
Introduction to Transport Phenomena: Mid-term Exam

Question 1 – Filling a tank from a river (10 points)

The water from a river is flowing into a cylindric tank through a smooth short pipe. Calculate the time t that takes to fill the tank up to $h = H$.

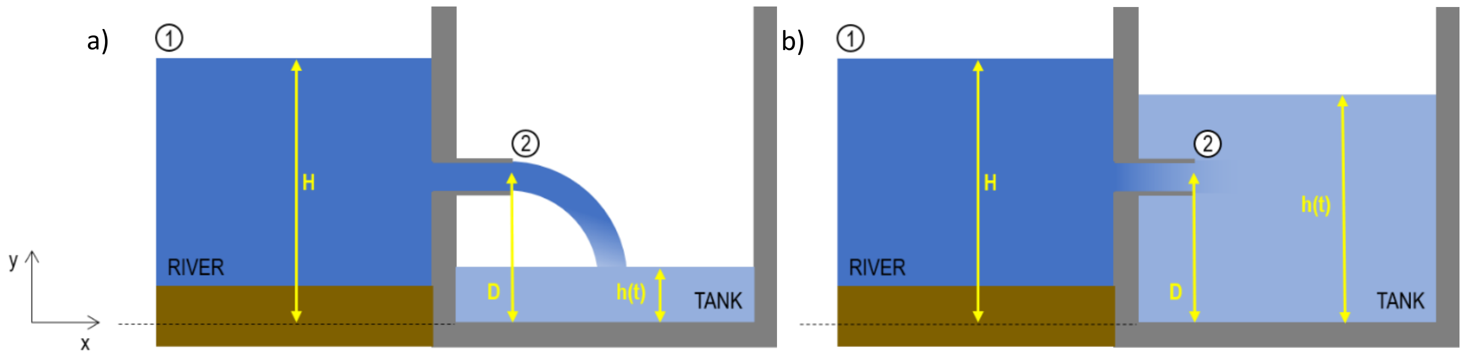
Tip: When $h > D$, the pressure at the exit of the short pipe is the hydrostatic pressure, which is equal to $\rho g h(t)$.

$A_{tank} = 1000\ m^2$ (surface area of the tank)

$D = 20\ m$

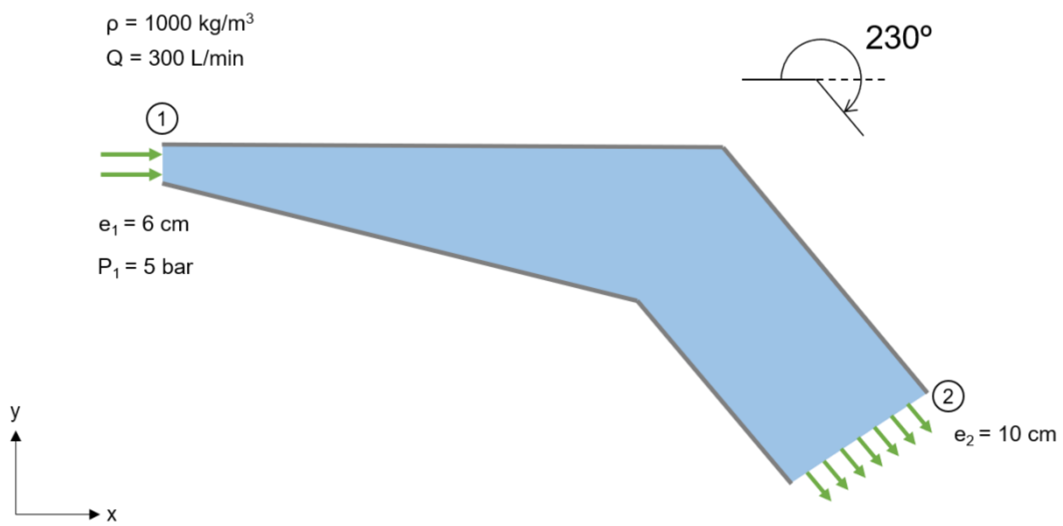
$H = 30\ m$

$A_{pipe} = 0.05\ m^2$ (cross-sectional area of the short pipe)



Question 2 – Holding an elbow in place (10 points)

An elbow placed flat on the floor is used to direct water which is flowing at a volumetric rate of 300 L/min. The water jet, square in section, changes from 6 cm edge in point 1 to 10 cm edge in point 2. Determine the absolute pressure at the outlet of the elbow and the value and direction of the anchoring force needed to hold the elbow in place.



