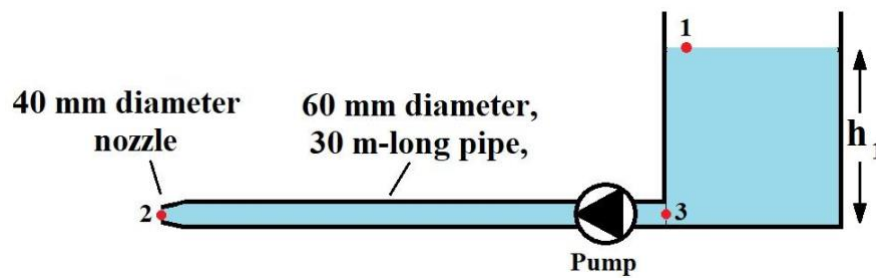


Mid-term exam simulation

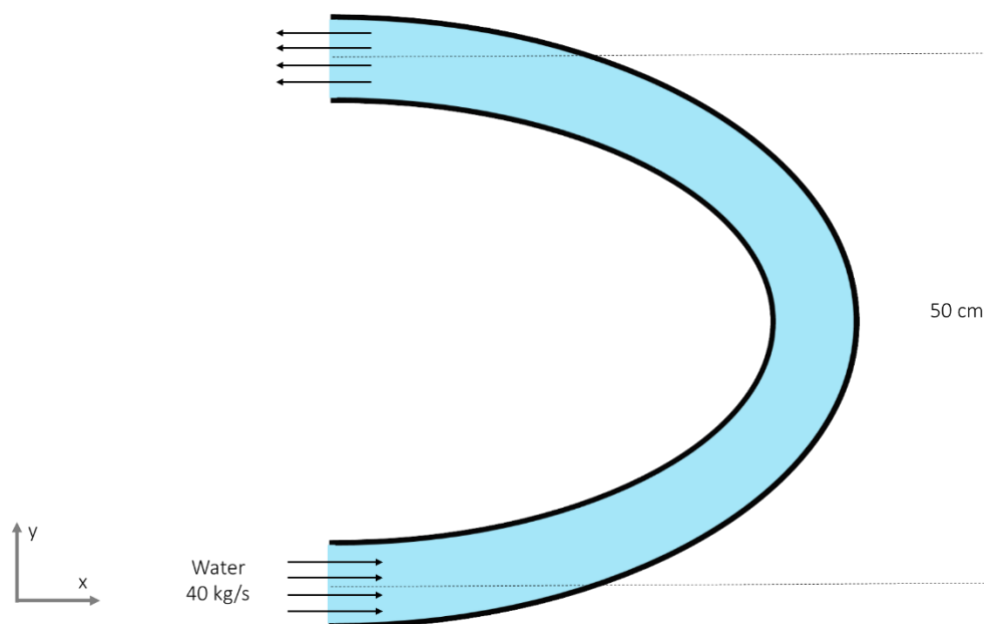
Question 1

In the system below, the pump provides a power of 25 kW and generates a flow rate of $0.04 \text{ m}^3/\text{s}$ in the pipe. The fluid is water so $\rho = 1000 \text{ kg/m}^3$ and $\mu = 1.69 \times 10^{-3} \text{ Pa} \cdot \text{s}$. Also take $g = 9.8 \text{ m/s}^2$. To calculate the friction pressure drop in the pipe, assume the roughness of the pipe $\varepsilon = 6 \times 10^{-6} \text{ m}$ and neglect the minor losses.

- Determine the height of water in the reservoir (h_1).
- Determine the flow rate if the pump is removed from the system.

