
Introduction to Transport Phenomena: Mid-term Exam

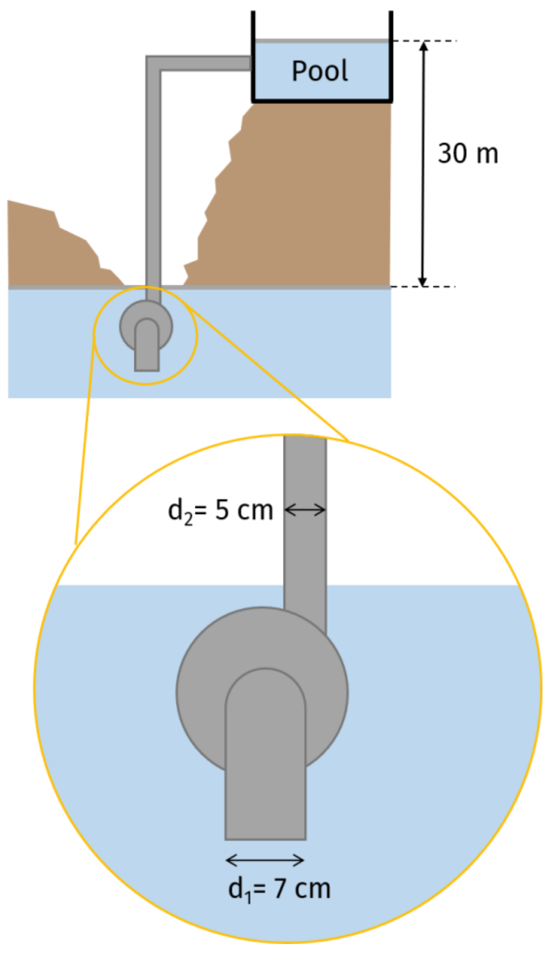
Question 1 – Filling a swimming pool (9 points)

A pool needs to be filled with underground water which is 30 m below the surface using a pump. The pump power is 3 kW and the efficiency is 70%. The diameter of the pipe is d_1 = 7 cm at the inlet of the pump and d_2 = 5 cm at the outlet of the pump. The irreversible head loss of the piping system is 5 m.

Determine:

a) The volumetric flow rate of water.
b) The pressure difference across the pump $\Delta P_{1,2}$.

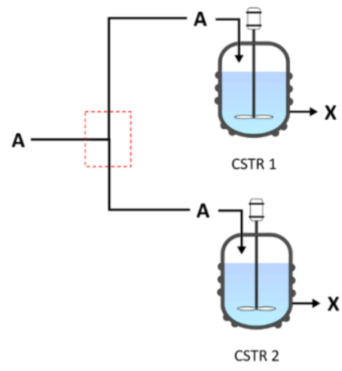
Assume the elevation difference between the pump inlet and the outlet to be negligible.



Question 2 – Holding a tee in place (12 points)

The chemical manufacturer for which you are working wants to increase the production volume of the molecule "X" produced by the dimerization of "A":

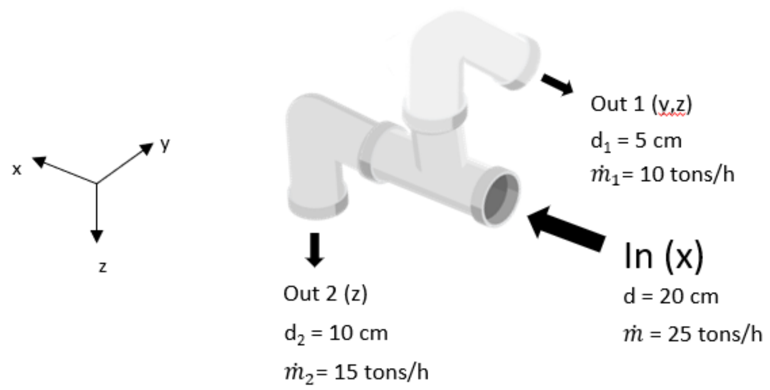
$$2A \rightarrow X$$



The reaction is currently realized in a single CSTR (continuous stirred-tank reactor). The company wants to install a brand new CSTR in parallel to increase the production of X as shown on the diagram on the right. In order to separate the flow of A, a complex tee must be installed at the location indicated by the red box.

