ChE-309 TP-9
Conduction in stationary and transient states

instructions for use, spring 2025

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1. Statement of purpose

Your employer, the Bergquist Company, supplies thermal materials for electronic applications and has recently developed two new alloys with different thermal characteristics. Your boss has asked you to fully characterize the thermal properties of these new alloys for the engineers who design heat sinks. Your task is to find the thermal conductivity, thermal diffusivity and heat capacity of the two alloys.

2. Theoretical basis.

Heat transfer through solids and stationary fluids occurs by conduction. The equation that quantifies the process of heat transfer by conduction is known as Fourier's Law.

$$q = -k\nabla T$$

Where q is the heat flux density [W m^{-2}], k is the thermal conductivity of the material [W m^{-1} K^{-1}], and ∇T is the temperature gradient.

In the one-dimensional case and in a state of equilibrium, the heat flow through in the direction x through a material with two ends (or sides, e.g. a wall) is given by the following expression:

$$Q = -kA\,\frac{(T_2 - T_1)}{x} \qquad \text{(eq. 1)}$$

where Q is the heat flow [W], x is the thickness of the material [m] in the direction of interest, T_2 is the temperature of the cold end of the material [K], T_1 is the temperature of the hot end of the material [K], and A is the area perpendicular to the direction of heat flow [m^2].

2.1 Thermal Resistance

Thermal resistance is a key concept in the evaluation of heat transfer. In heat transfer, we can consider the thermal resistance is:

$$R_T = \frac{(T_1 - T_2)}{Q}$$

Where (T_1 - T_2) is the temperature difference and Q is the heat flow. From equation 1, the heat conduction resistance can be determined according to:

$$R_{T,cond} = \frac{x}{kA}$$

at the interface between two materials: The mode of heat transfer between two materials, which are at different temperatures, can be by various mechanisms (conduction or convection). For the interface between the two solids, the physical contact between the two materials can vary depending on the roughness of the surface. This is shown in Figure 1.

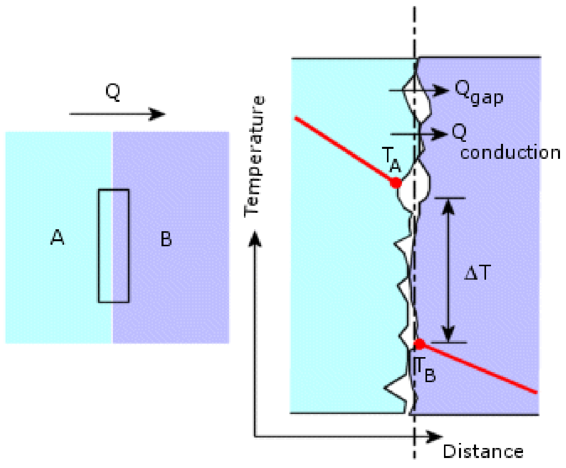

Figure 1. Through the interface between the two contact faces there are two modes of heat transfer. The first is conduction through solid to solid contact points ($Q_{conduction}$) which is very efficient. The second is conduction through the gap-filling gas (Q_{gap}) which, due to the low thermal conductivity of the gas, can be very low.

Independently of the overall heat transfer mechanism at the interface, a heat transfer coefficient can be defined which is expressed as follows:

$$Q = A\, h_r\ (T_A - T_B) \qquad \text{(eq. 2)}$$

Where, Q is the heat transfer rate [W], h_r is the heat transfer coefficient [W m^{-2} K^{-1}], A is the exchange surface [m^2], T_A is the temperature at the first surface [K], T_B is the temperature at the second surface [K]. Therefore, we can write an expression for resistance to heat transfer on contact:

$$R_{T,cont} = \frac{1}{h_r A}$$

Convection: The mode of heat transfer between a surface and a moving fluid, which are at different temperatures, is called convection. It is the result of the superposition of two physical phenomena: the energy transported by the random motion of the molecules (diffusion) and the energy transported by the fluid flow (macroscopic motion: advection). Convective heat transfer can be classified as forced or natural convection. Forced convection occurs when external means (a fan, pump or atmospheric wind) cause a flow or current. Natural convection occurs when the flow is induced by buoyant forces, which are the result of density differences caused by changes in fluid temperature.

