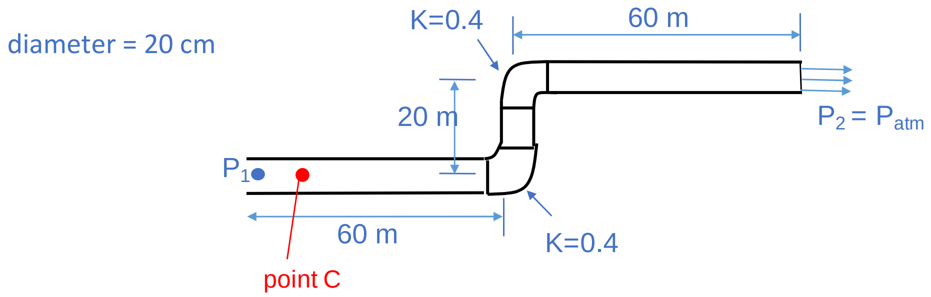
ChE 204 – Introduction to Transport Phenomena

Mid-term Exam: Questions

Question 1:

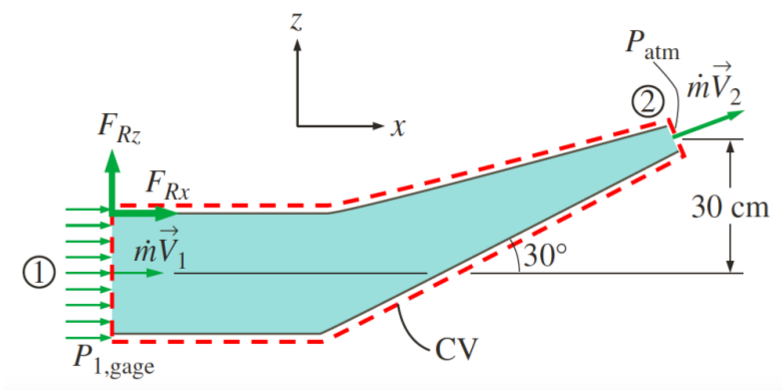
A pipe system carries water (ρ = 1000 kg/m^3; μ= 8.9 x 10^{-4} Pa·s) and discharges it as a free jet in the atmosphere, as shown in the following figure. Consider the pipe system to standing vertically, not laying on the ground.



a) What gauge pressure P_1 would be needed to provide a volumetric flow rate of 12 m^3/min of water? Assume a smooth pipe and include all the losses.
b) If the frictional losses inside the system were supposed to be compensated with a pump located at point C, what will be the required power of the pump?

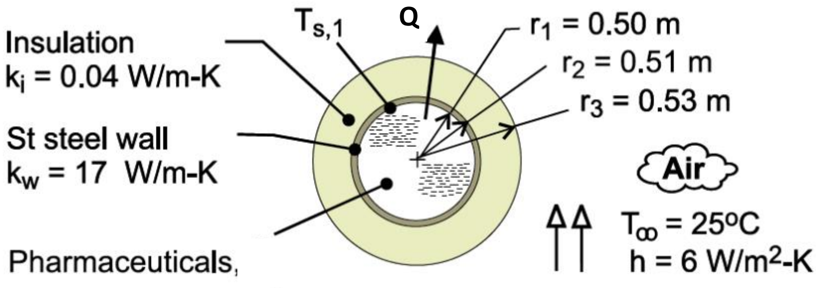
Question 2:

A reducing elbow is used to deflect water (ρ = 1000 kg/m^3) flow at a rate of 14 Kg/s in a horizontal pipe upward 30degree (see figure below). The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the center of the outlet and the inlet is 30 cm. The weight of the elbow and of the water in it is considered to be negligible. Determine (a) the gage pressure at the inlet of the elbow and b) the x- and z- components of the anchoring force needed to keep it in place Neglet friction.



Question 3:

The figure below illustrates the cross section of a cylindrical vessel with length L= 1 m

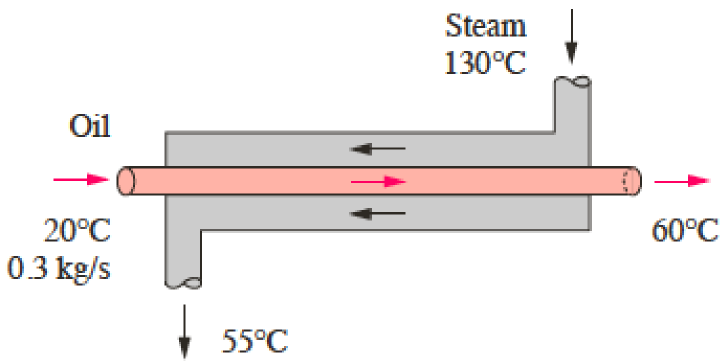


a) In this first question, consider the vessel above without the insulation layer, just imagine it is not there. In this case the inner surface temperature $T_{S,1}$ is 50 °C. Calculate the heat flow rate Q. What is the dominant resistance?

b) Now include the insulation layer, assume the heat flow rate Q to be the same and calculate the inner surface temperature $T_{S,1}$ in this case.

