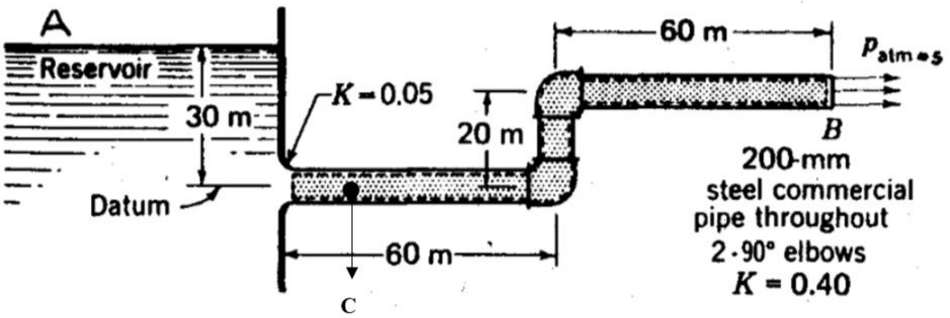
Mid-term Exam

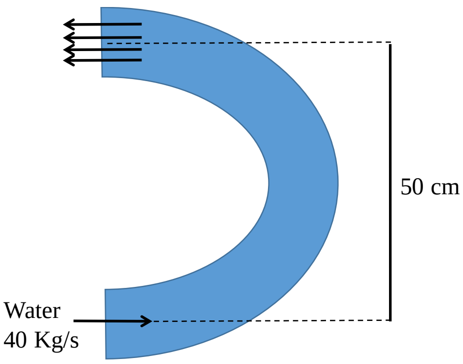
Question 1:

A flow of pipe system carries water (ρ = 1000 kg/m^3; μ= 8.9 x 10^{-4} Pa·s) from a reservoir and discharges it as a free jet in the atmosphere, as shown in the following figure.

a) Ignoring the frictional losses inside the system, if the pressure inside the reservoir (P_1) is 7 x 10^5 Pa, what will be the flow of the water coming out from the pipe? Consider the inner diameter of the pipe to be 200 mm.
b) What gauge pressure in the reservoir (P_1) is needed to provide a flow rate of 12 m^3/min of water? Assume a smooth pipe. In this part all of the losses in the system should be considered.
c) If the pressure of the reservoir is considered to be atmospheric and if the frictional loss is supposed to be compensated with a pump located at point C, what will be the required power of the pump considering the frictional losses inside the system?

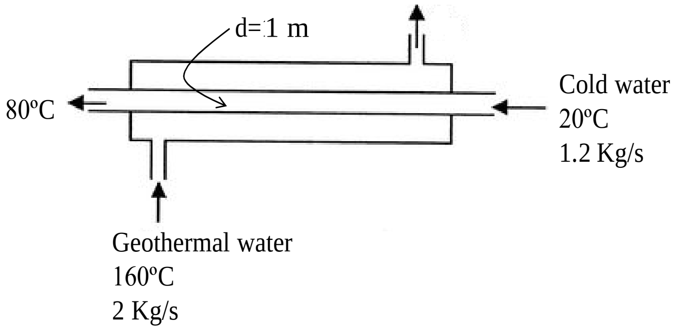


Question 2:



A 180° elbow is used to direct water flow upward at a rate of 40 Kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow is 2 kg. Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.

Question 3:



A counter-flow double pipe heat exchanger has to heat water from 20°C to 80°C at a rate of 1.2 Kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin walled and has a diameter of 1 m.

a) If the overall heat transfer coefficient of the heat exchanger is 640W/m^2 C determine the length of the heat exchanger required to achieve the desired heating. ($C_{p,water}$ = 4.187 kJ/ kg K).
b) Calculate the new overall heat transfer coefficient in the presence of a fouling layer with thickness 5 cm, assuming h_i = 1280 W/m^2.K and K_{foul}= 10 W/m^2.K.

