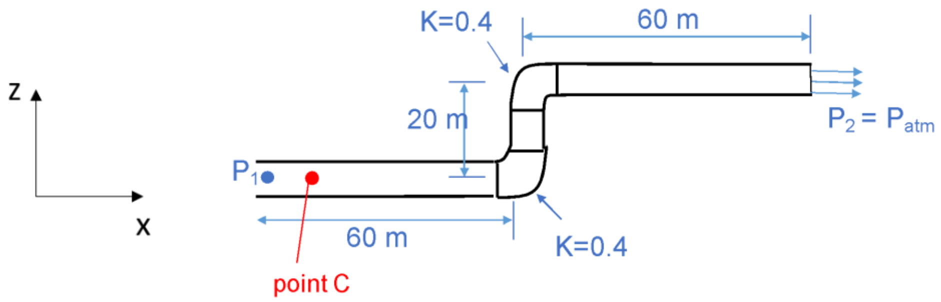
Introduction to Transport Phenomena: Mid-term Exam

Question 1 – Pipe system

A pipe system rises the level of water before discharging it as a free jet in the atmosphere, as shown in the following figure.

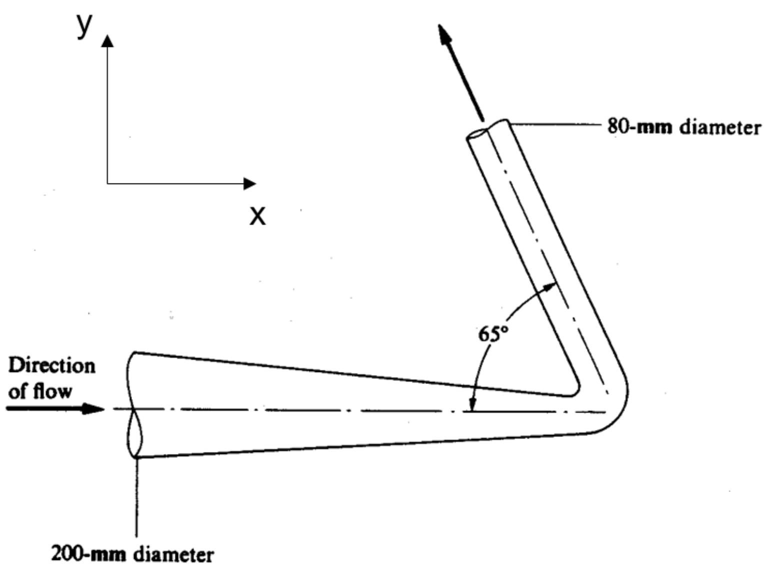


a) What gauge pressure P_1 would be needed to provide a volumetric flow rate of 12 m^3/min of water? Assume a hydraulically smooth pipe and include all the losses.
b) If the frictional losses inside the system were supposed to be compensated with a pump located at point C, what will be the required power of the pump?

Consider water to have a density of $1000\ kg/m^3$ and a dynamic viscosity of $8.9 \cdot 10^{-4}\ Pa \cdot s$, and gravity to be $9.8\ m/s^2$. The diameter of the pipe is 20 cm.

Question 2 – Holding a pipe in place

The reducing pipe bend depicted here is in a horizontal plane. Water enters the bend with a velocity of 3.5 m/s and a pressure of 280 kPa. Neglecting any energy losses in the bend, find the module and angle of the force required to hold the bend in place. $\rho = 1000\ kg/m^3$.



Question 3 - New Graphics Card

You would like to upgrade the graphics card (GPU) in your desktop computer. With your current setup, your GPU is contacted to a layer of aluminum with a layer of copper on top to conduct heat away as shown in the figure. The thermal conductivities of aluminum and copper are 206 and 385 W/m·K respectively. Do not consider convection within the GPU.