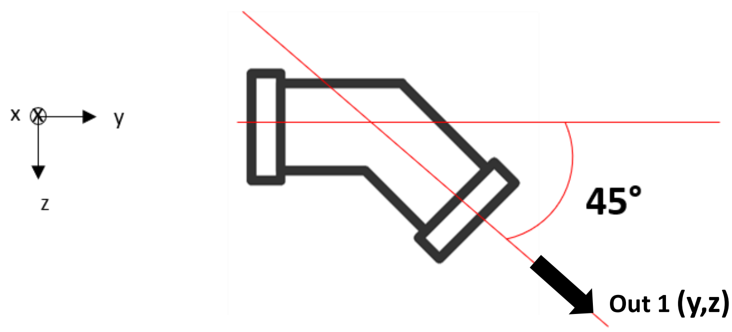
Your task is to calculate the value and direction of the anchoring force needed to hold the tee in place.

The tee design is given below:



The flow of A enters the tee in the **x-direction**. The fluid is then separated in an outlet stream 1, which exits in the **y- and z-directions**, and an outlet stream 2, which flows in the **z-direction**.

The projection of the outlet stream 1 in the y-z plane (i.e. seen in the **x-direction**) is given below:



25 tons/h of A enter the tee and are separated into 10 tons/h in the outlet 1 stream and 15 tons/h in the outlet 2 stream. The density of A is ρ = 950 kg/m^3 and it is an incompressible fluid. The pressure of the inlet stream in the tee is 3 bar (gauge value). You can neglect friction loss, elevation difference and gravity in the tee.

a) Calculate the fluid velocity in the inlet and the two outlet streams.
b) Calculate the pressure in the outlet 1 and outlet 2 streams.
c) Calculate the magnitude and the direction of the force needed to hold the complex tee in place and its modulus (i.e. $F_{rxn,x}$, $F_{rxn,y}$, $F_{rxn,z}$ and $F_{rxn,total}$).

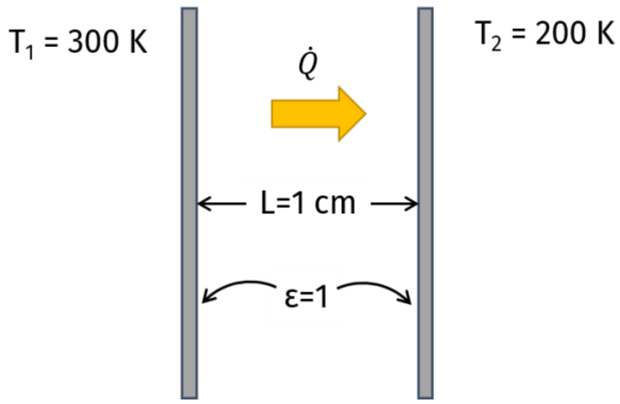
Question 3 – Heat transfer between two plates (7 points)

Consider steady state heat transfer between two large parallel plates with black surface (ε=1), which are each at constant temperature of T_1=300 K and T_2=200 K. The distance between the plates is L=1 cm, as shown in the figure. Determine the rate of heat transfer per unit surface area between the plates assuming the gap between the plates is:

a) Filled with atmospheric air (with no natural convection currents in the air between the plates)
b) Evacuated (vacuum)
c) Filled with opaque urethane insulation

The thermal conductivity at the average temperature of 250K is k=0.0219 W/mK for air and 0.026 W/mK for urethane insulation.

d) How would the heat transfer rate change in the presence of natural convection currents in the case (a)? Justify your answer.

