

# Membrane processes

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Lecture 13

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Reference book: Membrane Technology and Applications, R. W. Baker, Wiley 2012 (3rd ed.)



A single membrane stage for  $\text{CO}_2/\text{N}_2$  separation achieves  $\text{CO}_2$  recovery of 30% and  $\text{CO}_2$  purity of 60%. What do we expect if we add a second stage fed with the permeate of the single stage?

- a. Higher  $\text{CO}_2$  recovery, lower  $\text{CO}_2$  purity
- b. Lower  $\text{CO}_2$  recovery, higher  $\text{CO}_2$  purity
- c. Same  $\text{CO}_2$  recovery, higher  $\text{CO}_2$  purity
- d. Same  $\text{CO}_2$  purity, higher  $\text{CO}_2$  recovery

The CO<sub>2</sub> recovery from a single stage decreases when selectivity increases (with fixed CO<sub>2</sub> permeance) because of:

- a. The reduction in N<sub>2</sub> permeance
- b. The decrease of driving force for CO<sub>2</sub> flux
- c. The increase in purity
- d. All the above are correct.

# Question 3

The CO<sub>2</sub> purity from a single stage decreases when feed pressure increases because:

- a. The flux of CO<sub>2</sub> increases
- b. The flux of N<sub>2</sub> increases
- c. The CO<sub>2</sub> permeance is lower
- d. The N<sub>2</sub> permeance is higher

In a double stage, the total CO<sub>2</sub> recovery reaches a maximum and then it decreases when we increase the feed pressure of the first stage because:

- a. Beyond that pressure, membranes start to break
- b. The recovery is limited by the permeate flow of the second stage
- c. The driving force for the flux in the first stage first increases then decreases
- d. Purity achieves a plateau

# Question 5

Non-ideal phenomena:

- a. Damage the membranes
- b. Reduce the available driving force
- c. Do not allow to achieve high recovery
- d. Cause the presence of impurities in the permeate

# Question 6

Pressure drops can be minimized by:

- a. Increasing feed velocity
- b. Reducing feed velocity
- c. Reducing channel thickness
- d. Increasing membrane selectivity

# Question 7

The effect of concentration polarization is stronger at:

- a. Higher channel thickness
- b. Higher feed velocity
- c. Lower membrane permeance
- d. Higher membrane selectivity



# Question 8

If membranes have higher permeance, the concentration polarization effect:

- a. Increases because the flux from the bulk to the interface reduces
- b. Increases because the trans-membrane flux increases
- c. Decreases because the mass transfer coefficient is higher
- d. Does not change

# In-class exercise

# Exercise: Concentration polarization and Pressure drops

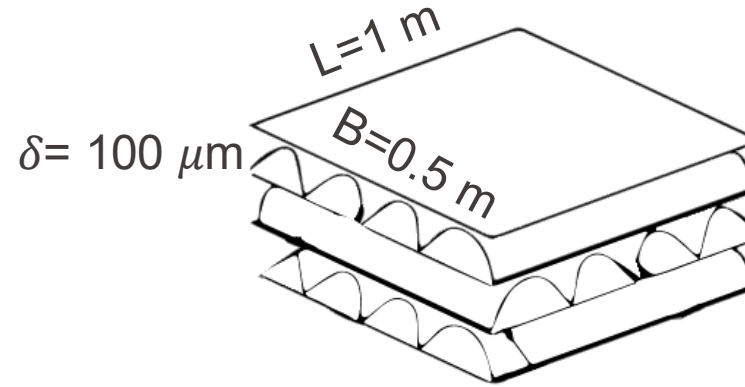
- A gas permeation module with cross-current flow arrangement is separating CO<sub>2</sub> from N<sub>2</sub> using a membrane with a CO<sub>2</sub>/N<sub>2</sub> selectivity of 16.9 and CO<sub>2</sub> permeability of  $3.3 \times 10^{-14}$  mole m<sup>-1</sup> s<sup>-1</sup> Pa<sup>-1</sup>. Feed to the membrane contains 20 mol% CO<sub>2</sub>. Feed and retentate pressure are 5.5 bar and a permeate pressure is 1 bar. Membrane thickness is 1.0 μm.
- Membrane area is 60 m<sup>2</sup> and  $Q_{\text{feed}}$  is 1 mol/s. The channel thickness is 100 μm. The dimensions of the membrane sheets are 0.5 m (width) x 1 m (length). Assume  $Re = 2000$ , density = 1.3 kg/m<sup>3</sup>,  $k = 0.5$  mol/m<sup>2</sup>s
- Calculate the outlet purity and recovery.