Question 3 – Iced water storage (11 points)

A spherical tank of stainless steel ($k = 15 \frac{W}{m \cdot °C}$) is used to store iced water at $T_{\infty 1} = 0°C$. The tank is in a room whose temperature is $T_{\infty 2} = 22°C$. The heat transfer between the outer surface of the tank and the room occurs by natural convection *and* radiation. The convective heat transfer coefficients inside and outside the tank are $h_1 = 80 \frac{W}{m^2 \cdot °C}$ and $h_2 = 10 \frac{W}{m^2 \cdot °C}$, respectively.

a) Draw the thermal circuit which represents all the heat transfer resistances in the system.

Please label the resistances and label the diagram with the temperatures $T_{\infty 1}, T_1, T_2$, and $T_{\infty 2}$ at the appropriate locations. Note T_1 and T_2 are the temperatures on the inner surface of the wall and the outer surface of the wall, respectively.

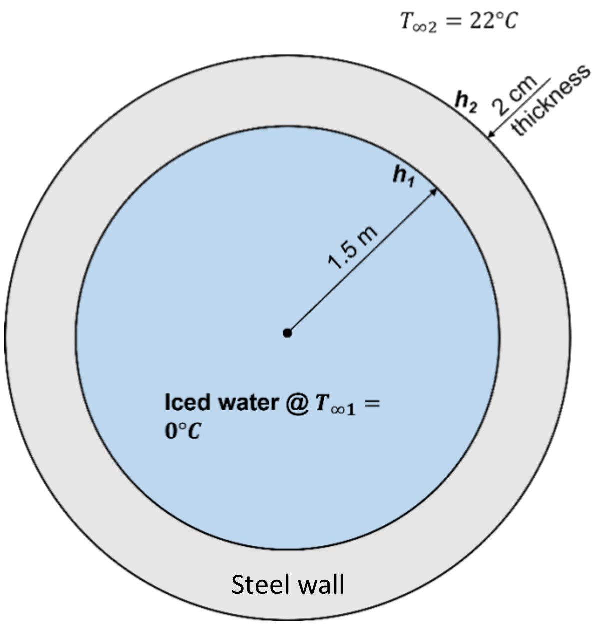
b) Calculate the areas for heat transfer on the inside and outside of the tank and determine the radiative heat transfer coefficient, h_{rad}.

Tip: $h_{rad} = \epsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$ where $\epsilon = \text{emissivity} = 1$, $\sigma = \text{Stefan} - \text{Boltzmann constant} = 5.67 * 10^{-8}\ W/m^2 \cdot K^4$. Assume a reasonable temperature for T_2 (e.g $5°C$), the temperature on the outer surface of the tank, as the actual temperature is unknown.

c) Calculate all the thermal resistances.

d) Determine the rate of heat transfer, *Q*, to the iced water in the tank.

e) Was the assumption for T_2 reasonable or not? Show with a calculation, not just words.



Question 4 – Heat exchanger qualification (9 points)

You are working for a chemical engineering firm and a client of yours is interested in replacing their current heat exchanger. But to choose a new one, they first must understand the properties of the one they are currently using… so they call you!

The diagram below represents the current heat exchanger of your client. The inlet and outlet temperatures and the volume flow rates of the hot and cold fluids are known. Assume steady-state and that all fluid properties are constant.