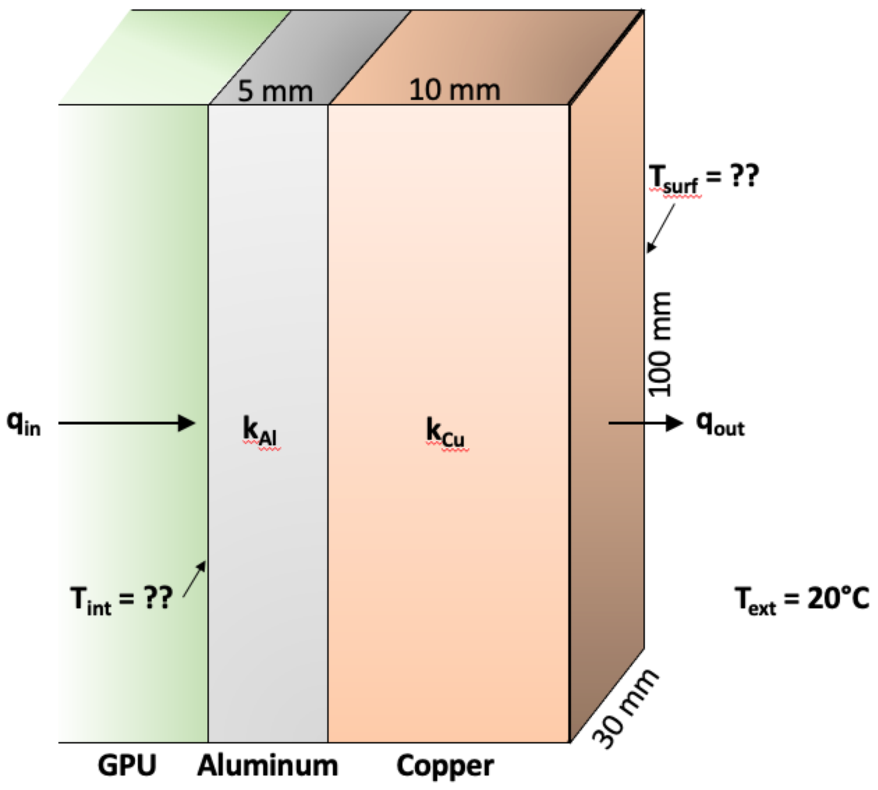
a) Draw the thermal circuit and the temperature profile of the system.

b) Knowing your current GPU produces 4 W in the form of heat, and assuming the fluid cooling the copper surface is air in natural convection, calculate T_{int} and the temperature at the interface between the aluminum and copper.

c) Using the natural convection of air to cool the copper surface, what is the maximum heat that your GPU can output with T_{int} equal to 85°C?

d) The GPU you would like to buy produces 50 W of heat (must be quite an upgrade!). Will it be okay if you change your cooling system to using a fan (forced convection)? And what about water cooling?

Heat transfer coefficients: Natural convection of air h = 25 W/m^2 ·K. Forced convection of air h = 250 W/m^2 ·K. Forced convection of water h = 900 W/m^2 ·K.



Question 4 – Opening a brewery

You and your colleagues decide it would be useful to make some money while continuing your studies and that you have already learned enough to open a new brewery. Of course, when it comes to fermenting your beer, it is necessary that you add the yeast at the proper temperature, so it does not die. Thus, you need to design a **counterflow** heat exchanger to cool the hot wort (unfermented beer) leaving the mash tun to 60°C ($T_{wort,out}$), the maximum temperature your yeast strain can survive. Assume that the wort has the same physical properties as water, i.e. $c_p = 4180\ J/kg$ and $\rho = 997\ kg/m^3$.

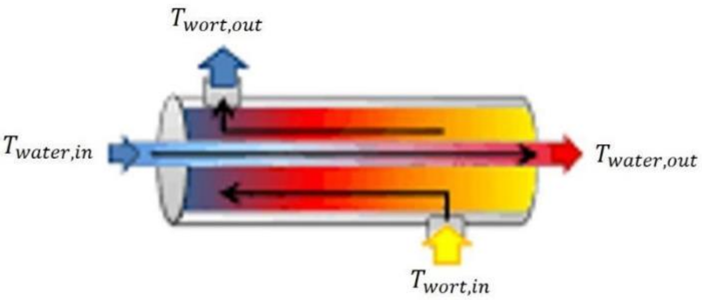
a) First, determine the heat transfer rate knowing the mass flow rate of the wort, $\dot{m} = 2\ kg/s$, and that the wort leaving the mash tun is 100 °C ($T_{wort,in}$).

b) You have ground water available at 20 °C for cooling. With a flow rate of $\dot{m} = 1.5\ kg/s$, at what temperature does it exit the heat exchanger?

c) Draw the temperature profile of the heat exchanger.

d) Assuming an overall heat transfer coefficient of $U = 650\ W/m^2 \cdot K$, what is the necessary heat transfer area?

e) If you have 5 cm diameter tubing available for the water stream as depicted in the figure, what length of tubing will you need for your heat exchanger?