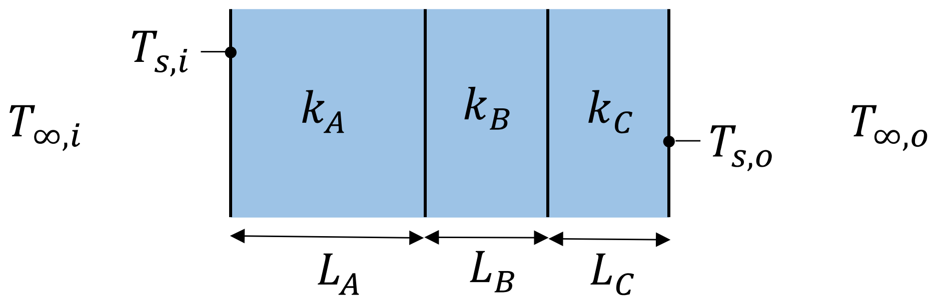
Question 4:

A cake is baked in a convection oven.

a) Calculate the heat flux at cake surface when it is just inserted in the oven if its initial temperature is $T_i = 24$ °C, oven air and wall are at the same temperature $T_{\infty,i} = T_{s,i} = 180$ °C and the forced convection transfer coefficient between the oven and the cake is 25 W/m^2.K.

b) Consider that the walls of the oven consist of L = 30-mm-**thick** layer of insulation characterized by k= 0.03W/m^2.K that is sandwiched between two **very thin** layers of sheet **metal**. The exterior surface of the oven is exposed to air at $T_{\infty,o} = 24$ °C with h_o = 2W/m^2.K. The interior oven air temperature is $T_{\infty,i} = T_{s,i} = 180$ °C. Neglecting radiation heat transfer, calculate the steady-state heat flux through the oven walls (**Tip:** use your engineering intuition to reasonably ignore what you don't have enough information about. Express your reasoning why you think it can be ignored)

c) Consider now a composite wall of the oven consisting of three materials, two of which are of known thermal conductivity, k_A=20 W/m^2.K and k_C=50 W/m^2.K, and known thickness, L_A= 0.30 m and L_C= 0.15 m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B= 0.15 m, but unknown thermal conductivity k_B. Under steady-state operating conditions, the outer surface temperature $T_{\infty,o}$= 24 °C, the inner surface temperature $T_{s,i}$=768 °C and the oven air temperature $T_{\infty,i}$=800 °C. The outside convection coefficient h_o is 2W/m^2.K while the internal convection coefficient must be calculated. Find the value of k_B corresponding to overall heat loss of 800 W/m^2.



Solutions

Question 1

a)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Ignore frictional losses: $\Delta P_f = 0$

Simplified Bernoulli equation according to the question:

$$P_1 + \cancel{\rho g h_1} + \cancel{\frac{1}{2}\rho v_1^2} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \cancel{\Delta P_f}$$

Solve equation for v_2:

$$v_2 = \left(2 * \frac{P_1 - P_2 - \rho g h_2}{\rho}\right)^{\frac{1}{2}}$$

Numerical solution:

$$v_2 = \left(2 * \frac{(7-1) * 10^5 Pa - 1000\,\frac{kg}{m^3} * 9.81\,\frac{m}{s^2} * 20m}{1000\,\frac{kg}{m^3}}\right)^{\frac{1}{2}} = 28.41\,\frac{m}{s}$$

Volumetric flow:

$$Q = A_2 * v_2$$

$$Q = \left(\frac{d_i}{2}\right)^2 * \Pi * \mathrm{v}_2$$

$$Q = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\,\frac{m}{s} = 0.892\,\frac{m^3}{s}$$

Mass flow:

$$\dot{m} = A_2 * v_2 * \rho$$

$$\dot{m} = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\,\frac{m}{s} * 1000\,\frac{kg}{m^3} = 892\,\frac{kg}{s}$$

Correction notes: Pressure term in Bernoulli equation for friction loss not required for point. Atmospheric pressure can be either 1bar or 1.013bar *(no big difference in numerical result). Mass flow and volumetric flow are both correct.*

b)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

$$P_1 + \cancel{\rho g h_1} + \cancel{\frac{1}{2}\rho v_1^2} = \cancel{P_2} + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

$$v_{avg} = v_2 = \frac{Q_2}{A_2}$$

$$Re = \frac{\rho * v_{avg} * D}{\mu}$$

$$Re = \frac{1000\,\frac{kg}{m^3} * 6.366\,\frac{m}{s} * 0.2m}{8.9 * 10^{-4} Pas} = 1.431\ [\,]$$

