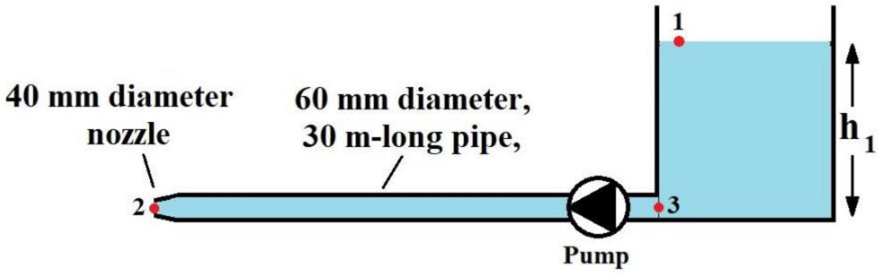
Mid-term exam simulation

Question 1

In the system below, the pump provides a power of $25\ kW$ and generates a flow rate of $0.04\ m^3/s$ in the pipe. The fluid is water so $\rho = 1000\ kg/m^3$ and $\mu = 1.69 \times 10^{-3}\ Pa \cdot s$. Also take $g = 9.8\ m/s^2$. To calculate the friction pressure drop in the pipe, assume the roughness of the pipe $\varepsilon = 6 \times 10^{-6}\ m$ and neglect the minor losses.

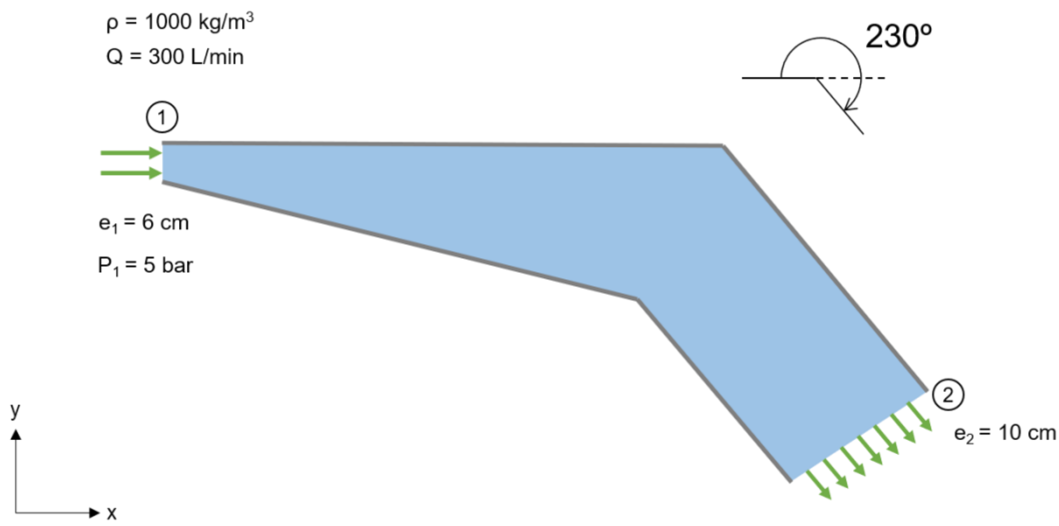
a) Determine the height of water in the reservoir (h_1).

b) Determine the flow rate if the pump is removed from the system.