Ideal case with length discretization:

$$N_{CO_2} = 3.3 \times 10^{-8} \text{ mol} / \text{m}^2 \text{ s Pa} = 98.5 \text{ GPU}$$

- Purity = 0.59
- Recovery = 0.35

- Only pressure drops



$$d_{hydro} = \frac{4A}{P} = \frac{4B \times \delta}{2(B + \delta)} = 0.0002 \text{ m}$$

$$\frac{dP_{feed}}{dx} = -f \frac{\rho}{2 d_{hydr}} v^2$$

$$v = \frac{Q_f [\text{mol/s}] RT}{P} \left[ \text{m}^3/\text{s} \right] \times \frac{1}{B \times \delta \times N_{channels}} = \frac{1 \times 8.31 \times 298}{5.5 \times 10^5} \times \frac{1}{0.5 \times 0.0001 \times 120} = 0.75 \text{ m/s}$$

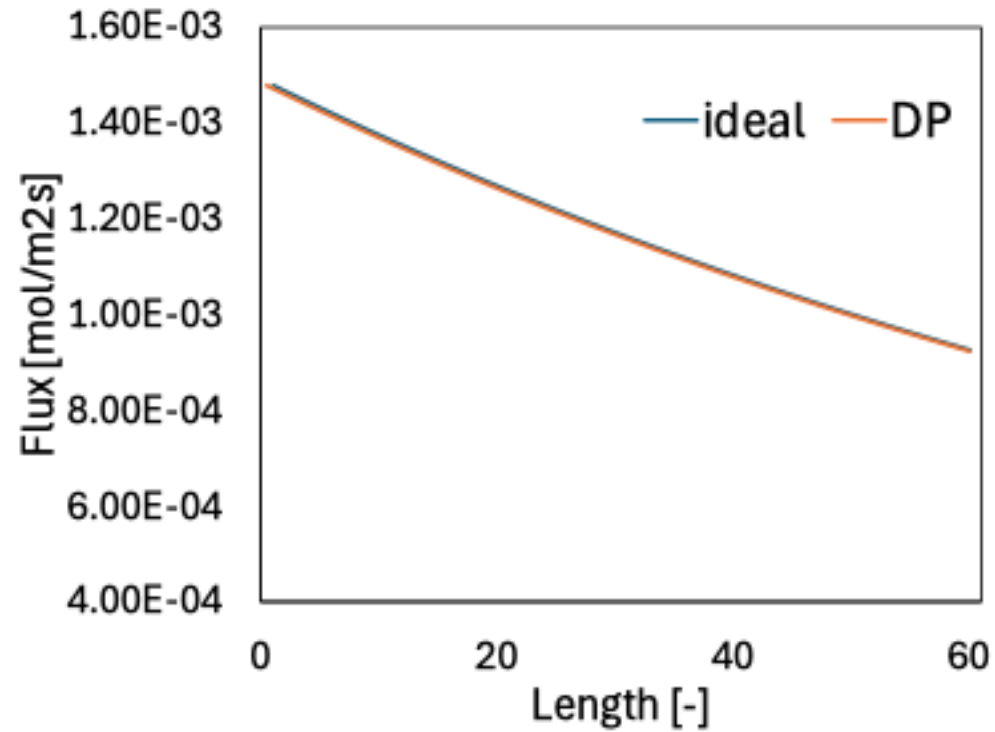
$$\frac{dP}{dx} = -\frac{f\rho}{2 d_{hydr}} v^2 = -\frac{64}{2000} \times \frac{1.3}{2 \times 0.0002} \times 0.75^2 = -58.6 \frac{\text{Pa}}{\text{m}} = -0.00059 \text{ bar/m}$$