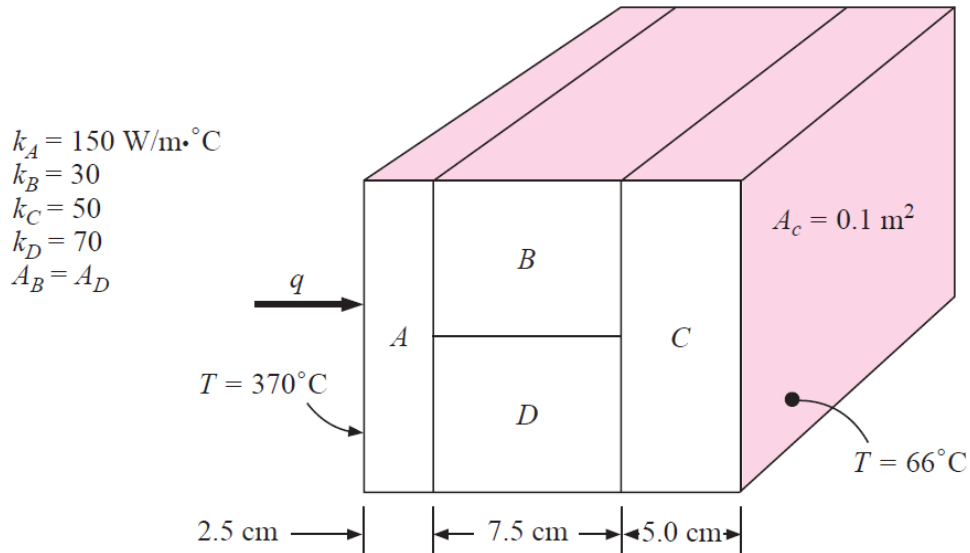
Question 2

When a flowing fluid changes direction, a force acts on the bend and tends to move it. In many cases the joints are sufficiently strong to prevent such movement, but for large pipes (i.e. those used in hydroelectric installations) large concrete anchorages are usually employed to keep the pipe-bends in place, and it is important to know how much force a support must withstand. In this case, a 180° elbow is used to direct water ($\rho = 1000 \text{ kg/m}^3$) flow upward at a mass flow rate of 40 kg/s . The diameter of the entire elbow is 10 cm . The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm . The weight of the elbow, including the water within it, is 2 kg . Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.

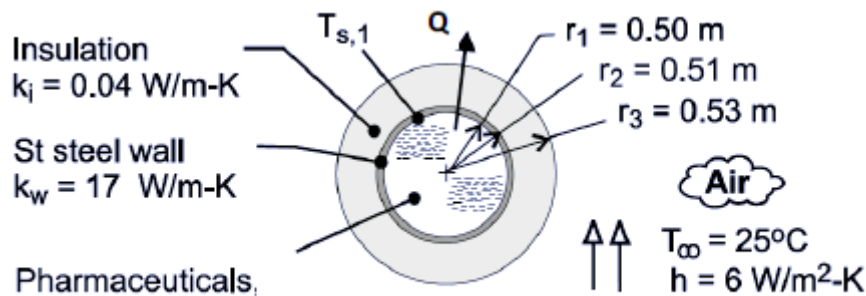


Question 3

Find the heat transfer flow rate (q) through the composite wall in the following figure. Assume one-dimensional heat flow in the direction of the bold arrow.

Question 4

The figure below illustrates the cross section of a cylindrical vessel with length $L = 1 \text{ m}$.



a) In this first question, consider the vessel above without the insulation layer, just imagine it is not there. In this case the inner surface temperature $T_{s,1} = 50^\circ\text{C}$. Calculate the heat flow rate Q . What is the dominant resistance?

b) Now include the insulation layer, assume the heat flow rate Q to be the same and calculate the inner surface temperature $T_{s,1}$ in this case.

Mid-term exam simulation solution

Question 1

a) We apply the Bernoulli's equation between point 1 and point 3. Keep in mind that for the same assumption made in the Torricelli's theorem $v_1 \ll v_3$. So, we write:

$$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_3 + \rho g h_3 + \frac{1}{2} \rho v_3^2$$

Then we apply the Bernoulli equation between point 2 and point 3, ($h_2 = h_3 = 0$).

$$P_3 + \frac{1}{2} \rho v_3^2 + P_{pump} = P_2 + \frac{1}{2} \rho v_2^2 + \Delta P_f$$

Comparing the two equations, we can write:

$$P_1 + \rho g h_1 + P_{pump} = P_2 + \frac{1}{2} \rho v_2^2 + \Delta P_f$$

$$P_{1,gauge} = P_{2,gauge} = 0$$

$$v_2 = \frac{Q}{A_2} = \frac{0.04 \frac{m^3}{s}}{\frac{\pi}{4} (0.04 m)^2} = 31.84 \frac{m}{s}$$

In order to find the ΔP_f , we need to find the friction factor:

$$Re = \frac{\rho * v_{pipe} * D}{\mu}$$

$$v_{pipe} = \frac{Q}{A_{pipe}} = \frac{0.04 \frac{m^3}{s}}{\frac{\pi}{4} (0.06 m)^2} = 14.15 \frac{m}{s}$$