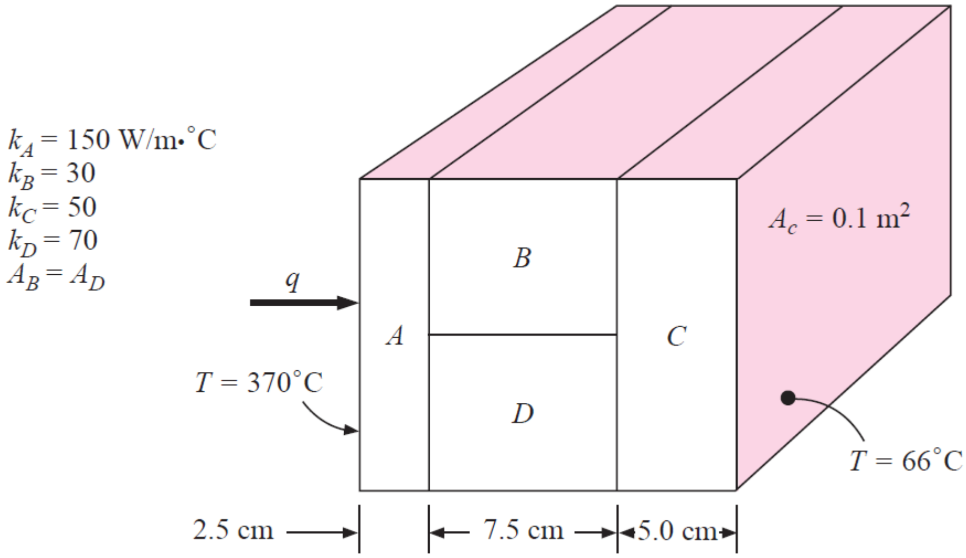
Question 2

When a flowing fluid changes direction, a force acts on the bend and tends to move it. In many cases the joints are sufficiently strong to prevent such movement, but for large pipes (i.e. those used in hydroelectric installations) large concrete anchorages are usually employed to keep the pipe-bends in place, and it is important to know how much force a support must withstand. In this case, n elbow placed flat on the floor is used to direct water which is flowing at a volumetric rate of 300 L/min. The water jet, square in section, changes from 6 cm edge in point 1 to 10 cm edge in point 2. Determine the absolute pressure at the outlet of the elbow and the value and direction of the anchoring force needed to hold the elbow in place.



Question 3

Find the heat transfer flow rate (q) through the composite wall in the following figure. Assume one-dimensional heat flow in the direction of the bold arrow.



Question 4

You are working for a chemical engineering firm and a client of yours is interested in replacing their current heat exchanger. But to choose a new one, they first must understand the properties of the one they are currently using… so they call you!

The diagram below represents the current heat exchanger of your client. The inlet and outlet temperatures and the volume flow rates of the hot and cold fluids are known. Assume steady-state and that all fluid properties are constant.

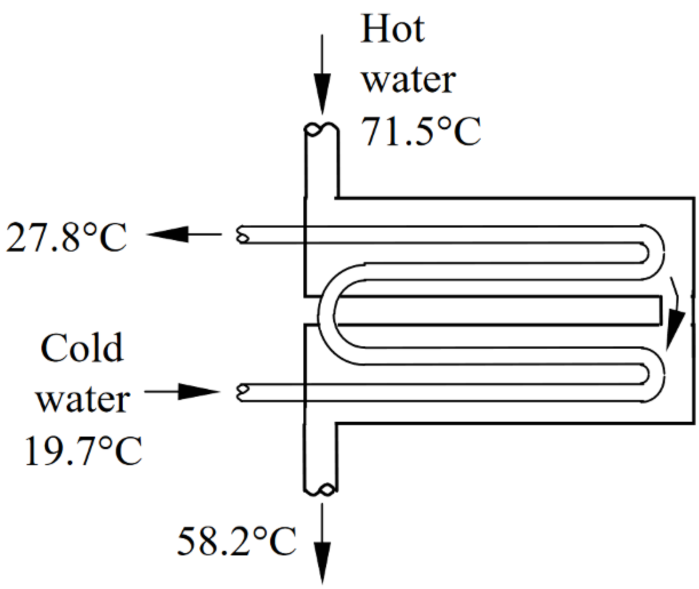
The flow rate of the hot and cold streams is 1.05 L/min and 1.55 L/min, respectively. The densities of hot and cold water are 980.5 and $997.3\ kg/m^3$, respectively. The specific heat is $4187\,\frac{J}{kg\cdot°C}$ for the hot-water stream and $4180\,\frac{J}{kg\cdot°C}$ for the cold-water stream. The cold pipe has an outer diameter of 5 cm and length of 10.19 m.

a) Calculate the mass flow rates for each stream and the rates of heat transfer from the hot water ($\dot{Q}_h$) and to the cold water ($\dot{Q}_c$). Note that the wall of the hot stream is not perfectly insulated.

b) Calculate the logarithmic mean temperature difference and the overall heat transfer coefficient.

c) Calculate the fraction of heat lost to the surroundings from the hot stream.

d) Your client would like to cool the hot stream five times faster (i.e. $5 * \dot{Q}_h$). Calculate the necessary heat transfer area and the cost of such a heat exchanger. Assume the same fraction of heat loss to the environment as calculated in part (b) when determining the necessary area. Also assume that the inlet and outlet temperatures and the overall heat transfer coefficient is the same. You have stainless steel tubing available with an outer diameter of 5 cm and it costs 8 CHF/m which will have its outer surface serve as the heat transfer area.