The profiles of  $X_{\text{CO}_2, \text{perm}}$  and  $J_{\text{CO}_2}$  have to be recalculated considering the P decrease.

$$X_{\text{CO}_2, \text{perm}}(x) = \frac{1 + (\alpha - 1) (\beta + X_{\text{CO}_2, \text{feed}}(x)) - \sqrt{\left[1 + (\alpha - 1) (\beta + X_{\text{CO}_2, \text{feed}}(x))\right]^2 - 4\alpha\beta(\alpha - 1) X_{\text{CO}_2, \text{feed}}(x)}}{2\beta(\alpha - 1)}$$

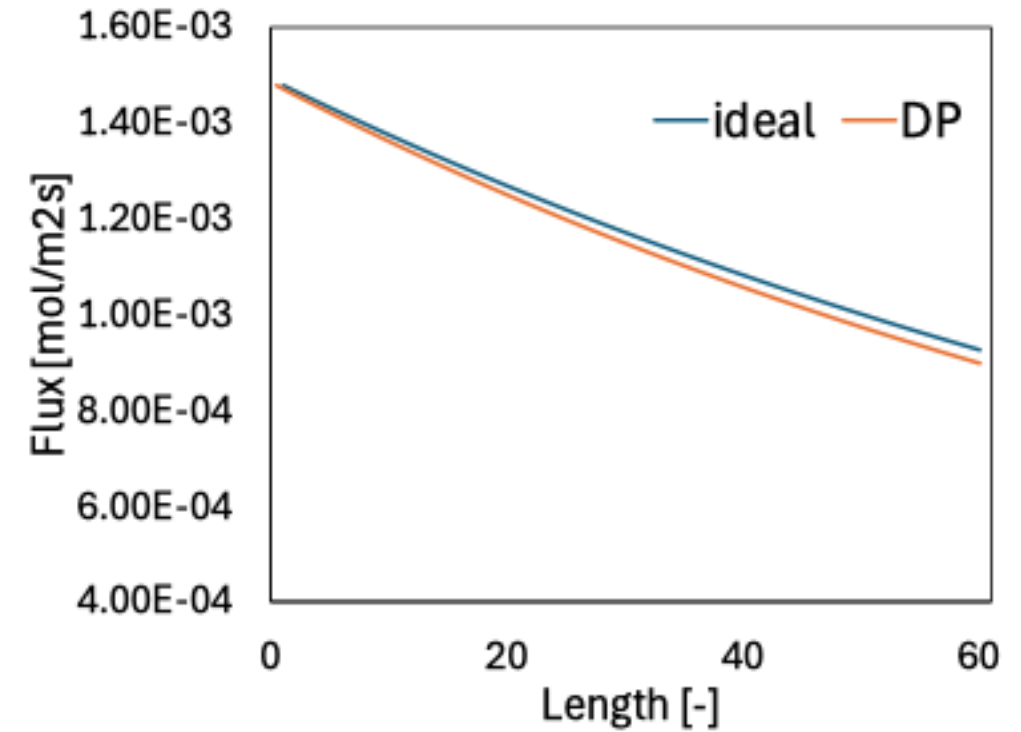
$$J_{\text{CO}_2}(x) = N_{\text{CO}_2} \left( P_{\text{feed}} X_{\text{CO}_2, \text{feed}}(x) - P_{\text{perm}} X_{\text{CO}_2, \text{perm}}(x) \right)$$