Irrespective of whether convection can be forced or natural, the convective heat flow between a surface and a fluid is given by Newton's law of cooling, which is expressed as follows:

$$Q = A\, h_c\ (T_s - T_\infty) \qquad \text{(eq. 3)}$$

Where Q is the heat transfer rate [W], h_c is the (convective) heat transfer coefficient [W m^{-2} K^{-1}], A is the heat exchange surface [m^2], T_s is the surface temperature [K], T_∞ is the temperature of the fluid far away from the surface [K]. Therefore, we can write an expression for resistance to convection heat transfer.

$$R_{T,conv} = \frac{1}{h_c A}$$

In general, the heat flow can be calculated by adding up all the thermal resistances:

$$Q = \frac{\Delta \mathrm{T}}{\sum R_T}$$

2.2 Dimensionless numbers for heat transfer

Dimensionless numbers are very important for heat transfer. They can quantify and compare transport phenomena to determine which ones are important and which ones can be neglected. The two most important numbers are the Nusselt number and the Biot number.

The Nusselt number (Nu) represents the ratio between total heat transfer and conduction transfer between two bodies or materials. If conduction is the main mode of transfer, then the Nusselt number will be of the order of the unit. In the case of convection (e.g. due to the displacement of a fluid in turbulent conditions), the heat transfer will take place mainly by displacement of the fluid and as a result the Nusselt number will tend towards ∞. It is defined as follows:

$$Nu = \frac{h_c L_c}{k}$$

Where h_c is the heat transfer coefficient, L_c is the characteristic length, and k is the thermal conductivity.

The Biot number (Bi) compares the heat transfer resistances inside and on the surface of a body. A Biot number value greater than 1 means that the conduction of heat inside the body is slower than at its surface, and that temperature gradients within the body are non-negligible. If the Biot number of a system is smaller than 1 (often $Bi < 0.1$ is used), it means that the internal resistance to heat transfer is negligible and therefore the temperature can be considered as uniform inside the body. It is defined as follows:

$$Bi = \frac{h\, L_c}{k}$$

with h as the overall heat transfer coefficient, L_c is the characteristic length, and k is the thermal conductivity.

Although these two dimensionless numbers seem to be defined in the same way, there is an important difference between them. (what is it?)

2.3 Transient heat transfer

While the above relationships are valid for most solids under imaginable conditions at steady state, sometimes in thermal systems temperatures change significantly with time. To understand how heat is conducted in non-isothermal systems as a function of time, we need to look at the general energy equation. For solids, the general equation of energy when combined with Fourier's Law of heat conduction becomes:

$$\rho c_p \frac{\partial T}{\partial t} = (\nabla \cdot k \nabla T) \qquad \text{(eq. 4)}$$

where c_p is the heat capacity of the material per unit mass (the specific heat). If we can assume that thermal conductivity, k, density and heat capacity are independent of temperature and position, then we can simplify by:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \qquad \text{(eq. 5)}$$

where $\alpha = k/\rho c_p$ is known as thermal diffusivity. This value is a very important material property for the design of heat sinks and heat exchangers that have to cope with large transient disturbances. It can be estimated if the material of an ideal geometric shape (e.g. a long cylinder or a sphere) is subjected to a known temperature variation. Then equation 5 can be solved. To solve this equation for T as a function of position and time is possible in this case 1D (using the method of separation of variables), but analytical solutions involve infinite series and implicit equations, which are difficult to evaluate. Consequently, simplified solutions have been prepared in tabular form. A commonly used ideal case is that of a sphere first at a temperature of T_i, and then quickly placed in an environment at a temperature of T_∞ (with h as the heat transfer coefficient between the sphere and the environment), as shown in Figure 2 below.

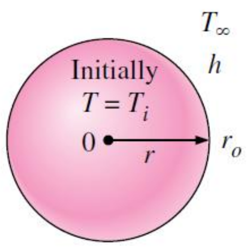

Figure 2. Diagram in our case of transient heat transfer.