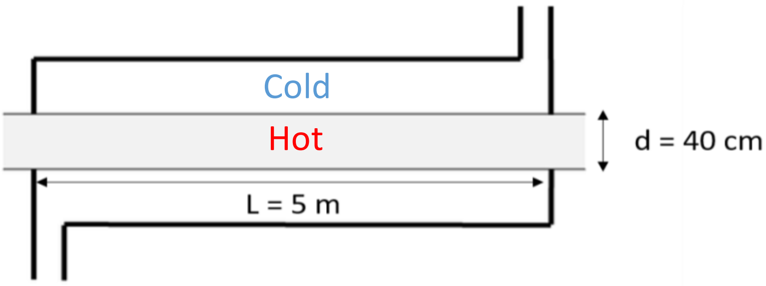


<u>Question 4 – Heat Exchanger Issues (12 points)</u>

A heat exchanger uses the hot stream from one reactor to pre-heat the feed of another reactor. The hot fluid is propylene glycol flowing at a rate of 1 kg/s. $T_{hot,in}$ is 65°C, $T_{hot,out}$ is 30°C. The cold fluid is octane (ρ = 703 kg/m^3). $T_{cold,in}$ is 0°C, $T_{cold,out}$ is 55°C .

The following specific heat capacities are given: $2.89\frac{J}{g \cdot K}$ for propylene glycol and $2.22\frac{J}{g \cdot K}$ for octane.

Assume steady-state and that all fluid properties are constant.



a) What is the volumetric flow rate of octane at the exit of the heat exchanger?

b) Based on the value of the outlet temperatures of the fluids, what is the operation mode of the heat-exchanger (co-current, counter current, can be both)? Justify your answer.

c) Calculate the overall heat transfer coefficient U.

d) An operator notices that the exit temperature of the octane stream is 48°C instead of 55°C. The operator quickly checks the other parameters, but sees that the flowrates of the hot and cold fluids are normal and that all the other temperatures are as usual. Calculate the percentage of performance loss of the heat exchanger (Hint: use the overall heat transfer coefficient). What could explain this loss of performance?

e) Another operator has mistakenly activated the wrong valve and is now feeding liquid pentane to the heat exchanger instead of liquid octane. Physical properties of liquid octane are T_{boil} = 125°C, T_{melt} = -57°C and of liquid pentane are: ρ = 703 kg/m^3, $C_p = 2.22\frac{J}{g \cdot K}$, T_{boil} = 36°C, T_{melt} = -130°C. What will happen and what should be done?

Introduction to Transport Phenomena: Mid-term Exam Solutions

Question 1 – Solutions (9 points)

a) The maximum flow rate of water (4.5 points)

Bernoulli equation in head terms to include energy gain from a pump and energy loss from friction (expressed in [m] terms):

$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + H_p = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g} + H_f$$

Where positions ① and ② are the free surface of the underground water reservoir and the pool respectively. Therefore, the following approximations can be made:

$v_1, v_2 \approx 0$ as velocities are negligible at the free surfaces

$P_1 = P_2 = P_{atm}$ as both surfaces are open to the atmosphere

$h_1 = 0$ as we take the free surface of the underground water reservoir as the reference level

Therefore:

$$H_p = h_2 + H_f$$

Using:

$$Power = Q\rho g H_p$$

$$Q = \frac{Power \cdot \eta}{(h_2 + H_f)\rho g} = \frac{3kW \cdot 0.7}{(30m + 5m) \cdot 9.8\frac{m}{s^2} \cdot 1000\frac{kg}{m^3}} = 6.12 \cdot 10^{-3}\frac{m^3}{s}$$

b) The pressure difference across the pump (4.5 points)

To calculate the difference in pressure between inlet and outlet of the pump, Bernoulli is applied between these two points, namely ③ and ④ respectively. The velocities at these points can be calculated from Q, as the volumetric flow rate has to be maintained through the system.