Mid-term exam simulation solutions

<u>Question 1</u>

a) We apply the Bernoulli's equation between point 1 and point 3. Keep in mind that for the same assumption made in the Torricelli's theorem $v_1 \ll v_3$. So, we write:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho \cancelto{0}{v_1^2} = P_3 + \rho g \cancelto{0}{h_3} + \frac{1}{2}\rho v_3^2$$

Then we apply the Bernoulli equation between point 2 and point 3, $(h_2 = h_3 = 0)$.

$$P_3 + \frac{1}{2}\rho v_3^2 + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

Comparing the two equations, we can write:

$$P_1 + \rho g h_1 + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

$P_{1,gauge} = P_{2,gauge} = 0$

$$v_2 = \frac{Q}{A_2} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04\,m)^2} = 31.84\,\frac{m}{s}$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v_{pipe} * D}{\mu}$$

$$v_{pipe} = \frac{Q}{A_{pipe}} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2} = 14.15\,\frac{m}{s}$$

Consider $v_{pipe} = v_3$

$$Re = \frac{1000\,\frac{kg}{m^3} * 14.15\,\frac{m}{s} * 0.06m}{1.69 * 10^{-3} Pas} = 5 * 10^5$$

$$\frac{\varepsilon}{D_{pipe}} = \frac{6 * 10^{-6}\,m}{0.06\,m} = 0.0001$$

Friction factor from Moody diagram: $f_f = 0.004$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{pipe}^2 \left((\frac{4f_f}{D_{pipe}} \sum_i L_i) + \sum_j K_j \right)$$

$$\Delta P_f = \frac{1}{2} * 1000 \frac{kg}{m^3} * \left(14.15 \frac{m}{s}\right)^2 * \left(4 * \frac{0.004}{0.06m} * 30 + 0\right) = 8 * 10^5 \ Pa$$

$$P_{pump} = \frac{power}{Q} = \frac{25 \times 10^3 W}{0.04 \frac{m^3}{s}} = 625000 \ Pa$$

$$h_1 \rho g = \frac{\rho v_2^2}{2} + \Delta P_f - P_{pump} = \frac{1000 \frac{kg}{m^3} * \left(31.8 \frac{m}{s}\right)^2}{2} + 8 * 10^5 - 625000 = 680620 \ Pa$$

$$\boldsymbol{h_1} = \frac{680620 Pa}{1000 \frac{kg}{m^3} * 9.81 \frac{m}{s^2}} = \mathbf{69.4\ m}$$

b)

$$Q = A_{pipe} * v_{pipe} = A_2 * v_2$$

$$v_2 = \frac{A_{pipe} * v_{pipe}}{A_2} = (\frac{D_{pipe}}{D_2})^2 * v_{pipe} = (\frac{0.06}{0.04})^2 * v_{pipe} = 2.25 * \ v_{pipe}$$

Without the pump: $P_{pump} = 0$ and we already calculated $h_1 = 69.4\ m$. So, we can write:

$$h_1 = 69.4\ m = \frac{v_2^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}} \sum_i L_i\right) = \frac{(2.25 \ v_{pipe})^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}} \sum_i L_i\right)$$

$$69.4\ m = \frac{{v_{pipe}}^2}{2 * 9.81 \frac{m}{s^2}} \left(5.06 + \frac{4 * 0.004 \ * 30\ m}{0.06\ m}\right)$$

$$\boldsymbol{v_{pipe}} = \sqrt{\frac{69.4 * 2 * 9.81\ m^2}{13.06\ s^2}} = \mathbf{10.21 \frac{m}{s}}$$

$$Q = v_{pipe} * A_{pipe} = 10.21 \frac{m}{s} * \frac{\pi}{4}(0.06\ m)^2 = 0.0288 \frac{m^3}{s}$$