In this case the exact solution for T(r,t) is:

$$\frac{T - T_i}{T_\infty - T_i} = \sum_{n=1}^{\infty} \frac{4(\sin\lambda_n - \lambda_n \cos\lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin\left(\frac{\lambda_n r}{r_0}\right)}{\frac{\lambda_n r}{r_0}}$$

where λ_n are the roots of $1 - \lambda_n \cot \lambda_n = Bi$, $Bi = hr_0/k$ (the Biot number) and $\tau = \alpha t/r_0^2$ (the Fourier number). The terms in this infinite series will quickly become smaller as n becomes large. The limit of $\tau > 0.2$ is used to define cases where only the first term is calculated because the error obtained is less than 2 percent. Thus, the following solution is simply used as an approximation to define the temperature in the center of the sphere as a function of time:

$$\text{Center of the sphere } (r = 0)\text{: } \frac{T - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \qquad \text{(eq. 6)}$$

(*Note the change on the left side of the equation from the previous equation*) Where the terms A_1 and λ_1 are available according to the number of Biot $Bi = hr_0/k$ by interpolation from the table below:

	Sphere	
Bi	λ_1	A_1
0.01	0.1730	1.0030
0.02	0.2445	1.0060
0.04	0.3450	1.0120
0.06	0.4217	1.0179
0.08	0.4860	1.0239
0.1	0.5423	1.0298
0.2	0.7593	1.0592
0.3	0.9208	1.0880
0.4	1.0528	1.1164
0.5	1.1656	1.1441
0.6	1.2644	1.1713
0.7	1.3525	1.1978
0.8	1.4320	1.2236
0.9	1.5044	1.2488
1.0	1.5708	1.2732
2.0	2.0288	1.4793
3.0	2.2889	1.6227
4.0	2.4556	1.7202
5.0	2.5704	1.7870
6.0	2.6537	1.8338
7.0	2.7165	1.8673
8.0	2.7654	1.8920
9.0	2.8044	1.9106
10.0	2.8363	1.9249
20.0	2.9857	1.9781
30.0	3.0372	1.9898
40.0	3.0632	1.9942
50.0	3.0788	1.9962
100.0	3.1102	1.9990
∞	3.1416	2.0000

To calculate thermal diffusivity, the experimental data for temperature versus time can be in the form of equation 6 using the appropriate values of A_1 and λ_1.

3. Practical Laboratory Exercises

3.1 Objectives

Your objective is to find a way to measure the thermal conductivity, thermal diffusivity and specific heat capacity of the two unknown alloys using the steady-state and transient systems.

3.2 Description of the systems

To fully characterize the thermal properties of the unknown materials we will use two assemblies: one for steady state properties and the other for transient properties.

The stationary state heat conduction unit consists of a heated module (electrical resistance) mounted on a test stand. The module consists of a cylindrical metal bar which allows the analysis of linear heat transfer. It also includes a series of connectors for taking temperature measurements along the cylinder. In order to keep the temperature gradient constant, a cooling system circulating water has been inserted on the right side of the module. The whole is insulated to minimize radial heat loss and thus promote axial heat transfer. The electrical power given by the heating element is controlled by a circuit which makes it possible to vary the maximum power with a resistance ranging from 0 to 100%.

The module is divided into three parts, one of which is interchangeable. These three parts are represented by the following figure:

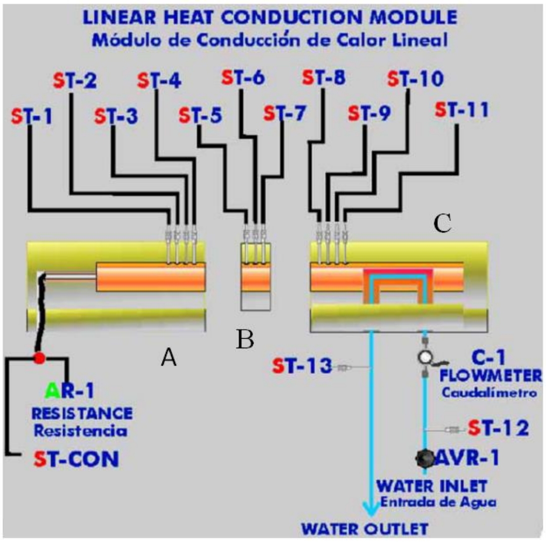

Figure 3: Schematic diagram of the plant for the steady state

They correspond to: A, the region where the resistive heater is located; B, the interchangeable element (unknown); C, the end of the cylinder with a water circulation cooling system that guarantees a constant heat gradient in the system.