■  $v = 0.75$  m/s



Purity = 0.59  
Recovery = 0.35

■  $v = 10$  m/s



Purity = 0.59  
Recovery = 0.348

- Only concentration polarization

$$\frac{X_1^p - X_1^{f,m}}{X_1^p - X_1^{f,b}} = \exp\left(\frac{J_1 + J_2}{k}\right)$$

We would need an iterative loop where fluxes and  $X_{\text{perm}}$  are recalculated in function of  $X^{f,m}$

Conservative assumption: CP index depends on the ideal flux at the entrance

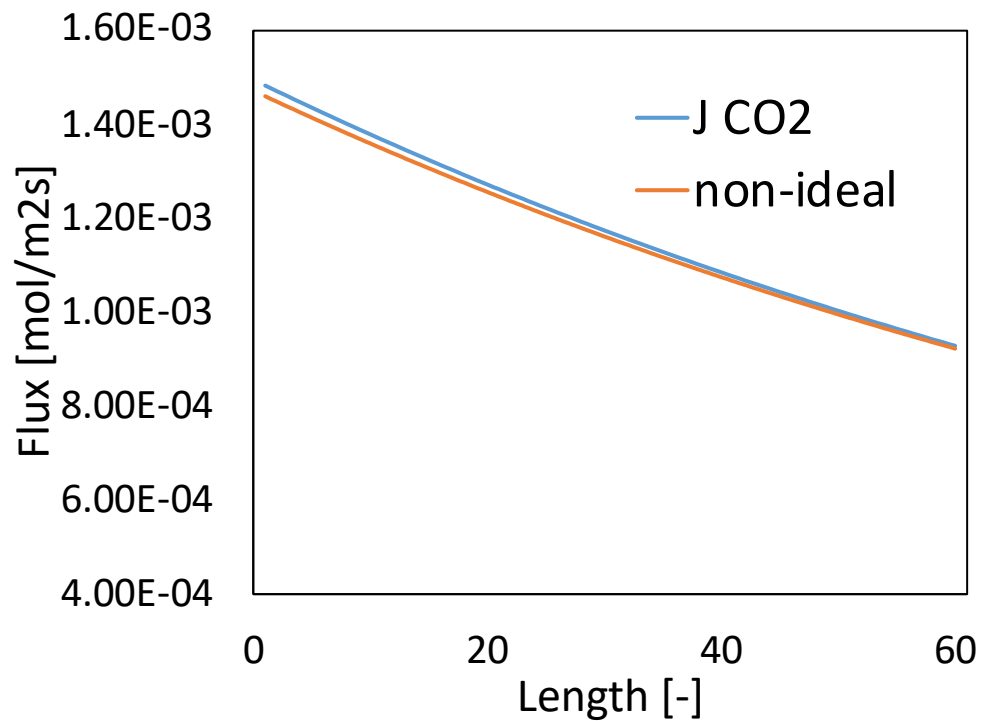
$$\begin{aligned} \frac{X_1^p - X_1^{f,m}}{X_1^p - X_1^{f,b}} &= \exp\left(\frac{J_1 + J_2}{k}\right) = \exp\left(\frac{J_{CO_2}[0] + J_{N_2}[0]}{k}\right) \\ &= \exp\left(\frac{0.00227}{0.5}\right) = 1.0045 \end{aligned}$$

For each element, we recalculate the  $X_1^{f,m}$  from  $X_1^{f,b}$  and  $X_1^p$  calculated as function of  $X_1^{f,b}$  -> we recalculate  $X_1^p$  and use this for fluxes and balances

A	dA	Qf	Xf,b	Xp (id)	Xf,m	Xp (real)	J CO2	J N2	J tot	Qr	Xr
0.5	0.5	1	0.2	Calc from Xf,b = 0.65	Calc from exp(J/k)	Calc from Xf,m	Calc from Xp,real	Calc from Xp,real	J CO2 + J N2	Calc from balance with J	Calc from balance with JCO2
1.5	0.5	Qr	Xr	Calc from Xf,b	Calc from exp(J/k)	Calc from Xf,m	Calc from Xp,real	Calc from Xp,real	J CO2 + J N2	Calc from balance with J	Calc from balance with JCO2