$$v_3 = \frac{Q}{A_3} = \frac{6.12 \cdot 10^{-3}\frac{m^3}{s}}{\frac{\pi}{4} \cdot (0.07m)^2} = 1.59\frac{m}{s}$$

$$v_4 = \frac{Q}{A_4} = \frac{6.12 \cdot 10^{-3}\frac{m^3}{s}}{\frac{\pi}{4} \cdot (0.05m)^2} = 3.11\frac{m}{s}$$

$$P_{pump} + P_3 + \frac{1}{2}\rho v_3^2 + \rho g h_3 = P_4 + \frac{1}{2}\rho v_4^2 + \rho g h_4$$

Because the height difference between inlet and outlet can be neglected:

$$P_3 - P_4 = \frac{1}{2}\rho(v_4^2 - v_3^2) - P_{pump} = \frac{1}{2}\rho(v_4^2 - v_3^2) - \frac{Power \cdot \eta}{Q}$$

$$P_3 - P_4 = \frac{1}{2}1000 \cdot ((3.11)^2 - (1.59)^2) - \frac{3000 \cdot 0.7}{6.12 \cdot 10^{-3}} = -339.57\ kPa$$

Question 2 – Solutions (12 points)

a) *Fluid velocities (2 points)*

Volumetric flow rate of A entering the tee:

$$Q_{in} = \frac{\dot{m}_{in}}{\rho} = \frac{6.95\ \left[\frac{kg}{s}\right]}{950\ \left[\frac{kg}{m^3}\right]} = 7.32 \cdot 10^{-3}\ \left[\frac{m^3}{s}\right]$$

The velocity in the inlet stream is therefore:

$$v_{in} = \frac{\dot{Q}_{in}}{A} = \frac{7.32 \cdot 10^{-3}\ \left[\frac{m^3}{s}\right]}{0.0314\ [\mathrm{m}^2]} = 0.23\ \left[\frac{m}{s}\right]\ where\ A = \frac{\pi d^2}{4}$$

The velocity in the outlet streams are:

$$v_{out1} = \frac{\dot{Q}_{out1}}{A} = \frac{2.78\ \left[\frac{kg}{s}\right]}{950\ \left[\frac{kg}{m^3}\right] \cdot 1.96 \cdot 10^{-3}\ [\mathrm{m}^2]} = 1.49\ \left[\frac{m}{s}\right]\ where\ A = \frac{\pi {d_1}^2}{4}$$

$$v_{out2} = \frac{\dot{Q}_{out2}}{A} = \frac{4.17\ \left[\frac{kg}{s}\right]}{950\ \left[\frac{kg}{m^3}\right] \cdot 7.85 \cdot 10^{-3}\ [\mathrm{m}^2]} = 0.56\ \left[\frac{m}{s}\right]\ where\ A = \frac{\pi {d_2}^2}{4}$$

b) *Pressure in the outlet 1 and outlet 2 streams (2 points)*

We can apply the Bernoulli equation between the inlet (point 1), the outlet 1 (point 2) and the outlet 2 (point 3) along streamlines.

Between 1 and 2:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

We neglect elevation difference so $h_1 = h_2$ and

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_{in}^2 - v_{out1}^2\right) = 298'971\ [Pa]$$

Between 1 and 3:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_3 + \rho g h_3 + \frac{1}{2}\rho v_3^2$$

We neglect gravity so $h_1 = h_3$ and

$$P_3 = P_1 + \frac{1}{2}\rho\left(v_{in}^2 - v_{out2}^2\right) = 299'876\ [Pa]$$

c) Reaction force (8 points)

Using the momentum balance equation to obtain the reaction force:

$$\sum F_{surface} + \sum F_{volume} = \sum_{i}^{N} \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i$$

$$where \sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

Volume and friction are neglected:

$$\sum F_{volume} = 0$$

$$\sum F_{friction} = 0$$

$$\sum_{i}^{N} F_{pressure} = -P_i . A_i . \widehat{n_i}$$

Therefore, the equation to solve is:

$$-P_i . A_i . \widehat{n_i} + \sum F_{reaction} = \sum_{i}^{N} \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i$$

Let's consider the inlet (point 1), the outlet 1 (point 2) and the outlet 2 (point 3).