Question 2

Because of the continuity equation, the velocities at the inlet and outlet of the elbow can be calculated by imposing:

$$Q = A_1 v_1 = A_2 v_2 = 300\ \frac{L}{min} \cdot \frac{1\ m^3}{1000\ L} \cdot \frac{1\ min}{60\ s} = 5 \cdot 10^{-3}\, \frac{m^3}{s}$$

$$A_1 = (0.06)^2\ m^2 \ \rightarrow\ v_1 = 1.39\ m/s$$

$$A_2 = (0.1)^2\ m^2 \ \rightarrow\ v_2 = 0.5\ m/s$$

Apply Bernoulli between point 1 and 2 to obtain the pressure at point 2:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = 5 \cdot 10^5\ Pa + \frac{1}{2} \cdot 1000\, \frac{kg}{m^3} \cdot (1.39^2 - 0.5^2)\, \frac{m^2}{s^2} = 500841\ Pa$$

Using the momentum balance equation to obtain the reaction force:

$$\sum F_{surface} + \sum F_{volume} = \sum_i^N \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n})\, dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

Volume and friction are neglected:

$$\sum F_{volume} = 0$$

$$\sum F_{friction} = 0$$

$$\sum_i^N F_{pressure} = -P_i . A_i . \widehat{n_i}$$

Solving for x:

$$F_{reaction-x} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \bar{v}_1 (\bar{v}_1 \cdot \widehat{n_1}) A_1 + \rho \bar{v}_2 (\bar{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-x} + P_1 A_1 - P_2 A_2 \cos(50) = -\rho v_1^2 A_1 + \rho v_2^2 A_2 \cos(50)$$

$$F_{reaction-x} + 5 \cdot 10^5\ (0.06)^2 - 500841\ (0.1)^2 \cos(50) = -1000 \cdot (1.39^2)(0.06)^2 + 1000 \cdot (0.5^2)(0.1)^2 \cos(50)$$

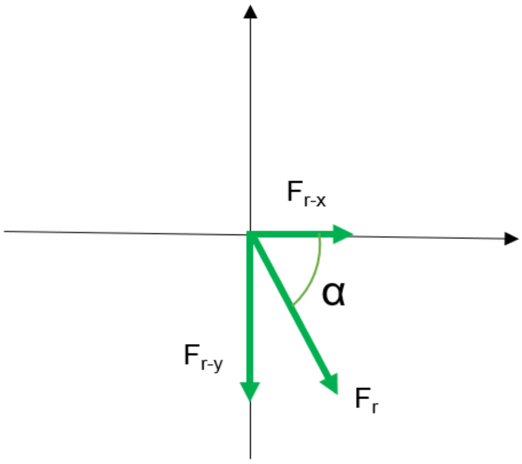
$$F_{reaction-x} = -1800 + 3219.3 - 6.9 + 1.6 = 1414.0\ N$$

Solving for y:

$$F_{reaction-y} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \bar{v}_1 (\bar{v}_1 \cdot \widehat{n_1}) A_1 + \rho \bar{v}_2 (\bar{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-y} + P_2 A_2 cos(40) = -\rho v_2^2 A_2 \cos(40)$$

$$F_{reaction-y} + 500841 \cdot (0.1)^2 cos(40) = -1000 (0.5^2)(0.1)^2 \cos(40)$$



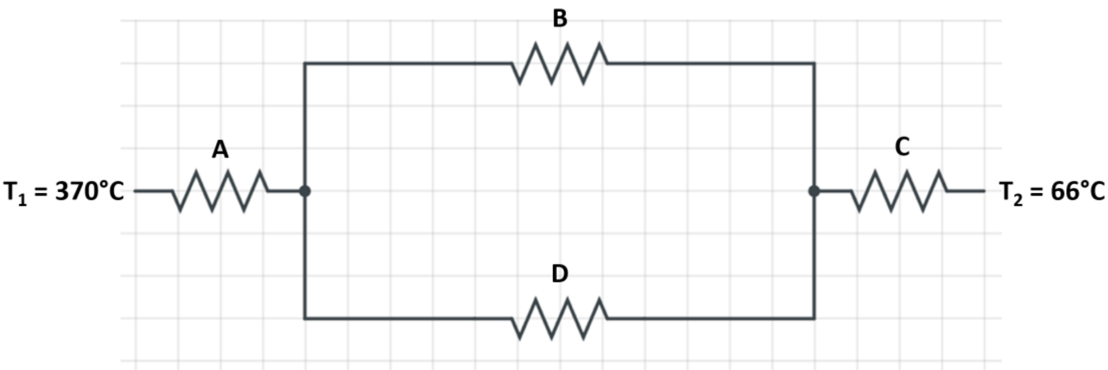
$$F_{reaction-y} = -3836.7 - 1.9 = -3836.6\ N$$