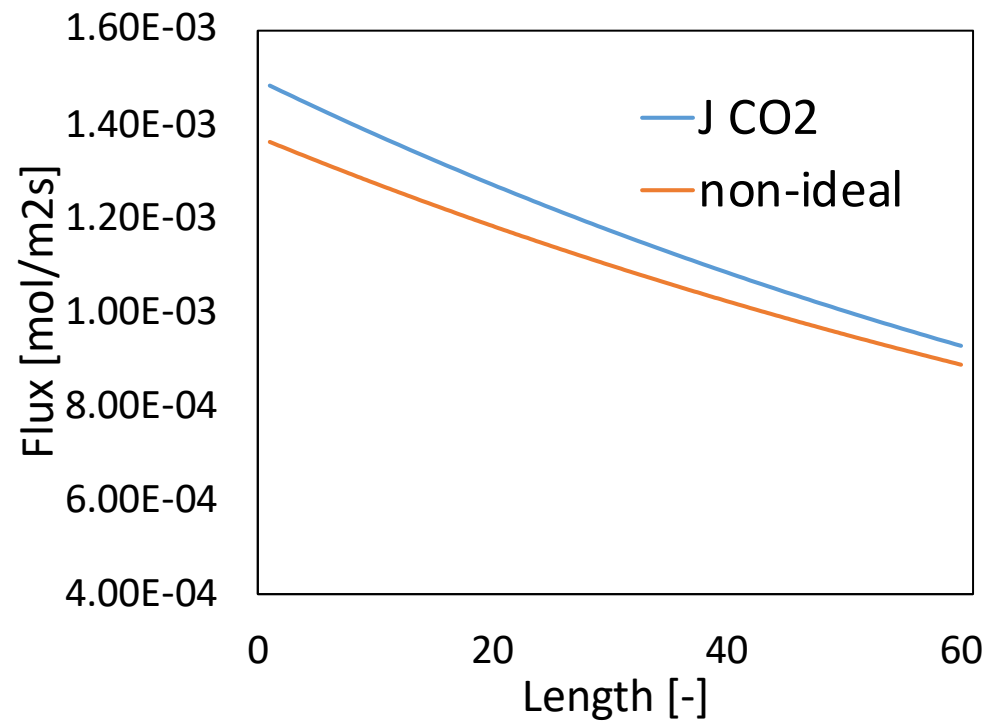
This is an approximation valid for limited concentration polarization