Question 4:

Engine oil (Cp=2100 J/ kg °C) is to be heated from 20°C to 60°C at a rate of 0.3 Kg/s in a 2-cm-diameter thin walled copper tube by condensing steam outside at a temperature of 130°C. The exit temperature of the steam is 55°C For an overall heat transfer coefficient of 650 W/m^2°C, determine a) the rate of heat transfer and b) the length of the tube required to achieve it.



ChE 204 – Introduction to Transport Phenomena

Mid-term Exam: Solutions

Question 1:

a)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Applying mass conservation at point 1 and point 2

$$\rho v_1 A_1 = \rho v_2 A_2 = \dot{m}$$

Since $A_1 = A_2$, we get $v_1 = v_2 = v$

$$P_1 + \rho g h_1 + \cancel{\frac{1}{2}\rho v_1^2} = P_2 + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \Delta P_f$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v * D}{\mu}$$

The volumetric flow rate, $\dot{v} = \frac{\dot{m}}{\rho}$, thus

$$v = \frac{\dot{v}}{\rho A} = \frac{\frac{12}{60}\,(\frac{m^3}{s})}{\frac{\pi (0.2)^2\,(m^2)}{4}} = 6.36\ m/s$$

$$Re = \frac{1000\,\frac{kg}{m^3} * 6.366\,\frac{m}{s} * 0.2m}{8.9 * 10^{-4} Pas} = 1.431\ * 10^6$$

Friction factor from Moody diagram:

$$f_f = 0.0025$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v^2 \left((\frac{4 f_f}{D} \sum_i L_i) + \sum_j K_j \right)$$

Loss coefficients:

$$\sum_{i} L_i = 60m + 20m + 60m = 140m$$

$$\sum_{j} K_j = 0.4 + 0.4 = 0.8$$

$$\Delta P_f = \frac{1}{2} * 1000 \frac{kg}{m^3} * \left(6.366 \frac{m}{s}\right)^2 * \left(4 * \frac{0.0025}{0.2m} * 140 + 0.8\right) = 1.59 * 10^5 Pa = 1.59 bar$$

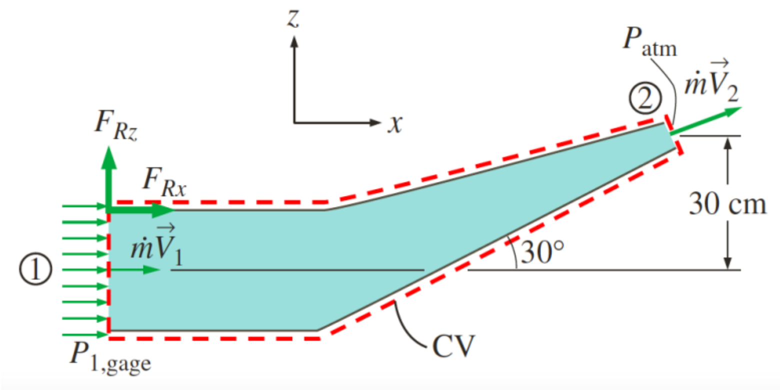
$$P_1 = P_2 + \rho g(h_2 - h_1) + \Delta P_f = 1 * 10^5 + (1000 * 9.81 * 20) + 1.59 * 10^5 = 4.55 * 10^5 Pa = 4.55 bar$$

b)

The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f * Q = 1.59 * 10^5 Pa * 0.2 \frac{m^3}{s} = 31.8 kW$$

Question 2:



a) Applying continuity equation at points 1 and 2 to calculate the inlet and outlet velocities:

$$v_1 = \frac{\dot{m}}{\rho A_1} = \frac{14\left(\frac{kg}{s}\right)}{1000\left(\frac{kg}{m^3}\right) \times 0.0113\left(\frac{m^2}{s}\right)} = 1.24\frac{m}{s}$$

$$v_2 = \frac{\dot{m}}{\rho A_2} = \frac{14\left(\frac{kg}{s}\right)}{1000\left(\frac{kg}{m^3}\right) \times 7 \times 10^{-4}\left(\frac{m^2}{s}\right)} = 20\frac{m}{s}$$