<u>Solving for x:</u>

$$F_{reaction,x} - P_1 . A_1 . \widehat{n_1} = \rho \bar{v}_1 (\bar{v}_1 \cdot \widehat{n_1}) \cdot A_1$$

$$F_{reaction,x} + P_1 A_1 = -\rho v_1^2 A_1$$

$$F_{reaction,x} + 3 \cdot 10^5 \ (0.0314) = -950 \cdot (0.23^2)(0.0314)$$

$$F_{reaction,x} = -9420 - 1.6 = -9421.6 \ N$$

<u>Solving for y:</u>

$$F_{reaction,y} - P_2 . A_2 . \widehat{n_2} = \rho \bar{v}_2 (\bar{v}_2 \cdot \widehat{n_2}) \cdot A_2$$

$$F_{reaction,y} - P_2 A_2 \cos(45) = \rho v_2^2 A_2 \cos(45)$$

$$F_{reaction,y} - 298'971(1.96 \cdot 10^{-3})0.707 = 950 \cdot (1.49^2)(1.96 \cdot 10^{-3})0.707$$

$$F_{reaction,y} = 414 + 3 = 417 \ N$$

<u>Solving for z:</u>

$$F_{reaction,z} - P_2.A_2.\widehat{n_2} - P_3.A_3.\widehat{n_3} = \rho\bar{v}_2(\bar{v}_2 \cdot \widehat{n_2})A_2 + \rho\bar{v}_3(\bar{v}_3 \cdot \widehat{n_3}) \cdot A_3$$

$$F_{reaction,z} - P_2A_2\cos(45) - P_3.A_3 = \rho v_2^2 A_2 \cos(45) + \rho v_3^2 A_3$$

$$F_{reaction,z} - 298'971(1.96 \cdot 10^{-3})0.707 - 299'876(7.85 \cdot 10^{-3}) = 950(1.49^2)(1.96 \cdot 10^{-3})0.707 + 950(0.56^2)(7.85 \cdot 10^{-3})$$

$$F_{reaction,z} = 414 + 2354 + 3 + 2.3 = 2773.3\ N$$

Modulus of the force:

$$F_r = \sqrt{F_{r,x}^2 + F_{r,y}^2 + F_{r,z}^2} = 9830\ N$$

<u>Question 3 – Solutions (7 points)</u>

a) *Filled with atmospheric air (2 points)*

If between the two plates there is still air, conduction and radiation are going to be the means for heat transfer.

$$\dot{Q}_{cond} = \frac{k_{air} \cdot A \cdot (T_1 - T_2)}{Y} = \frac{0.0219 \frac{W}{K \cdot m} \cdot (1\ m^2) \cdot (300 - 200)K}{0.01\ m} = 219\ W$$

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_1^4 - T_2^4) = 1 \cdot \left(5.67 \cdot 10^{-8} \frac{W}{m^2K^4}\right) \cdot (1\ m^2) \cdot (300^4 - 200^4)K^4 = 368\ W$$

$$\dot{Q}_{total} = \dot{Q}_{cond} + \dot{Q}_{rad} = 587\ W$$

b) *Evacuated (vacuum) (2 points)*

As conduction and convection require matter for heat transfer, if the space is evacuated there is only going to be radiation.

$$\dot{Q}_{rad} = \varepsilon\sigma A(T_1^4 - T_2^4) = 1 \cdot \left(5.67 \cdot 10^{-8} \frac{W}{m^2K^4}\right) \cdot (1\ m^2) \cdot (300^4 - 200^4)K^4 = 368\ W$$

c) *Filled with urethane insulation (2 points)*

Opaque insulation blocks the radiation, as only transparent or semitransparent solids are able to transmit radiation.

$$\dot{Q}_{cond} = \frac{k_{urethane} \cdot A \cdot (T_1 - T_2)}{Y} = \frac{0.026 \frac{W}{K \cdot m} \cdot (1\ m^2) \cdot (300 - 200)K}{0.01\ m} = 260\ W$$

d) *Convection currents (1 point)*

The heat transfer would be higher if there are natural convection currents between the plates, as the convection currents will have the same direction as conduction.