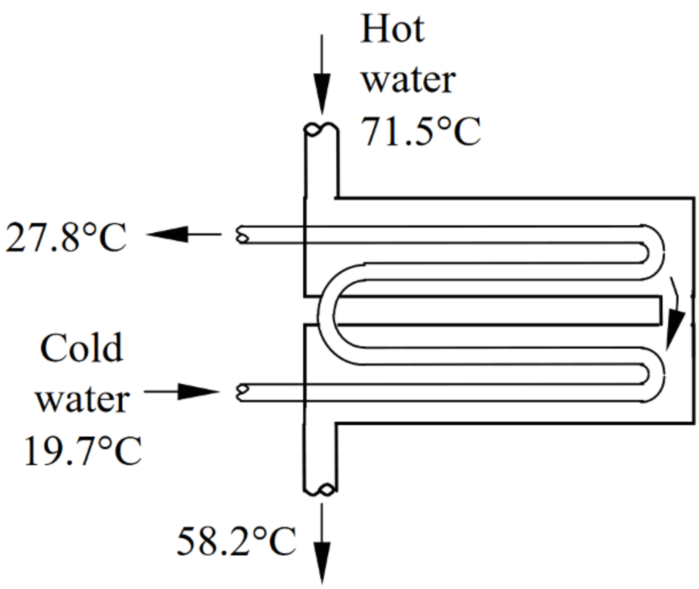
The flow rate of the hot and cold streams is 1.05 L/min and 1.55 L/min, respectively. The densities of hot and cold water are 980.5 and 997.3 kg/m^3, respectively. The specific heat is $4187 \frac{J}{kg{\cdot}°C}$ for the hot-water stream and $4180 \frac{J}{kg{\cdot}°C}$ for the cold-water stream. The cold pipe has an outer diameter of 5 cm and length of 10.19 m.

a) Calculate the mass flow rates for each stream and the rates of heat transfer from the hot water ($\dot{Q}_h$) and to the cold water ($\dot{Q}_c$). Note that the wall of the hot stream is not perfectly insulated.

b) Calculate the logarithmic mean temperature difference and the overall heat transfer coefficient.

c) Calculate the fraction of heat lost to the surroundings from the hot stream.

d) Your client would like to cool the hot stream five times faster (i.e. $5 * \dot{Q}_h$). Calculate the necessary heat transfer area and the cost of such a heat exchanger. Assume the same fraction of heat loss to the environment as calculated in part (b) when determining the necessary area. Also assume that the inlet and outlet temperatures and the overall heat transfer coefficient is the same. You have stainless steel tubing available with an outer diameter of 5 cm and it costs 8 CHF/m which will have its outer surface serve as the heat transfer area.



Solutions

Question 1. (10 points)

The water from a river is flowing into a cylindric tank through a smooth short pipe. Calculate the time *t* that takes to fill the tank up to h=H

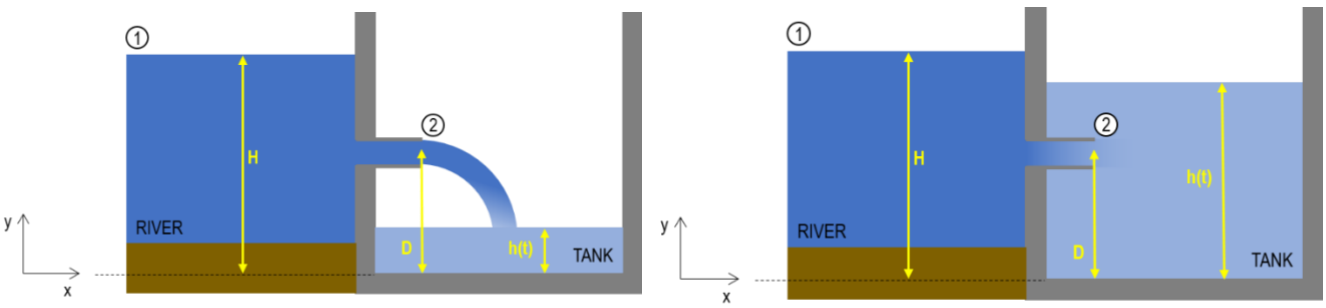
Tip: When h > D, the pressure at the exit of the short pipe is the hydrostatic pressure, which is equal to $\rho g h(t)$

$A_{tank} = 1000\ m^2$(surface area of the tank)

$$D = 20\ m$$

$$H = 30\ m$$

$A_{pipe} = 0.05\ m^2$ (cross-sectional area of the short pipe)



Solution.