■  $k = 0.5 \text{ mol/m}^2\text{s}$



Purity = 0.59  
Recovery = 0.349

■  $k = 0.1 \text{ mol/m}^2\text{s}$



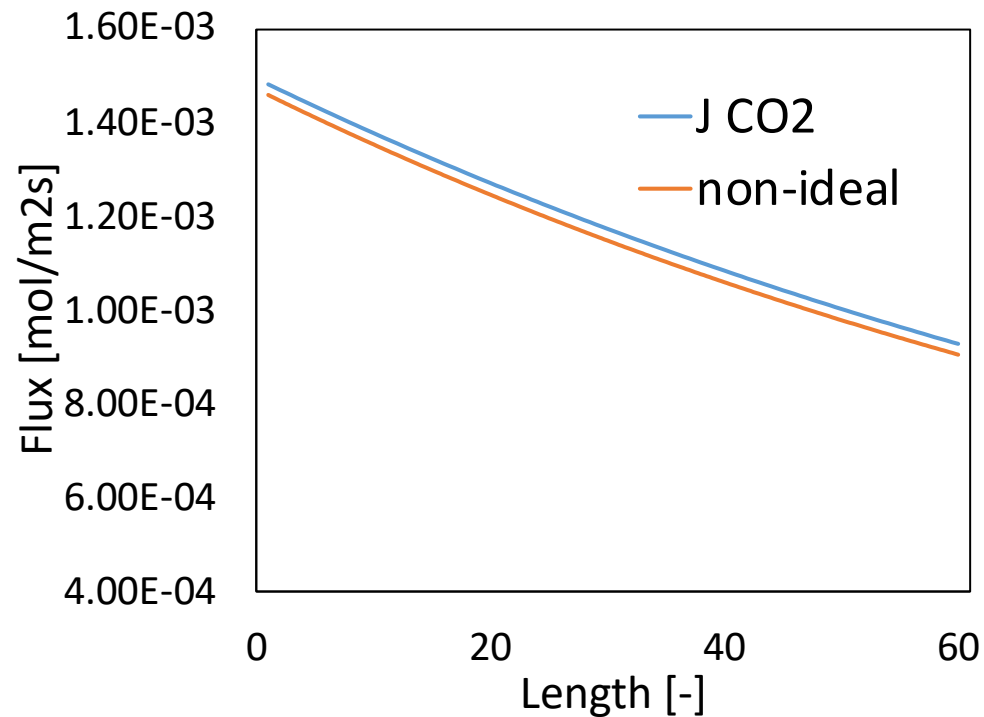
Purity = 0.58  
Recovery = 0.33



- Concentration polarization and pressure drops

- $k = 0.5 \text{ mol/m}^2\text{s}$

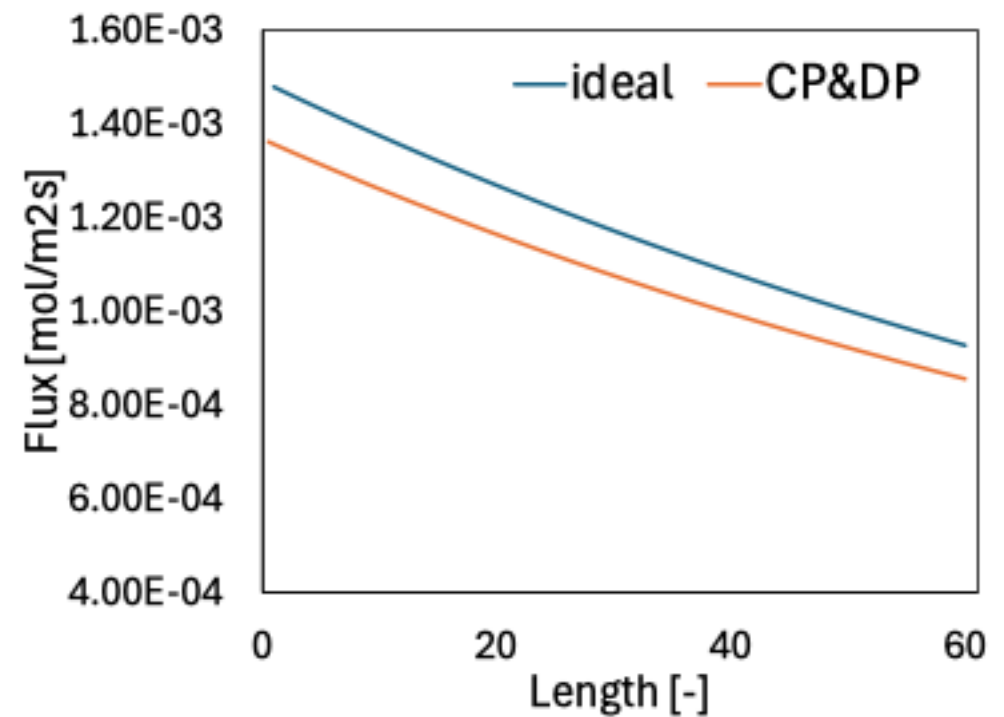