Consider $v_{pipe} = v_3$

$$Re = \frac{1000 \frac{kg}{m^3} * 14.15 \frac{m}{s} * 0.06 m}{1.69 * 10^{-3} Pa s} = 5 * 10^5$$

$$\frac{\varepsilon}{D_{pipe}} = \frac{6 * 10^{-6} m}{0.06 m} = 0.0001$$

Friction factor from Moody diagram: $f_f = 0.004$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2} \rho v_{pipe}^2 \left(\left(\frac{4f_f}{D_{pipe}} \sum_i L_i \right) + \sum_j K_j \right)$$

$$\Delta P_f = \frac{1}{2} * 1000 \frac{kg}{m^3} * \left(14.15 \frac{m}{s} \right)^2 * \left(4 * \frac{0.004}{0.06m} * 30 + 0 \right) = 8 * 10^5 Pa$$

$$P_{pump} = \frac{power}{Q} = \frac{25 \times 10^3 W}{0.04 \frac{m^3}{s}} = 625000 Pa$$

$$h_1 \rho g = \frac{\rho v_2^2}{2} + \Delta P_f - P_{pump} = \frac{1000 \frac{kg}{m^3} * \left(31.8 \frac{m}{s} \right)^2}{2} + 8 * 10^5 - 625000 = 680620 Pa$$

$$h_1 = \frac{680620 Pa}{1000 \frac{kg}{m^3} * 9.81 \frac{m}{s^2}} = \mathbf{69.4 m}$$

b)

$$Q = A_{pipe} * v_{pipe} = A_2 * v_2$$

$$v_2 = \frac{A_{pipe} * v_{pipe}}{A_2} = \left(\frac{D_{pipe}}{D_2} \right)^2 * v_{pipe} = \left(\frac{0.06}{0.04} \right)^2 * v_{pipe} = 2.25 * v_{pipe}$$

Without the pump: $P_{pump} = 0$ and we already calculated $h_1 = 69.4 m$. So, we can write:

$$h_1 = 69.4 m = \frac{v_2^2}{2g} + \frac{v_{pipe}^2}{2g} \left(\frac{4f_f}{D_{pipe}} \sum_i L_i \right) = \frac{(2.25 v_{pipe})^2}{2g} + \frac{v_{pipe}^2}{2g} \left(\frac{4f_f}{D_{pipe}} \sum_i L_i \right)$$

$$69.4 m = \frac{v_{pipe}^2}{2 * 9.81 \frac{m}{s^2}} \left(5.06 + \frac{4 * 0.004 * 30 m}{0.06 m} \right)$$

$$v_{pipe} = \sqrt{\frac{69.4 * 2 * 9.81 m^2}{13.06 s^2}} = \mathbf{10.21 \frac{m}{s}}$$

$$Q = v_{pipe} * A_{pipe} = 10.21 \frac{m}{s} * \frac{\pi}{4} (0.06 m)^2 = 0.0288 \frac{m^3}{s}$$

Question 2

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^N \int_{A_i} \rho v(v \cdot n) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i \cdot A_i \cdot n_i$$

$$\sum F_{volume} = m_{elbow} \cdot g$$

We need to calculate the gauge pressure so we apply Bernoulli's principle between the inlet point (1) and the discharge point (0)

$$P_1 + \frac{1}{2} \rho v^2 = P_0 + \frac{1}{2} \rho v^2 + \rho g h_0$$

We know that mass flow rate, $\dot{m} = \rho A v$

$$\text{Thus, velocity of water in the pipe inlet, } v_1 = \frac{\dot{m}}{\rho A} = \frac{40}{1000 \times (\pi \times (0.1)^2) / 4} = 5.09 \text{ m/s}$$

Applying conservation of mass between the inlet point (1) and the discharge point (0)

$$A_1 v_1 = A_0 v_0$$

Since the area of pipe is not changing,

$$A_1 = A_0 = A$$

Hence, $v_1 = v_0 = v = 5.09 \text{ m/s}$

By replacing the values in the Bernoulli's equation:

$$P_1 + 0.5 \times 1000 \times (5.09)^2 = P_0 + 0.5 \times 1000 \times (5.09)^2 + 1000 \times 9.81 \times 0.5$$

$$P_1 - P_0 = 4.9 \text{ kPa}$$

We consider the pressures as gauge pressures, thus

$$P_{0gauge} = 0 \quad P_{1gauge} = 4.9 \text{ KPa}$$

In order to determine the anchoring force, we need to resolve the forces along x and y directions.