We approach this problem by dividing it into two parts. First, we calculate the time needed for to fill the tank up to D and then from D to H.

First, let's calculate the time it takes to fill the tank up to the pipe:

$$time = \frac{volume}{Q} = \frac{A_{tank} \times D}{A_{pipe} \times v_2}$$

We need to calculate v_2, so we apply Bernoulli's theorem between points 1 and 2:

$$\rho g h_1 + P_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + P_2 + \frac{1}{2}\rho v_2^2 \qquad (1)$$

At this point, we have that:

- h_1 is H.
- h_2 is D.
- $P_{1,gauge} = P_{2,gauge} = 0\ atm$
- The level of water in the river is nominally not going to be affected by the filling of the tank, so we have that $v_1 \to 0$. Another way to see it is through the continuity equation: as the

surface area of the river (A_1) is going to be extremely bigger than the area of the pipe (A_{pipe}), v_1 will tend to be so small that it can be neglected.

Therefore, if substituted in (1), we obtain:

$$gH = gD + \frac{1}{2}v_2^2$$

$$v_2 = \sqrt{2g(H-D)} = \sqrt{2 \times 9.8 \times (30-20)} = 14\, m/s$$

$$time = \frac{A_{tank} \times D}{A_{pipe} \times v_2} = \frac{1000 \times 20}{0.05 \times 14} = 28571\ seconds \approx 8\ hours$$

Once the pipe is submerged in the water, we have to account that P_2 is no longer atmospheric pressure, but that it is under hydrostatic pressure. The hydrostatic pressure is going to increase as the water level raises in the tank, so it will be time dependent:

$$P_2 = \rho g(h(t) - D)$$

We can again describe the Bernoulli equation between points 1 and 2:

$$\rho g h_1 + P_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + P_2 + \frac{1}{2}\rho v_2^2$$

Now we have that:

- h_1 is H.
- h_2 is D.
- $P_{gauge} = P_1 = 0\ atm$
- $v_1 \to 0.$
- $P_2 = \rho g(h(t) - D)$

$$gH = gD + g(h(t) - D) + \frac{1}{2}v_2^2$$

$$v_2^2 = 2 \times (gH - gD - gh(t) + gD)$$

$$v_2 = \sqrt{2g(H - h(t))}$$

The *volumetric flow rate* must be equal to the *rate of water volume increase in the tank*:

$$v_2 \times A_{pipe} = A_{tank} \times \frac{dh}{dt}$$

$$dt = \frac{A_{tank}}{A_{pipe}}\frac{dh}{v_2} = \frac{A_{tank}}{A_{pipe}}\frac{dh}{\sqrt{2g(H-h(t))}}$$

Boundaries:

time: 0 ⍰ t

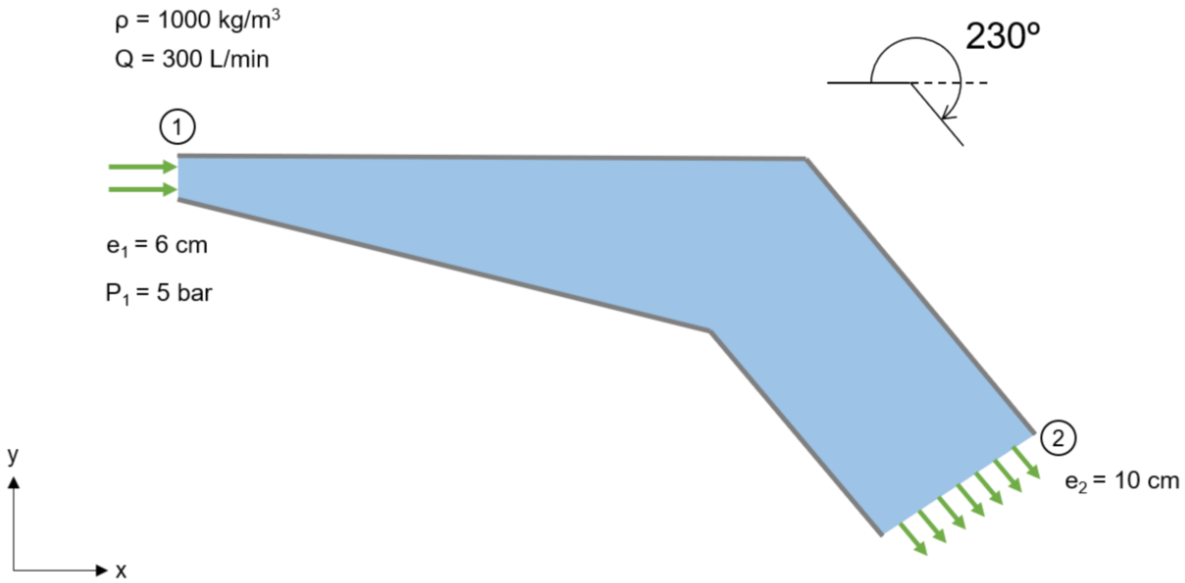
height: D ⍰ H

$$\int_0^t dt = \frac{A_{tank}}{A_2}\int_D^H \frac{dh}{\sqrt{2g(H-h)}}$$

$$t = \frac{A_{tank}}{A_{pipe}}\frac{\sqrt{2g(H-D)}}{g} = \frac{1000}{0.05}\frac{\sqrt{2\times 9.8\times(30-20)}}{9.8} = 89443\ seconds \approx 24.8\ hours$$

Question 2. (10 points)

An elbow placed flat on the floor is used to direct water which is flowing at a volumetric rate of 300 L/min. The water jet, square in section, changes from 6 cm edge in point 1 to 10 cm edge in point 2. Determine the absolute pressure at the outlet of the elbow and the value and direction of the anchoring force needed to hold the elbow in place. The elbow is



Solution.

Because of the continuity equation, the velocities at the inlet and outlet of the elbow can be calculated by imposing:

$$Q = A_1 v_1 = A_2 v_2 = 300\ \frac{L}{min}\cdot\frac{1\ m^3}{1000\ L}\cdot\frac{1\ min}{60\ s} = 5\cdot 10^{-3}\frac{m^3}{s}$$

$$A_1 = (0.06)^2\ m^2\ \rightarrow\ v_1 = 1.39\ m/s$$

$$A_2 = (0.1)^2\ m^2\ \rightarrow\ v_2 = 0.5\ m/s$$

Apply Bernoulli between point 1 and 2 to obtain the pressure at point 2:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = 5\cdot 10^5\ Pa + \frac{1}{2}\cdot 1000\frac{kg}{m^3}\cdot(1.39^2 - 0.5^2)\frac{m^2}{s^2} = 500841\ Pa$$

Using the momentum balance equation to obtain the reaction force:

$$\sum F_{surface} + \sum F_{volume} = \sum_i^N \int_{A_i} \rho \underline{v}(\underline{v} \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