Introduction to Transport Phenomena: Mid-term Exam solutions

Question 1 (10 points)

a)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Applying mass conservation at point 1 and point 2,

$$\rho v_1 A_1 = \rho v_2 A_2 = \dot{m}$$

Since $A_1 = A_2$, we get $v_1 = v_2 = v$ (because we are considering a steady state system)

$$P_1 + \rho g h_1 + \cancel{\frac{1}{2}\rho v_1^2} = P_2 + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \Delta P_f$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v * D}{\mu}$$

The volumetric flow rate, $\dot{v} = \frac{\dot{m}}{\rho}$, thus

$$v = \frac{\dot{v}}{\rho A} = \frac{\frac{12}{60}\ (\frac{m^3}{s})}{\frac{\pi(0.2)^2\ (m^2)}{4}} = 6.36\ m/s$$

$$Re = \frac{1000\,\frac{kg}{m^3} * 6.366\,\frac{m}{s} * 0.2m}{8.9 * 10^{-4} Pas} = 1.431\ * 10^6$$

Friction factor from Moody diagram:

$$f_f = 0.0025$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v^2 \left((\frac{4f_f}{D}\sum_{i=1}^{3} \square L_i) + \sum_{j=1}^{2} \square K_j \right)$$

Loss coefficients:

$$\sum_{i=!}^{3} \square L_i = 60m + 20m + 60m = 140m$$

$$\sum_{j=1}^{2} \square K_j = 0.4 + 0.4 = 0.8$$

$$\Delta P_f = \frac{1}{2} * 1000\,\frac{kg}{m^3} * \left(6.366\,\frac{m}{s}\right)^2 * \left(4 * \frac{0.0025}{0.2m} * 140 + 0.8\right) = 1.59 * 10^5 Pa$$

$$P_1 = P_2 + \rho g(h_2 - h_1) + \Delta P_f = 1 * 10^5 + (1000 * 9.81 * 20) + 1.59 * 10^5 = 4.55 * 10^5 Pa$$

b)

The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f * Q = 1.59 * 10^5 Pa * 0.2\,\frac{m^3}{s} = 31.8kW$$

Question 2 (10 points)

$$v_1 = 3.5\frac{m}{s}$$

$$P_1 = 280\ kPa$$

In order to determine the anchoring force, we need to resolve the forces along x and y directions.

$$\Sigma F_{reaction} + \Sigma F_{pressure} + \Sigma F_{friction} = \Sigma F_{surface}$$

$$\Sigma F_{friction} = 0$$

$$\Sigma F_{reaction} + (-P_1 A_1 \hat{n}_1) + (-P_2 A_2 \hat{n}_2) = \int_{A_1} \rho v_1 (v_1 . \hat{n}_1) dA_1 + \int_{A_2} \rho v_2 (v_2 . \hat{n}_2) dA_2$$

We need to calculate P_2 and v_2.

Because of the continuity equation: $A_1 v_1 = A_2 v_2$

$$3.5 \times 0.2^2 = v_2 \times 0.08^2$$

$$v_2 = 21.875\ m/s$$

Regarding P_2, we apply Bernoulli's principle:

$$P_1 + \frac{1}{2}\rho . v_1^2 + \rho . g . h_1 = P_2 + \frac{1}{2}\rho . v_2^2 + \rho . g . h_2$$

$$280000 + 0.5 \times 1000 \times 3.5^2 = P_2 + 0.5 \times 1000 \times 21.875^2$$

$$P_2 = 46.86\ kPa$$

Resolving the forces along the +x direction:

$$\Sigma F_{reaction,x} - P_1 A_1 (-1) - P_2 A_2 (-1) cos 65^o = -\rho v_1^2 A_1 - \rho v_2^2 A_2 cos 65^o$$

$$\Sigma F_{reaction,x} = -P_1 A_1 - P_2 A_2 cos 65^o - \rho v_1^2 A_1 - \rho v_2^2 A_2 cos 65^o$$

$$= -A_1 (P_1 + \rho v_1^2) - A_2 cos 65^o (P_2 + \rho v_2^2)$$

$$= -3.14(0.1)^2 (280000 + 1000 \times 3.5^2) - 3.14(0.04)^2 \times 0.42 \times (46860 + 1000 \times 21.875^2)$$

$$= -10200.04\ N = -10.2\ kN$$

Resolving the forces along the +y direction:

$$\Sigma F_{reaction,y} + 0 - P_2 A_2 (+1) sin65^o = 0 + \rho v_2^2 A_2 sin65^o$$

$$\Sigma F_{reaction,y} = A_2 sin65^o (P_2 + \rho v_2^2)$$

$$= 3.14 \times (0.04)^2 \times 0.9(46860 + 1000 \times 21.875^2)$$

$$= 2375.53\ N = 2.375\ kN$$

The total net reaction force corresponds to:

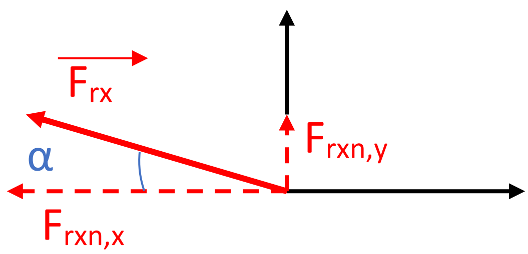
$$F_{rxn} = \sqrt{(F_{rxn,x}^2 + F_{rxn,y}^2)} = ((-10200.04)^2 + (2375.53)^2)^{\frac{1}{2}}$$

$$= 10473\ N = 10.473\ kN$$

And it is directed along angle α:

$$tg\ \ \alpha = \frac{F_{rxn,y}}{F_{rxn,x}}$$

$$\alpha = arctg\ \frac{2.375}{10.2} = 13.1°$$



Question 3 (10 points)

a) thermal circuit should have 3 resistances, 2 conductive and 1 convective.

b)

$$Q = \frac{\Delta T}{\Sigma R_{Th}} = \frac{(T_{int} - T_{ext})}{\frac{L_{Al}}{k_{Al}A} + \frac{L_{Cu}}{k_{Cu}A} + \frac{1}{h_{ext}A}}$$

$$Q * \left(\frac{L_{Al}}{k_{Al}A} + \frac{L_{Cu}}{k_{Cu}A} + \frac{1}{h_{ext}A}\right) + T_{ext} = T_{int}$$

$$\frac{Q}{A} * \left(\frac{L_{Al}}{k_{Al}} + \frac{L_{Cu}}{k_{Cu}} + \frac{1}{h_{ext}}\right) + T_{ext} = T_{int}$$

$$\frac{4}{0.1 * 0.03} * \left(\frac{0.005}{206} + \frac{0.01}{385} + \frac{1}{25}\right) + 20 = 73.4°C$$

T at the interface between aluminum and copper

$$Q = \frac{\Delta T}{\Sigma R_{Th}} = \frac{\left(T_{int} - T_{Al/Cu}\right)}{\frac{L_{Al}}{k_{Al}A}}$$

$$\left(Q * \frac{L_{Al}}{k_{Al} * A} - T_{int}\right) * (-1) = T_{Al/Cu} = (4 * \frac{0.005}{206 * 0.1 * 0.03} - 73.4) * (-1) = 73.36°C$$

c)

$$Q = \frac{(T_{int} - T_{ext})}{\frac{L_{Al}}{k_{Al}A} + \frac{L_{Cu}}{k_{Cu}A} + \frac{1}{h_{ext}A}} = \frac{(85 - 20)}{(0.1 * 0.03)\left(\frac{0.005}{206} + \frac{0.01}{385} + \frac{1}{25}\right)} = 4.87\ W$$

d) All we must do is test the different values of h and see what we get for T_{int} when the value of $Q = 50$.

$$Q * \left(\frac{L_{Al}}{k_{Al}A} + \frac{L_{Cu}}{k_{Cu}A} + \frac{1}{h_{ext}A}\right) + T_{ext} = T_{int}$$

$$\frac{50}{0.1 * 0.03} * \left(\frac{0.005}{206} + \frac{0.01}{385} + \frac{1}{250}\right) + 20 = 87.5°C$$

$$\frac{50}{0.1 * 0.03} * \left(\frac{0.005}{206} + \frac{0.01}{385} + \frac{1}{900}\right) + 20 = 39.4°C$$

With a fan (forced convection of air), the GPU still gets a bit too hot, but with force convection of water, it stays cool enough.

Question 4 (10 points)

a)

$$\dot{Q} = \dot{m}_{wort} C_p \left(T_{h,wort} - T_{c,wort}\right) = 2 * 4180 * (100 - 60) = 334.4\ kW$$

b)

$$333.4\ kW = 1.5 * 4180 * (T_{h,water} - 20)$$

$$\frac{333400}{1.5 * 4180} + 20 = T_{h,water} = 73.3°C$$

c) temperature profile of heat exchanger

d)

$$\Delta T_1 = 60 - 20 = 40°C$$

$$\Delta T_2 = 100 - 73.3 = 26.7°C$$

$$\Delta T_{LM} = \frac{(\Delta T_1 - \Delta T_2)}{ln\ ln\ \left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{40 - 26.7}{ln\ ln\ \left(\frac{40}{26.7}\right)} = 32.9°C$$

$$Q = UA\Delta T_{LM}$$

$$A = \frac{Q}{U\Delta T_{LM}} = \frac{333400}{650 * 32.9} = 15.6m^2$$

e)

The tubing we have available has a diameter of 5 cm.

$$A = 2\pi rL = 2\pi \left(\frac{d}{2}\right) L = \pi dL$$

$$\frac{A}{\pi d} = L$$

$$L = \frac{15.6}{\pi * 0.05} = 99\ m$$