Friction factor from Moody diagram:

$$f_f = 0.011$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2 \left(\frac{4 f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

Loss coefficients:

$$\sum_i L_i = 60m + 20m + 60m = 140m$$

$$\sum_j K_j = 0.05 + 0.4 + 0.4 = 0.85$$

also accepted:

$$\sum_j K_j = 0.05 + 0.4 = 0.45$$

$$\Delta P_f = \frac{1}{2} * 1000\,\frac{kg}{m^3} * \left(6.366\,\frac{m}{s}\right)^2 * \left(4 * \frac{0.011}{0.2m} * 140m + 0.85\right) = 6.4 * 10^5 Pa = 6.4 bar$$

$$P_1 = \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f = 8.56 * 10^5 Pa = 8.56 bar$$

Correction notes: Pressure term for friction loss in Bernoulli equation required for point. Gauge pressure is only considered for numerical solution of P_1. If you did not cross out ambient pressure, but did all the rest correctly, you get deducted only 1/2 Point.

c)

The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f * Q = 6.4 * 10^5 Pa * 0.2 \frac{m^3}{s} = 128 kW$$

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Question 2

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v.n) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . n_i$$

$$\sum F_{volume} = m_{water} . g + m_{elbow} . g$$

Velocity of the water in the pipe is 5.09 m/s

Applying Bernoulli's principle between the inlet and the discharge points,

$$P_1 + 0.5 \times 1000 \times (5.09)^2 + 0 = P_o + 0.5 \times 1000 \times (5.09)^2 + 1000 \times 9.81 \times 0.5$$

$$P_1 - P_o = 4.9\ kPa$$

Resolving the forces in the +x direction using gauge pressure,

$$\sum F_{pressure} = -4.9 \times 10^3 \times \pi \times 100 \times 10^{-4} \times -\frac{1}{4} = 38.48\ N$$

$$\sum F_{volume.x} = 0$$

Therefore,

$$\sum F_{surface} - \sum F_{pressure} = \sum F_{reaction}$$

$$\sum F_{surface.x} = -\left[1000 \times \pi \times \frac{100}{4} \times 10^{-4} \times (5.09)^2\right] - \left[1000 \times \pi \times \frac{100}{4} \times 10^{-4} \times (5.09)^2\right]$$

$$= -406.96\ N$$

$$\sum F_{reaction.x} = -406.96 - 38.48 = -445.44\ N$$

Now resolving in the y direction,

$$\sum F_{pressure.y} = 0$$

$$\sum F_{reaction.y} = -F_{volume.y} = \pi \times 100 \times \frac{10^{-4}}{4} \times \pi \times \frac{2.5}{100} \times 1000 \times 9.8 = 60.45\ N$$

Weight of the bend is 2 kg. This additional weight must also be supported by the anchoring force.

Therefore,

$$F_{reaction.y.total} = 60.45 + (2 \times 9.8) = 80.05N$$

Note: Answer sheets that have considered 2 kgs as the weight including that of the water will also be awarded full marks.

$$F_{net} = ((445.44)^2 + (80.05)^2)^{\frac{1}{2}} = 452.57N$$

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Question 3

a) Overall heat transfer coefficient, U = 640 $\frac{W}{m^2}$
   Heat transfer = $m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right) = m_{hot}c_{p_{hot}}(T_{hot_{in}} - T_{hot_{out}})$

$$T_{hot_{out}} = T_{hot_{in}} - \frac{m_{cold}(T_{cold_{out}} - T_{cold_{in}})}{m_{hot}}$$

$T_{hot_{out}} = 160 - \left(1.2 * \frac{60}{2}\right) = 124℃$

$T_{LMTD} = \frac{(160-80)-(124-20)}{\ln\left(\frac{160-80}{124-20}\right)} = 91.5℃$