Volume and friction are neglected:

$$\sum F_{volume} = 0$$

$$\sum F_{friction} = 0$$

$$\sum_i^N F_{pressure} = -P_i . A_i . \hat{n_i}$$

Solving for x:

$$F_{reaction-x} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \underline{v}_1 (\underline{v}_1 \cdot \widehat{n_1}) A_1 + \rho \underline{v}_2 (\underline{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-x} + P_1 A_1 - P_2 A_2 \cos\cos(50) = -\rho v_1^2 A_1 + \rho v_2^2 A_2 \cos(50)$$

$$F_{reaction-x} + 5 \cdot 10^5 (0.06)^2 - 500841 (0.1)^2 \cos\cos(50) = -1000 \cdot (1.39^2)(0.06)^2 + 1000 \cdot (0.5^2)(0.1)^2 \cos\cos(50)$$

$$F_{reaction-x} = -1800 + 3219.3 - 6.9 + 1.6 = 1414.0\, N$$

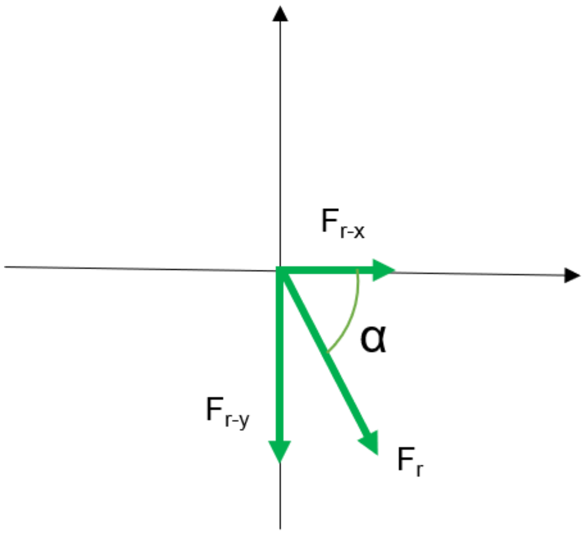
Solving for y:

$$F_{reaction-y} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \underline{v}_1 (\underline{v}_1 \cdot \widehat{n_1}) A_1 + \rho \underline{v}_2 (\underline{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-y} + P_2 A_2 \cos(40) = -\rho v_2^2 A_2 \cos(40)$$

$$F_{reaction-y} + 500841 \cdot (0.1)^2 \cos(40) = -1000(0.5^2)(0.1)^2 \cos\cos(40)$$

$$F_{reaction-y} = -3836.7 - 1.9 = -3836.6\, N$$



$$F_r = \sqrt{F_{r-x}^2 + F_{r-y}^2} = 4088.9\ N$$

$$arctg\ \alpha = \frac{F_{r-y}}{F_{r-x}} = 69.8°$$

Question 3 – Iced water storage (11 points)

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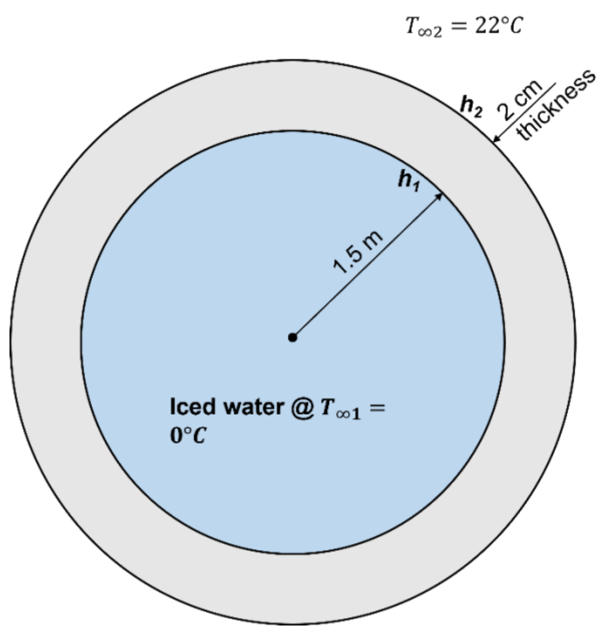
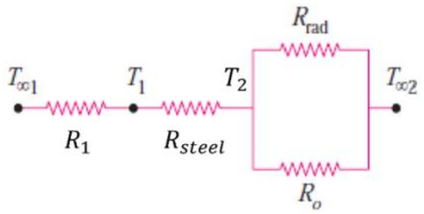