We use Bernoulli equation to calculate the pressures. Taking the center of the inlet cross section as the reference level (z_1=0) and writing the pressure value in gauge, $P_{2,gauge}$ = 0, the Bernoulli equation for the streamline going through the center of the elbow is expressed as:

$$P_{1,gauge} + \frac{\rho v_1^2}{2} + \rho g z_1 = P_{2,gauge} + \frac{\rho v_2^2}{2} + \rho g z_2$$

$$P_{1,gauge} - P_{2,gauge} = \frac{\rho(V_2^2 - V_1^2)}{2} + \rho g(z_2 - z_1)$$

$$P_{1,gauge} - 0 = \frac{1000\left(\frac{kg}{m^3}\right) \times (20^2 - 1.238^2)\left(\frac{m^2}{s^2}\right)}{2} + 1000\left(\frac{kg}{m^3}\right) \times 9.81\left(\frac{m}{s^2}\right) \times 0.3(m)$$

$P_{1,gauge} = 202.2\ kPa$

b)

$$F_{rxn,x} + \left(-P_{1,gauge} A_1 \widehat{n_1}\right) + \left(-P_{2,gauge} A_2 \widehat{n_2}\right) = \int_{\square}^{\square} \square\, \rho V_1 (V_1 . \widehat{n_1}) dA_1 + \int_{\square}^{\square} \square\, \rho V_2 (V_2 . \widehat{n_2}) dA_2$$

$$F_{rxn,x} + \left(-P_{1,gauge} A_1 (-1)\right) + \left(-P_{2,gauge} A_2 (1) cos30°\right) = \rho V_1^2 A_1 (-1) + \rho V_2^2 A_2 (1)\ cos\ cos\ 30°$$

$$F_{rxn,x} = -\left(202.2 \times 10^3 (Pa) \times 113 \times 10^{-4} (m^2)\right) + 0 - \left(10^3 \left(\frac{kg}{m^3}\right) \times 1.238^2 \left(\frac{m^2}{s^2}\right) \times 113 \times 10^{-4} (m^2)\right) + \left(10^3 \left(\frac{kg}{m^3}\right) \times 20^2 \left(\frac{m^2}{s^2}\right) \times 7 \times 10^{-4} (m^2)\right)$$

$F_{rxn,x} = -2053N$

Similarly, the momentum equation for steady flow in the z-direction can be written as:

$$F_{rxn,z} + \left(-P_{1,gauge} A_1 \widehat{n_1}\right) + \left(-P_{2,gauge} A_2 \widehat{n_2}\right) = \int_{\square}^{\square} \square\, \rho V_1 (V_1 . \widehat{n_1}) dA_1 + \int_{\square}^{\square} \square\, \rho V_2 (V_2 . \widehat{n_2}) dA_2$$

However, there is no momentum terms in the z-component at the inlet.

$$F_{rxn,z} + 0 + (-P_2 A_2 (1) sin30°) = 0 + \rho V_2^2 A_2 (1)\ sin\ sin\ 30°$$

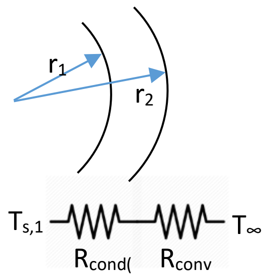
$$F_{rxn,z} = 0 + (10^3 \left(\frac{kg}{m^3}\right) \times 20^2 \left(\frac{m^2}{s^2}\right) \times\ 7 \times 10^{-4} (m^2) \times sin\ sin\ 30°\ )$$

$F_{rxn,z} = 144\ N$

Question 3:

Assumptions: (1) Steady-state, (2) one-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

(a)



$$R_{cond(1)} = \frac{l}{kA} = \int_{r_1}^{r_2} \frac{dr}{k_w\, 2\pi r l} = \frac{1}{2\pi l k_w}(ln\, ln\, \frac{r_2}{r_1}) = \frac{1}{2\pi \times 1m \times \left(\frac{17W}{m}.K\right)} \times \frac{0.51m}{0.5m})$$

