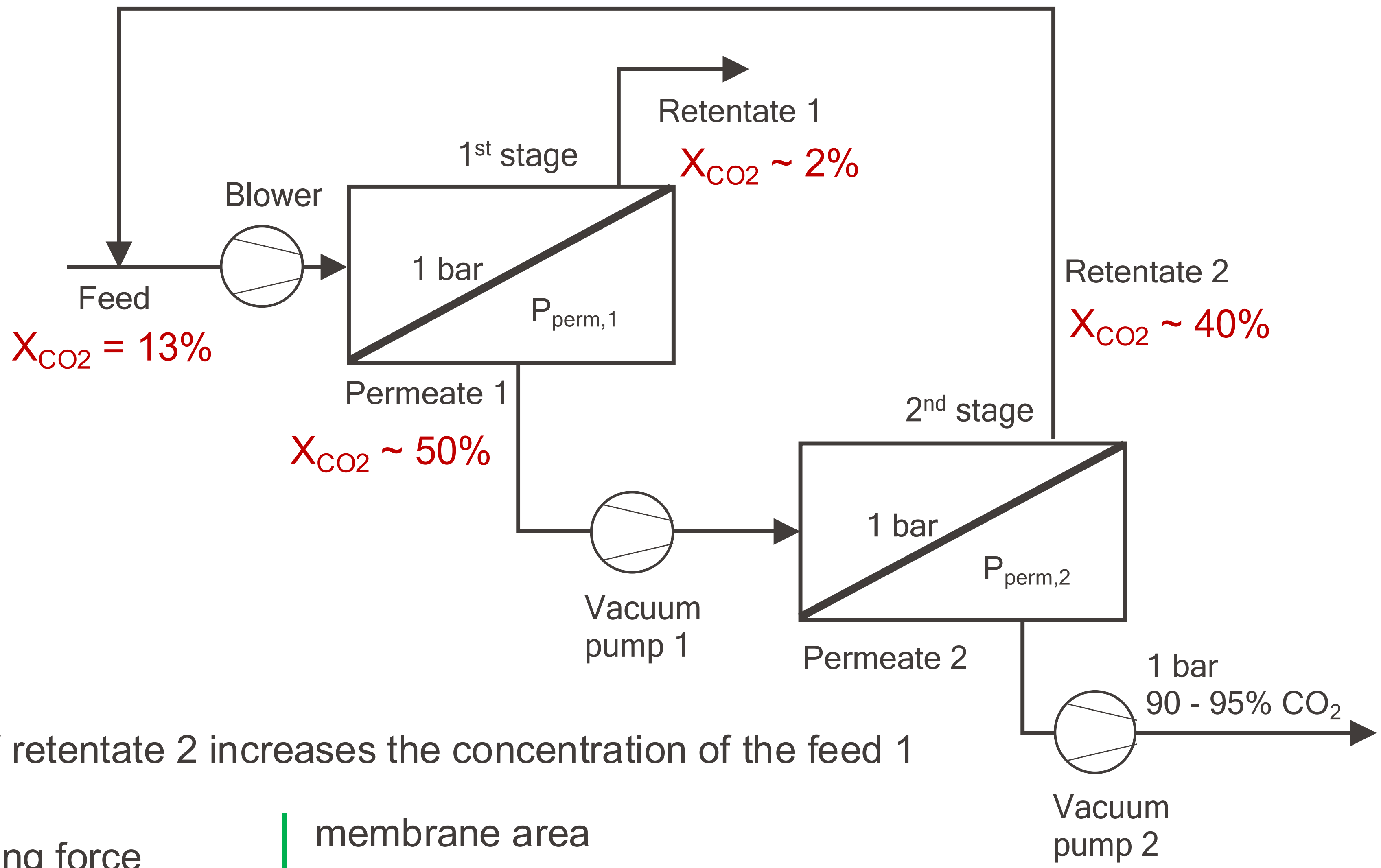
$$Q_{feed,2} = Q_{perm,1}$$

$$X_{CO_2,feed,2} = X_{CO_2,perm,1}$$

Higher driving force in the 2nd stage



Recycle design



Recycle of retentate 2 increases the concentration of the feed 1

↑ driving force

↓ membrane area, energy

Part 3

Agenda

- How to model a membrane stage?
- How to design multi-stage processes?
- Results at variable membrane **permeance and selectivity**
- Results at variable **feed concentration and flow rate**
- Results at variable **feed and permeate pressure**
- Results at variable **membrane areas**

next lecture

Building the model for a single-stage process

Inputs:
- feed concentration
- feed flow rate
- feed pressure
- permeate pressure
- membrane area

Parameters:
- membrane permeance
- membrane selectivity
- physical properties
- geometrical properties

class **single stage**

Outputs:
- permeate concentration
- permeate flow rate
- retentate concentration
- retentate flow rate

method solver:

- flux definition $J_i(x) = \boldsymbol{N}_i \left(P_{feed}\, X_{i,feed}(x) - P_{perm}\, X_{i,perm}(x) \right)$
- total flux definition $J(x) = \sum_{components} J_i(x)$
- mass balance on the retentate $Q_r(x) = Q_f(x) - J(x)\, dA$
- mass balance on the permeate $Q_{P,out} = Q_{feed} - Q_{r,out}$
- next element feed definition $Q_f(x+1) = Q_r(x)$

Building the model for a double-stage process

Inputs:
- feed concentration
- feed flow rate
- feed pressure (1 and 2)
- permeate pressure (1 and 2)
- membrane area (1 and 2)

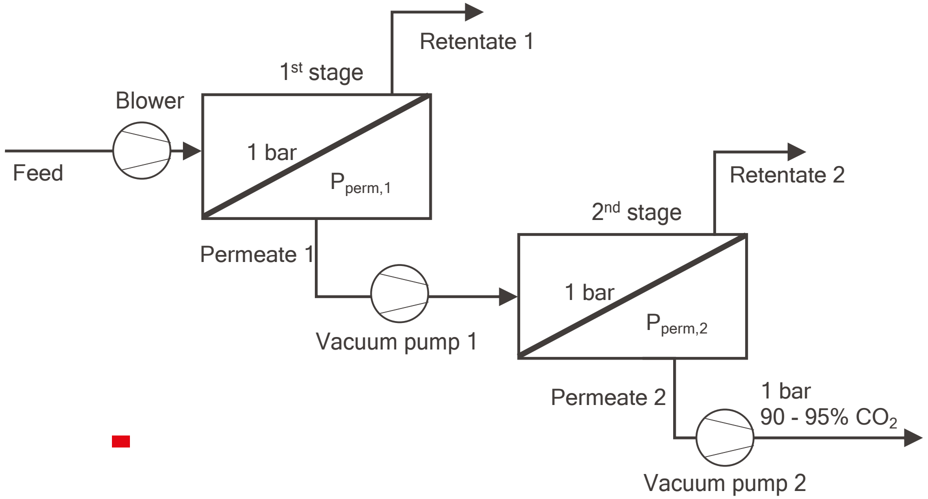
Parameters:
- membrane permeance
- membrane selectivity
- physical properties
- geometrical properties

class double stage

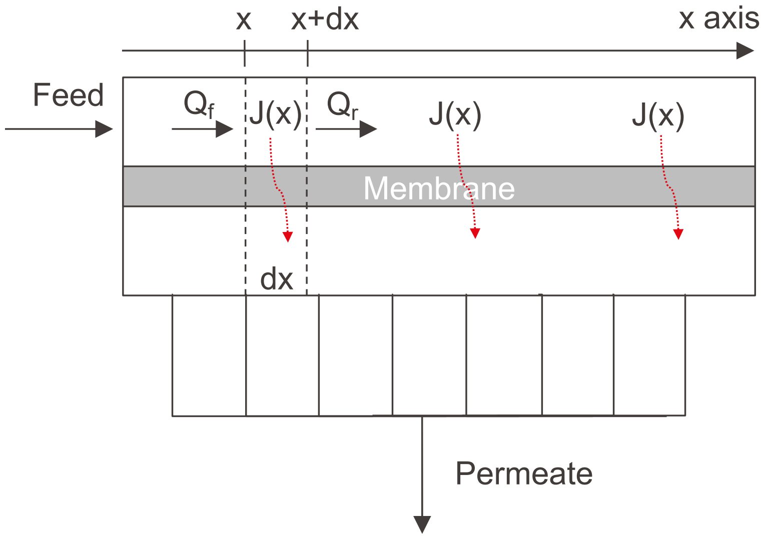
Outputs:
- permeate concentration (1 and 2)
- permeate flow rate (1 and 2)
- retentate concentration (1 and 2)
- retentate flow rate (1 and 2)

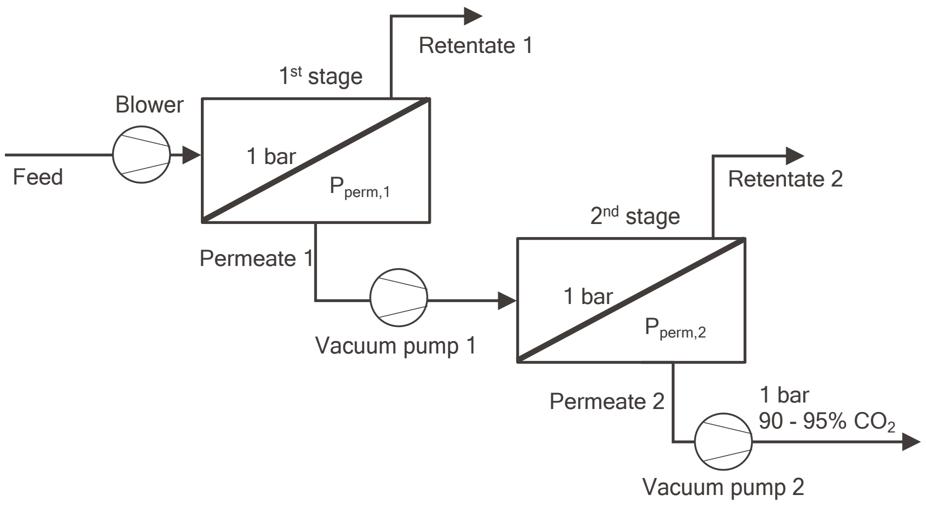
method solver:

- def stage 1 as ***class*** single stage (inputs stage 1, parameters stage 1)
- def feed stage 2 = permeate stage 1
- def stage 2 as ***class*** single stage (inputs stage 2, parameters stage 2)



Simulations - inputs

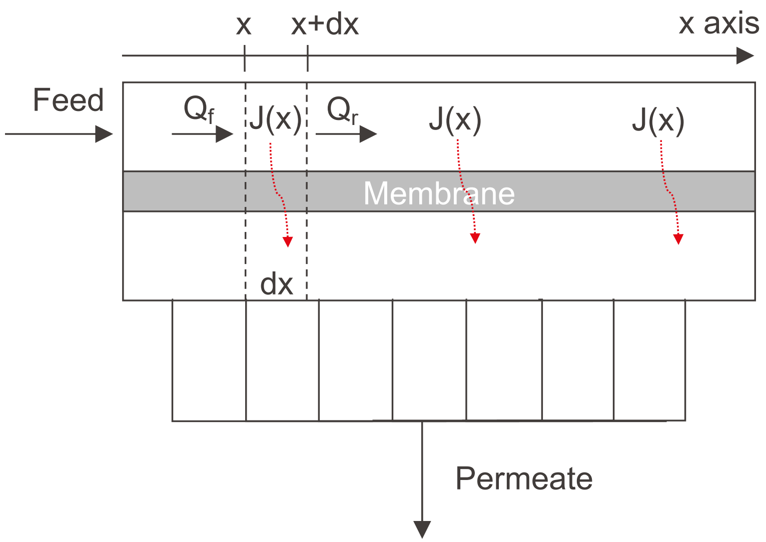




P_{feed}	1 bar
$P_{permeate}$	0.1 bar
$X_{feed,CO2}$	0.1
Q_{feed}	2.5 mol/s
$A_{membrane}$	10 m^2
CO_2 Permeance	10000 GPU
CO_2/N_2 Selectivity	30

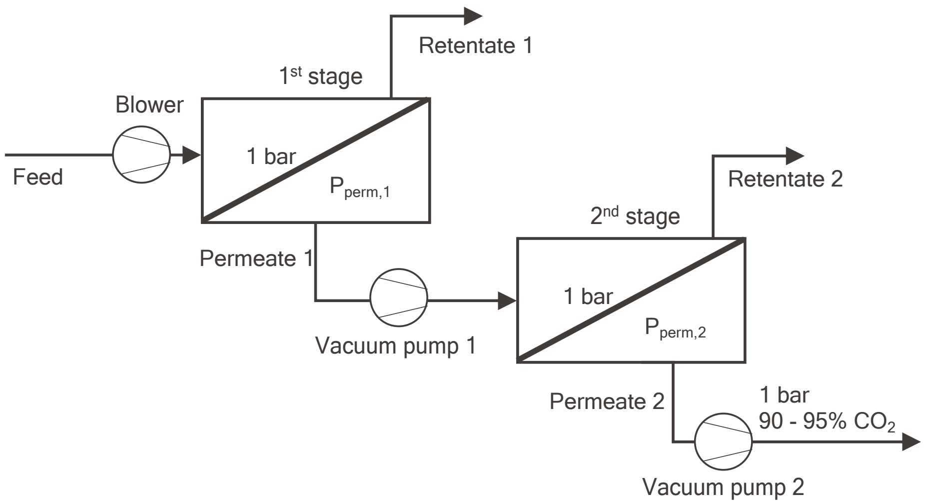
P_{feed}	1 bar
$P_{permeate,1}$	0.1 bar
$P_{permeate,2}$	0.2 bar
$X_{feed,CO2}$	0.1
Q_{feed}	2.5 mol/s
$A_{membrane,1}$	10 m^2
$A_{membrane,2}$	5 m^2
CO_2 Permeance	10000 GPU
CO_2/N_2 Selectivity	30

Simulations - outputs



$$Purity = X_{CO2,p,out}$$

$$Recovery = \frac{Q_{P,out}X_{CO2,p,out}}{Q_{feed,in}X_{CO2,feed,in}}$$



$$Purity = X_{CO2,p,out,stage2}$$

$$Recovery = \frac{Q_{P,out,stage2}\ X_{CO2,p,out,stage2}}{Q_{feed,in}\ X_{CO2,feed,in}}$$