Question 3 – Solution

a)



b)

$$A_1 = \pi D_1^2 = \pi 3^2 = 28.3\ m^2$$

$$A_2 = \pi D_2^2 = \pi 3.04^2 = 29.0\ m^2$$

$$h_{rad} = \epsilon\sigma(T_2^2 + T_{\infty 2}^2)(T_2 + T_{\infty 2})$$

As suggested in the problem statement, we will make a guess regarding T_2. Let's assume is $5°C$

$$h_{rad} = 1 * 5.67 * \frac{10^{-8}W}{m^2 \cdot K^4} * ((295K)^2 + (278K)^2) * (295K + 278K) = 5.34\ W/(m^2 \cdot °K)$$

c)

$$R_{conv,in} = \frac{1}{h_1 A_1} = \frac{1}{(80W/(m^2 \cdot °C))(28.3m^2)} = 0.000442°C/W$$

$$R_{steel} = \frac{r_2 - r_1}{4\pi k r_1^2} = \frac{1.52\ m - 1.50\ m}{4\pi\left(\frac{15W}{m \cdot °C}\right)(1.50m)^2} = 0.000047°C/W$$

$$R_{conv,out} = \frac{1}{h_2 A_2} = \frac{1}{\left(\frac{10W}{m^2 \cdot °C}\right)(29.0m^2)} = 0.00345°C/W$$

$$R_{rad} = \frac{1}{h_{rad} A_2} = \frac{1}{\left(\frac{5.34W}{m^2 \cdot °C}\right)(29.0m^2)} = 0.00646°C/W$$

If we wish, we can combine the two parallel resistances of $R_{conv,out}$ and R_{rad} into one R_{out}

$$1/R_{out} = \frac{1}{R_{conv,out}} + \frac{1}{R_{rad}} = \frac{1}{0.00345} + \frac{1}{0.00646} = 444.7\ W/°C$$

$$R_{out} = 0.00225°C/W$$

d)

$$Q = \frac{T_{\infty 2} - T_{\infty 1}}{R_{total}} = \frac{(22-0)°C}{\frac{0.00274°C}{W}} = 8029\ W$$

e) To check our assumption, we can determine the outer surface temperature of the tank using our calculated Q:

$$Q = \frac{T_{\infty 2} - T_2}{R_{out}} \rightarrow T_2 = T_{\infty 2} - Q * R_{out} = 4°C$$

Yes, our assumption was okay.

Question 4 – Heat exchanger qualification (9 points)

You are working for a chemical engineering firm and a client of yours is interested in replacing their current heat exchanger. But to choose a new one, they first must understand the properties of the one they are currently using… so they call you!

The diagram below represents the current heat exchanger of your client. The inlet and outlet temperatures and the volume flow rates of the hot and cold fluids are known. Assume steady-state and that all fluid properties are constant.

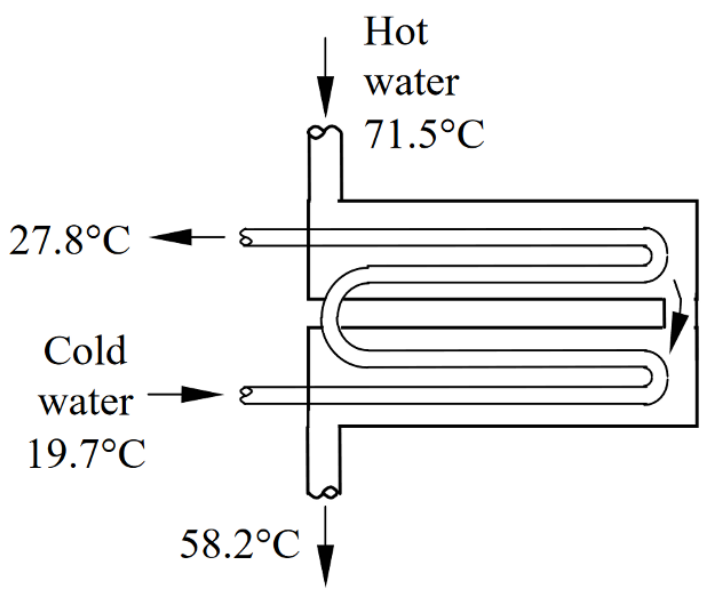
The flow rate of the hot and cold streams is 1.05 L/min and 1.55 L/min, respectively. The densities of hot and cold water are 980.5 and 997.3 kg/m^3, respectively. The specific heat is 4187 $\frac{J}{kg\cdot°C}$ for the hot-water stream and 4180 $\frac{J}{kg\cdot°C}$ for the cold-water stream. The cold pipe has an outer diameter of 5 cm and length of 10.19 m.