The module has a total of eleven temperature taps, each 10 mm apart, which enable an ideal temperature profile to be obtained for each model.

The system for transient thermal properties consists of a bath of high-temperature water, T_∞, which is slowly recirculated in order to maintain the homogeneous temperature in the bath. In this bath a sphere of the unknown material (at a lower temperature, T_i) is placed, and the temperature in the center of the sphere can be measured as a function of time. For geometry and flow in this system, the heat transfer coefficient, *h*, is known (ask the assistant).

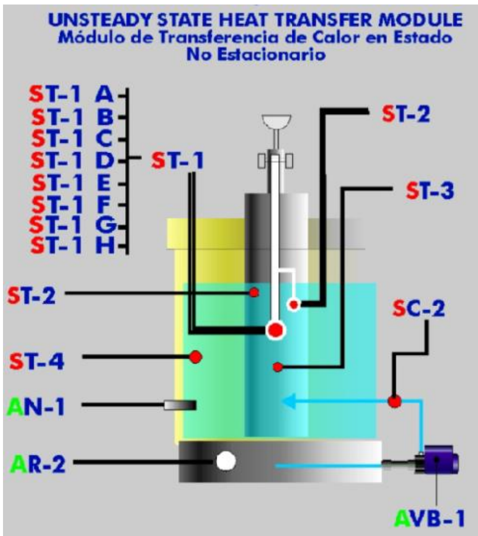

Figure 4: Installation diagram for transients

3.2 Experimental procedure

<u>*Steady State Conduction:*</u>

1. In the program, select the CL module and **check that the correct sensors are connected**.
2. Select the interchangeable cylinder (unknown material) to be analyzed (part B of the module) and install it.
3. Connect the temperature sensors to this cylinder and check that all the sensors are fixed on the module.
4. Open the water valve for cooling and check that the water flows (SC-2 flow meter > 1 L/min).
5. Set a power of ca. 20 W (SW-1) with the AR-1 controller to heat the resistor.
6. Wait until the system is in a steady state (constant temperatures, ~30-60 min).

Calculate the thermal conductivity of part B.

Determine the heat transfer coefficient between parts A and B. *How could we improve this value?*

Using the same method, determine the thermal conductivity of the second unknown cylinder.

Transient Conduction:

1. In the program, select the EI module and check that the correct sensors are connected.
2. Heat the bath to 70°C with the PID and set the water flow rate to (*ask the person in charge of the work)* L/min.
3. While waiting for the bath to heat up, choose a sphere to test and attach it to the cover.
4. Place the sphere and the temperature sensor in an ice bath or cold water.
5. When the water is at 70°C, start taking values (at least every second) and place the cover on the water bath. Check that the ball and the sensor are in the central cylinder.
6. Allow temperatures to stabilize (~5-10 min).

Using the final temperature at the center of the sphere as the infinite temperature, calculate the thermal diffusivity of the material used by making the necessary simplifications.

What type(s) of approximation(s) did you make? *Explain why*.

Compare your result (by calculation or graphically) with other possible approximations.

Using the same method, determine the thermal diffusivity of the second sphere.

Can you determine what those unknown alloys are?

4. Characteristics

Alloys:

Density Alloy#1: $\rho_1 \sim 7.98\ g/cm^3$

Density Alloy#2: $\rho_2 \sim 8.59\ g/cm^3$

Assembly 1:

Cylinder diameter: $d = 25\ mm$

Assembly 2:

Sphere diameter: $d = 40\ mm$

Coefficient of heat transfer: h = ask the person in charge of the TP