$$F_r = \sqrt{F_{r-x}^2 + F_{r-y}^2} = 4088.9\ N$$

$$arctg\ \alpha = \frac{F_{r-y}}{F_{r-x}} = 69.8°$$

Question 3

The first thing we do for these heat transfer problems is to draw the thermal circuit:



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

$$R_{tot} = \frac{L_A}{k_A A_A} + \frac{1}{\left(\frac{k_B A_B}{L_B}\right) + \left(\frac{k_D A_D}{L_D}\right)} + \frac{L_C}{k_C A_C}$$

$$R_{tot} = \frac{0.025}{150 * 0.1} + \frac{1}{\left(\frac{30 * 0.05}{0.0075}\right) + \left(\frac{70 * 0.05}{0.0075}\right)} + \frac{0.05}{50 * 0.1} = 0.026667\ K \cdot W^{-1}$$

$$q = -\frac{\Delta T}{R_{tot}} = -\frac{66 - 370}{0.026667} = 11399.8\ W \approx 11.4\ kW$$

Question 4

a)

$$\dot{m}_h = \rho_h \dot{V}_h = \frac{980.5kg}{m^3} * \frac{0.00105m^3}{60s} = 0.0172\,\frac{kg}{s}$$

$$\dot{m}_c = \rho_c \dot{V}_c = \frac{997.3kg}{m^3} * \frac{0.00155m^3}{60s} = 0.0258\,\frac{kg}{s}$$

$$\dot{Q}_h = \left[\dot{m}c_p(T_{in} - T_{out})\right]_h = \left(\frac{0.0172kg}{s}\right)\left(\frac{4187kJ}{kg\cdot°C}\right)(71.5°C - 58.2°C) = 957.8\,W$$

$$\dot{Q}_c = \left[\dot{m}c_p(T_{in} - T_{out})\right]_c = \left(\frac{0.0258kg}{s}\right)\left(\frac{4180kJ}{kg\cdot°C}\right)(27.8°C - 19.2°C) = 873.5\,W$$

b)

$$\Delta T_1 = T_{h,in} - T_{c,out} = 71.5°C - 27.8°C = 43.7°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 58.2°C - 19.7°C = 38.5°C$$

$$\Delta T_{lm} = \frac{(\Delta T_1 - \Delta T_2)}{ln ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = 41°C$$

Area can be calculated from the length and outer diameter of the cold pipe given.

$$U = \frac{\dot{Q}_{hc,m}}{A\Delta T_{lm}} = \frac{\left(\left(\frac{957.8 + 873.5}{2}\right)W\right)}{1.6m^2 * 41°C} = 13.96\frac{W}{m^2\cdot°C}$$

c)

$$f_{loss} = \frac{\dot{Q}_h - \dot{Q}_c}{\dot{Q}_h} = \frac{957.8 - 873.5}{957.8} = 0.088 = 8.8\%$$

d)

$$\dot{Q}_h * 5 = 4789\,W \rightarrow 4789\,W * 91.2\%$$
$$= 4368\,W\ heat\ flow\ via\ heat\ exchanger\ in\ both\ directions$$

$$A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{4368W}{13.96\left(\frac{W}{m^2°C}\right)*41°C} = 7.63\,m^2$$

Length of pipe necessary is

$$L = \frac{7.63m^2}{2 * \pi * 0.025} = 48.6\,m$$

$$48.6\,m * 8\frac{CHF}{m} = 388.73\,CHF$$