a) Calculate the mass flow rates for each stream and the rates of heat transfer from the hot water ($\dot{Q}_h$) and to the cold water ($\dot{Q}_c$). Note that the wall of the hot stream is not perfectly insulated.

b) Calculate the logarithmic mean temperature difference and the overall heat transfer coefficient.

c) Calculate the fraction of heat lost to the surroundings from the hot stream.

d) Your client would like to cool the hot stream five times faster (i.e. $5 * \dot{Q}_h$). Calculate the necessary heat transfer area and the cost of such a heat exchanger. Assume the same fraction of heat loss to the environment as calculated in part (b) when determining the necessary area. Also assume that the inlet and outlet temperatures and the overall heat transfer coefficient is the same. You have stainless steel tubing available with an outer diameter of 5 cm and it costs 8 CHF/m which will have its outer surface serve as the heat transfer area.



Question 4 – Solution

a)

$$\dot{m}_h = \rho_h \dot{V}_h = \frac{980.5kg}{m^3} * \frac{0.00105m^3}{60s} = 0.0172\,\frac{kg}{s}$$

$$\dot{m}_c = \rho_c \dot{V}_c = \frac{997.3kg}{m^3} * \frac{0.00155m^3}{60s} = 0.0258\,\frac{kg}{s}$$

$$\dot{Q}_h = \left[\dot{m}c_p(T_{in} - T_{out})\right]_h = \left(\frac{0.0172kg}{s}\right)\left(\frac{4187kJ}{kg\cdot°C}\right)(71.5°C - 58.2°C) = 957.8\,W$$

$$\dot{Q}_c = \left[\dot{m}c_p(T_{in} - T_{out})\right]_c = \left(\frac{0.0258kg}{s}\right)\left(\frac{4180kJ}{kg\cdot°C}\right)(27.8°C - 19.2°C) = 873.5\,W$$

b)

$$\Delta T_1 = T_{h,in} - T_{c,out} = 71.5°C - 27.8°C = 43.7°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 58.2°C - 19.7°C = 38.5°C$$

$$\Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{ln ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = 41°C$$

Area can be calculated from the length and outer diameter of the cold pipe given.

$$U = \frac{\dot{Q}_{hc,m}}{A\Delta T_{lm}} = \frac{\left(\left(\frac{957.8 + 873.5}{2}\right)W\right)}{1.6m^2 * 41°C} = 13.96\,\frac{W}{m^2\cdot°C}$$

c)

$$f_{loss} = \frac{\dot{Q}_h - \dot{Q}_c}{\dot{Q}_h} = \frac{957.8 - 873.5}{957.8} = 0.88 = 8.8\%$$

d)

$$\dot{Q}_h * 5 = 4789\,W \rightarrow 4789\,W * 91.2\%$$
$$= 4368\,W\ heat\ flow\ via\ heat\ exchanger\ in\ both\ directions$$

$$A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{4368W}{1117\left(\frac{W}{m^2°C}\right)*41°C} = 0.095\,m^2$$

Length of pipe necessary is

$$L = \frac{0.095m^2}{2 * \pi * 0.025} = 0.61\,m$$

$$0.61\,m * 8\,\frac{CHF}{m} = 4.86\,CHF$$