Heat transfer = UA(T_{LMTD}) = $m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)$

Area, $A = 2 * pi * r * L$

$$L = \frac{m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{UT_{LMTD} * 2 * pi * r}$$

$\boldsymbol{L = \frac{1.2*4187*60}{640*91.5*2*pi*0.5} = 1.639\ m}$ ℃

b) Tube is thin walled and hence the resistance to heat transfer from the walls of the tube is neglected.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$h_o = \frac{1}{\left(\frac{1}{U} - \frac{1}{h_i}\right)}$=1280 W m^{-2}K

So, the new U,

$$\frac{1}{r_o U_{new}} = \frac{1}{(r_o - \delta_{foul})h_i} + \frac{1}{h_o r_o} + \frac{\ln\left(\frac{r_o}{(r_o - \delta_{foul})}\right)}{k_{foul}}$$

$$\frac{1}{U_{new}} = \left(\frac{1}{(0.5 - 0.05) * 1280} + \frac{1}{1280 * 0.5} + \frac{\ln\left(\frac{0.5}{(0.5 - 0.05)}\right)}{10} \right) * 0.5$$

$$\boldsymbol{U_{new} = 144.59\ Wm^{-2}K}$$

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Question 4 (10Pts)

Question 4:

a) $Q = hA\Delta T \Rightarrow \frac{Q}{A} = h\Delta T = 25 * (180-24) = 3900 \ W/m^2$

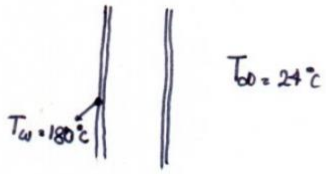
b) $Q = UA\Delta T \Rightarrow \frac{Q}{A} = U\Delta T = \frac{\Delta T}{\frac{1}{U}}$

$\frac{1}{U} = \frac{1}{h_i} + \frac{L}{k} + \frac{1}{h_o}$ (2 Assump)

1- We can neclect the conductive heat transfer resistance of thin layers of metal, for their high conductivity and very small thickness.

2- We can neglect the internal heat transfer coefficient, for the temp of wall is given

$\Rightarrow \frac{Q}{A} = \frac{180-24}{\frac{0.03}{0.03} + \frac{1}{2}} = \frac{156}{1.5} = 104 \ W/m^2$



c)

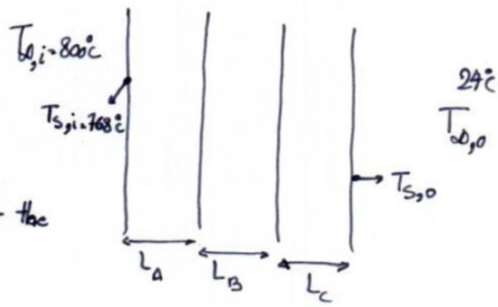
Part a) $Q = h_i A\Delta T \Rightarrow \frac{Q}{A} = h_i \Delta T \Rightarrow 800 = h_i * (800-768) \Rightarrow$

$h_i = 25 \ W/m^2.°C$

Part b) $Q = UA\Delta T \Rightarrow \frac{Q}{A} = \frac{\Delta T}{\frac{1}{U}}$

The internal heat transfer coefficient can be neglected, for the temperature of wall has been given.

$\frac{1}{U} = \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_o}$



$$\frac{1}{U} = \frac{\Delta T}{\frac{Q}{A}} = \frac{768-24}{800} = 0.93 \ m^2 \cdot °C / W$$

$$\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_o} = 0.93 \Rightarrow \frac{0.3}{20} + \frac{0.15}{k_B} + \frac{0.15}{50} + \frac{1}{2} = 0.93$$

$$\Rightarrow \frac{0.15}{k_B} = 0.93 - 0.5 - 0.015 - 0.0003 \Rightarrow \frac{0.15}{k_B} = 0.4147 \Rightarrow$$

$$k_B = \frac{0.15}{0.4147} = \boxed{0.3617 \ W/m°C}$$

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