- $v = 0.75 \text{ m/s}$



Purity = 0.59  
Recovery = 0.349

- $k = 0.1 \text{ mol/m}^2\text{s}$

- $v = 10 \text{ m/s}$

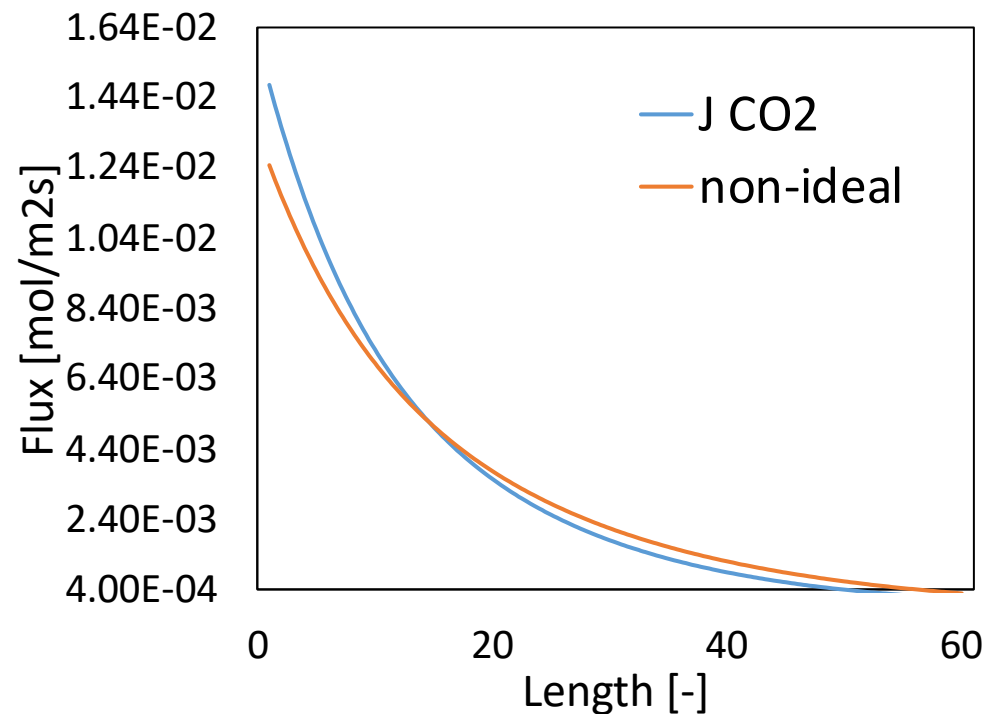


Purity = 0.575  
Recovery = 0.326

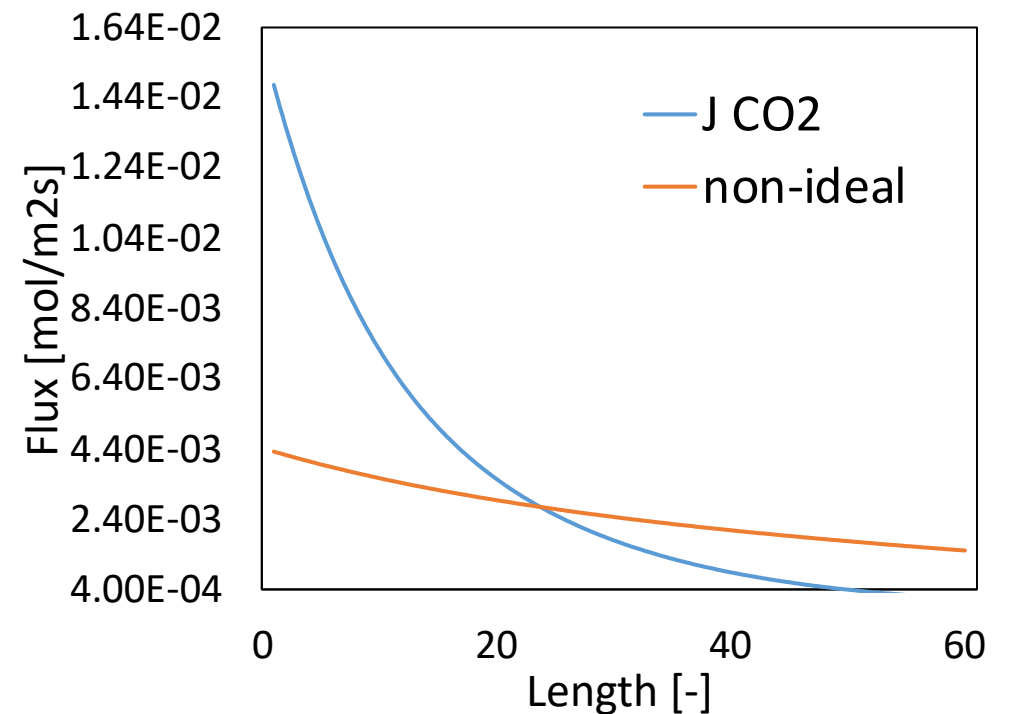
- Concentration polarization and pressure drops with higher permeance