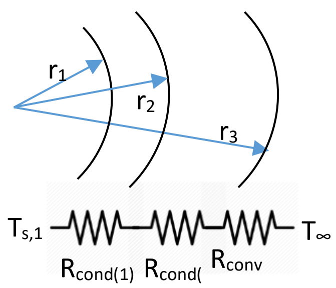
$$R_{cond(1)} = 1.85 \times 10^{-4} K/W$$

$$R_{conv} = \frac{1}{2\pi r_2 l h} = \frac{1}{2\pi(0.51m) \times 1m \ \times 6\ W/m^2.K} = 5.2 \times 10^{-2} K/W$$

$R_{conv} > R_{cond(1)}$ ⍰ The dominant resistance is due to convection.

$$\dot{Q} = \frac{T_{s,1} - T_\infty}{\frac{1}{2\pi l k_w}(ln\, ln\, \frac{r_2}{r_1}) + \frac{1}{2\pi r_2 l h}} = \frac{(50-25)\ oC}{1.85 \times 10^{-4} K/W + 5.2 \times 10^{-2} K/W} = 479\ W$$

(b) With the insulation,



$$R_{cond(1)} = \frac{l}{kA} = \int_{r_1}^{r_2} \frac{dr}{k_w\ 2\pi rl} = \frac{1}{2\pi lk_w}(ln\ ln\ \frac{r_2}{r_1}) = \frac{1}{2\pi \times 1m \times \left(\frac{17W}{m}.K\right)} \times \frac{0.51m}{0.5m})$$

$$R_{cond(1)} = 1.85 \times 10^{-4} K/W$$

$$R_{cond(2)} = \frac{l}{kA} = \int_{r_2}^{r_3} \frac{dr}{k_i\ 2\pi rl} = \frac{1}{2\pi lk_i}(ln\ ln\ \frac{r_3}{r_2}) = \frac{1}{2\pi \times 1m \times \left(\frac{0.04W}{m}.K\right)} \times \frac{0.53m}{0.51m})$$

$R_{cond(2)} = 1.53 \times 10^{-1} K/W$

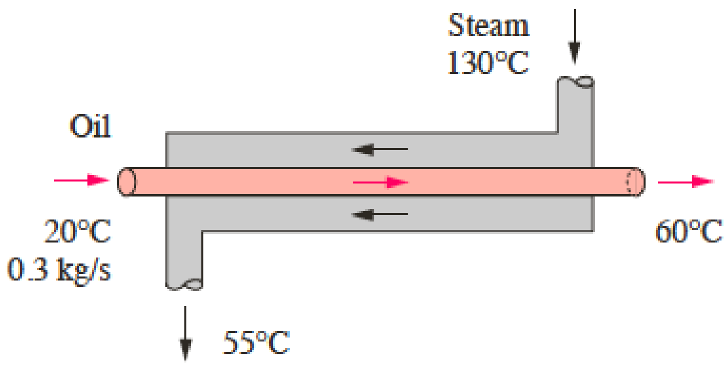
$$R_{conv} = \frac{1}{2\pi r_3 lh} = \frac{1}{2\pi(0.53m) \times 1m\ \times 6\ W/m^2.K} = 5.0 \times 10^{-2} K/W$$

$$\dot{Q} = \frac{T_{s,1} - T_\infty}{\Sigma R}$$

$$T_{s,1} = T_\infty + \dot{Q}[\Sigma R] = T_\infty + \dot{Q}\left[R_{cond(1)} + R_{cond(2)} + R_{conv}\right]$$

$$T_{s,1} = 25\ oC + 479W[1.85 \times 10^{-4} + 1.53 \times 10^{-1} + 5.0 \times 10^{-2}]\frac{K}{W} = 122.3\ oC$$

Question 4:



a) All the parameters required to find the rate of heat transfer to the cold fluid have been given.

$$= 0.3 * 2100 * (60 - 20)$$

$$= 25.2\ kW$$

b) The inlet and outlet temperatures of the hot fluid have also been given. As we have all four temperatures to calculate the ΔT_m, we will do so using the formula for counter current heat exchangers:

Where

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\ ln\ \left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$\Delta T_1 = T_{h,i} - T_{c,o}$ and $\Delta T_2 = T_{h,o} - T_{c,i}$

Putting in the values we get

$$\Delta T_{lm} = \frac{(130 - 60) - (55 - 20)}{ln\ ln\left(\frac{(130-60)}{(55-20)}\right)}$$

$$= 50.5\ K$$

From

We get,

Area of the heat exchanger = 0.77 m^2

$$A = 2 * \pi * r * l$$

$$= 2 * \pi * 0.01 * l$$

$$l = 12.22\ m$$