<u>Question 4 – Solutions (12 points)</u>

a) *Volumetric flow rate (3 points)*

$$\dot{m}_h = 1\left[\frac{kg}{s}\right]$$

$$c_{p,h} = 2890\ \left[\frac{J}{kg\cdot K}\right]$$

$$\dot{Q}_h = \left[\dot{m}c_p(T_{in} - T_{out})\right]_h = 1\left[\frac{kg}{s}\right]2890\left[\frac{J}{kg\cdot K}\right](65°C - 30°C) = 101.15\ [kW]$$

No heat loss → $\dot{Q}_h = -\dot{Q}_c$

$$\dot{Q}_c = -\dot{Q}_h = -\ 101.15\ kW$$

$$-\ 101.15\ kW = \left[\dot{m}c_p(T_{in} - T_{out})\right]_c = \dot{m}_c\cdot 2220\left[\frac{J}{kg\cdot K}\right](0°C - 55°C)$$

$$\dot{m}_c = \frac{-\ 101.15\ [kW]}{-55[K]\cdot 2220\left[\frac{J}{kg\cdot K}\right]} = 0.83\left[\frac{kg}{s}\right]$$

$$\dot{V}_c = \frac{0.83\left[\frac{kg}{s}\right]}{703\left[\frac{kg}{m^3}\right]} = 1.18\cdot 10^{-3}\left[\frac{m^3}{s}\right] = 1.18\left[\frac{L}{s}\right]$$

b) *Co-current, counter current or both? (1.5 point)*

Since $T_{c,out} > T_{h,out}$ the heat exchanger is necessarily counter current.

c) *Overall heat transfer coefficient (3 points)*

$$\Delta T_1 = T_{h,in} - T_{c,out} = 65°C - 55°C = 10°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°C - 0°C = 30°C$$

$$\Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = 18.2\ [K]$$

The exchange area is the outer surface of the tube in which the hot stream is flowing.

$$A = \pi\cdot d\cdot L = 3.14\cdot 0.4[m]\cdot 5[m] = 6.28\ [m^2]$$

$$U_{th} = \frac{\dot{Q}}{A\Delta T_{lm}} = \frac{101.15\ [\mathrm{kW}]}{6.28\ [m^2]\cdot 18.2[K]} = 885\left[\frac{W}{m^2\cdot K}\right]$$

d) *Loss of performance (3 points)*

The loss of performance is caused by fouling, which adds an additional resistance to the heat transfer in the heat exchanger.

$$\Delta T_1 = T_{h,in} - T_{c,out} = 65°C - 48°C = 17°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°C - 0°C = 30°C$$

$$\Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = 22.9\ [K]$$

The new overall heat transfer coefficient is:

$$U = \frac{\dot{Q}}{A\Delta T_{lm}} = \frac{101.15\ [\text{kW}]}{6.28\ [m^2] \cdot 22.9[K]} = 703\ \left[\frac{W}{m^2 \cdot K}\right]$$

$$\frac{U}{U_{th}} = \frac{703\ \left[\frac{W}{m^2 \cdot K}\right]}{885\ \left[\frac{W}{m^2 \cdot K}\right]} = 79.4\%$$

The heat exchanger works at 79.4% of its theoretical performance or, in other words, the loss of performance is 20.6%.

e) Liquid pentane (1.5 points)

Since C_p and ρ are similar, the final temperature of pentane is expected to be the same as octane: 48°C (or 55°C if we use the theoretical performance, it does not matter here). This temperature is higher than the boiling point of pentane. Therefore, some heat will be used to evaporate pentane and there will be pressure built up in the heat exchanger. The operation of the heat exchanger should be stopped to avoid overpressure, which could lead to damage or an explosion.