$$\sum F_{surface} + \sum F_{pressure} = \sum F_{reaction}$$

Resolving the forces along the +x direction using gauge pressure:

$$\begin{aligned} \sum F_{pressure,x} &= -P_{1gauge} \cdot A_1 \cdot n_1 - \cancel{P_{0gauge} \cdot A_0 \cdot n_0} = -P_{1gauge} \cdot A_1 \cdot (-1) = +P_{1gauge} A_1 \\ \sum F_{pressure,x} &= \left(\pi \times \frac{(0.1)^2}{4} \right) (4.9 \times 10^3) = 38.48 \text{ N} \end{aligned}$$

$$\sum F_{volume,x} = 0$$

Therefore,

$$\sum F_{surface,x} = \rho v_1 A_1 (v_1 \cdot n) + \rho v_0 A_0 (v_0 \cdot n) = \rho v_1 A_1 (+v_1)(-1) + \rho v_0 A_0 (-v_0)(+1)$$

$$\rho v_1 A_1 = \rho v_0 A_0 = \dot{m} \quad \rightarrow$$

$$\sum F_{surface,x} = \dot{m}(-v_1) + \dot{m}(-v_0) = \dot{m}(-v_1 - v_0)$$

$$\sum F_{surface,x} = 40 \times (-5.09 - 5.09) = -407.2 \text{ N}$$

$$\sum F_{reaction,x} = \sum F_{surface,x} - \sum F_{pressure,x} = -407.2 - 38.48 = -445.68 \text{ N}$$

Now resolving in the y direction,

$$\sum F_{pressure,y} = 0$$

The weight of the elbow, including the water within it, is 2 kg. Therefore,

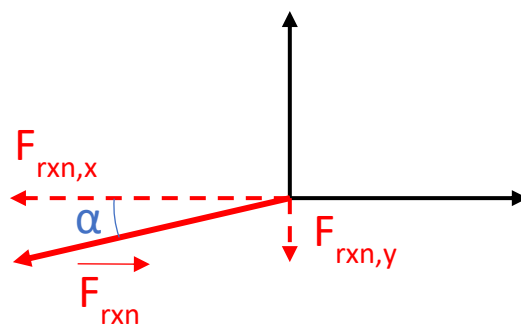
$$\sum F_{reaction,y,total} = -F_{volume,y} = -m_{elbow} \cdot g = -2 \times 9.8 = -19.6 \text{ N}$$

$$F_{net} = ((445.68)^2 + (19.6)^2)^{\frac{1}{2}} = 446.11 \text{ N}$$

And it is directed along angle α :

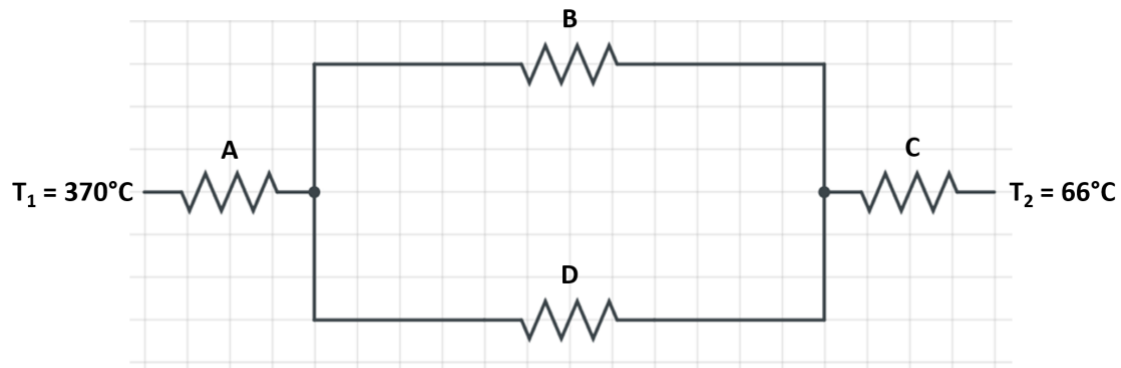
$$\tan \alpha = \frac{F_{rxn,y}}{F_{rxn,x}}$$

$$\alpha = \arctg \frac{19.6}{445.68} = 2.51^\circ$$



Question 3

The first thing we do for these heat transfer problems is to draw the thermal circuit:



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

$$R_{tot} = \frac{L_A}{k_A A_A} + \frac{1}{\left(\frac{k_B A_B}{L_B}\right) + \left(\frac{k_D A_D}{L_D}\right)} + \frac{L_C}{k_C A_C}$$

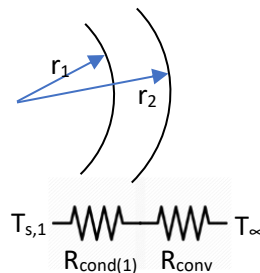
$$R_{tot} = \frac{0.025}{150 * 0.1} + \frac{1}{\left(\frac{30 * 0.05}{0.0075}\right) + \left(\frac{70 * 0.05}{0.0075}\right)} + \frac{0.05}{50 * 0.1} = 0.026667 \text{ K} \cdot \text{W}^{-1}$$

$$q = -\frac{\Delta T}{R_{tot}} = -\frac{66 - 370}{0.026667} = 11399.8 \text{ W} \approx 11.4 \text{ kW}$$

Question 4

Assumptions: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

(a)



$$R_{cond(1)} = \frac{l}{kA} = \int_{r_1}^{r_2} \frac{dr}{k_w 2\pi r l} = \frac{1}{2\pi l k_w} (\ln \ln \frac{r_2}{r_1}) = \frac{1}{2\pi \times 1\text{m} \times \left(\frac{17\text{W}}{\text{m}} \cdot \text{K}\right)} \times \frac{0.51\text{m}}{0.5\text{m}}$$

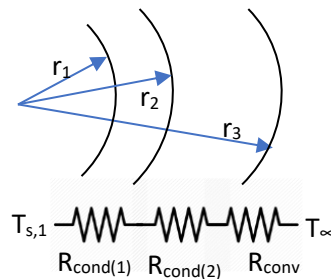
$$R_{cond(1)} = 1.85 \times 10^{-4} \text{ K/W}$$

$$R_{conv} = \frac{1}{2\pi r_2 l h} = \frac{1}{2\pi(0.51m) \times 1m \times 6 W/m^2.K} = 5.2 \times 10^{-2} K/W$$

$R_{conv} > R_{cond(1)} \rightarrow$ The dominant resistance is due to convection.

$$\dot{Q} = \frac{T_{s,1} - T_{\infty}}{\frac{1}{2\pi l k_w} (\ln \ln \frac{r_2}{r_1}) + \frac{1}{2\pi r_2 l h}} = \frac{(50 - 25) ^\circ C}{1.85 \times 10^{-4} K/W + 5.2 \times 10^{-2} K/W} = 479 W$$

(b) With the insulation,



$$R_{cond(1)} = \frac{l}{kA} = \int_{r_1}^{r_2} \frac{dr}{k_w 2\pi r l} = \frac{1}{2\pi l k_w} (\ln \ln \frac{r_2}{r_1}) = \frac{1}{2\pi \times 1m \times (\frac{17W}{m}.K)} \times \frac{0.51m}{0.51m}$$

$$R_{cond(1)} = 1.85 \times 10^{-4} K/W$$

$$R_{cond(2)} = \frac{l}{kA} = \int_{r_2}^{r_3} \frac{dr}{k_i 2\pi r l} = \frac{1}{2\pi l k_i} (\ln \ln \frac{r_3}{r_2}) = \frac{1}{2\pi \times 1m \times (\frac{0.04W}{m}.K)} \times \frac{0.53m}{0.51m}$$

$$R_{cond(2)} = 1.53 \times 10^{-1} K/W$$

$$R_{conv} = \frac{1}{2\pi r_3 l h} = \frac{1}{2\pi(0.53m) \times 1m \times 6 W/m^2.K} = 5.0 \times 10^{-2} K/W$$

$$\dot{Q} = \frac{T_{s,1} - T_{\infty}}{\Sigma R}$$

$$T_{s,1} = T_{\infty} + \dot{Q}[\Sigma R] = T_{\infty} + \dot{Q}[R_{cond(1)} + R_{cond(2)} + R_{conv}]$$

$$T_{s,1} = 25 ^\circ C + 479W[1.85 \times 10^{-4} + 1.53 \times 10^{-1} + 5.0 \times 10^{-2}] \frac{K}{W} = 122.3 ^\circ C$$