$$N_{CO_2} = 3.3 \times 10^{-7} \text{ mol} / \text{m}^2 \text{ s Pa} = 985 \text{ GPU}$$

- $k = 0.5 \text{ mol/m}^2 \text{ s}$
- $v = 0.75 \text{ m/s}$

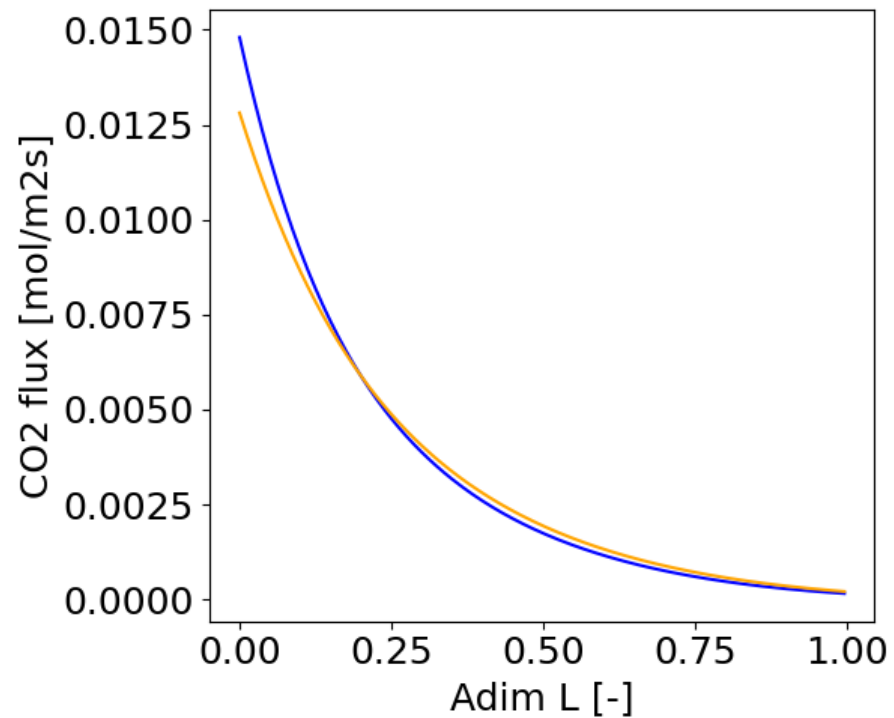


- $k = 0.1 \text{ mol/m}^2 \text{ s}$
- $v = 2 \text{ m/s}$



- Profiles from the rigorous model

- $k = 0.5 \text{ mol/m}^2\text{s}$



- $k = 0.1 \text{ mol/m}^2\text{s}$

