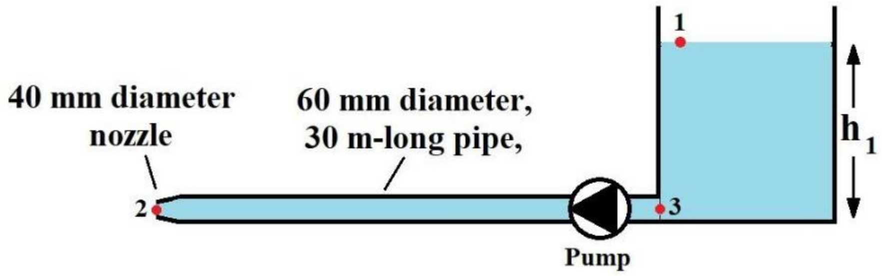
Introduction to Transport Phenomena: mid-term exam simulation

Question 1 (Bernoulli)

In the system below, the 25 kW pump generates a flow rate of 0.04 m^3/s in the pipe. The fluid is water so ρ=1000kg/m^3 and μ=1.69 x 10^{-3} Pa*s. Take g=9.81m/s^2. When calculating the pressure drop due to friction, assume the roughness of the pipe ε = 6 x 10^{-6} m and neglect the minor losses.

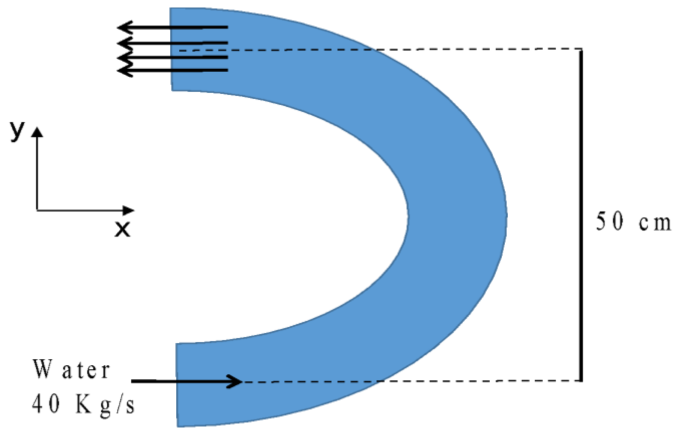
a) Determine the height of water in the reservoir (h_1)

b) Determine the flow rate if the pump is removed from the system.



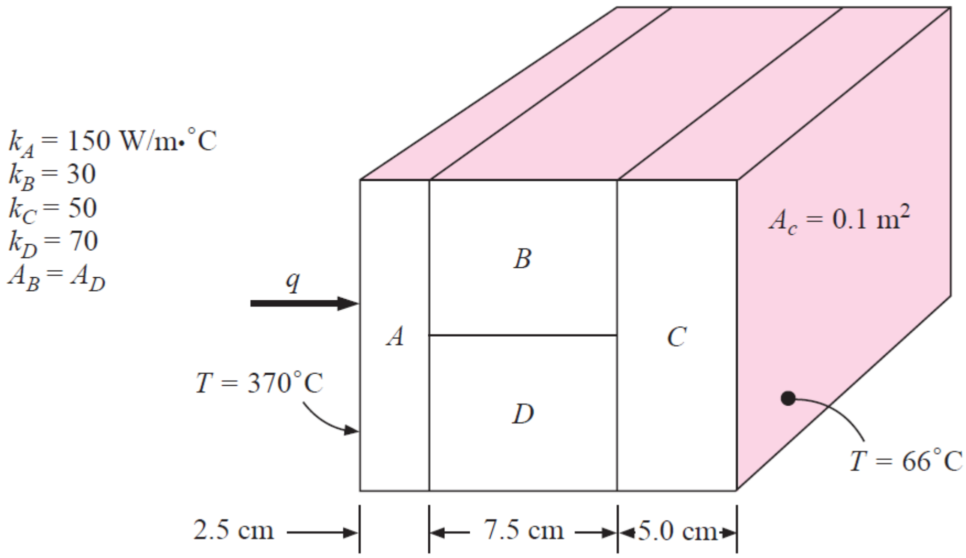
Question 2 (Momentum balance)

When a fluid changes direction, a force acts in the bend and tends to move it. In many cases the joints are sufficiently strong to prevent such movement, but for large pipes (i.e. those used in hydroelectric installations) large concrete anchorages are usually employed to keep the pipe-bends in place, and it is important to know how much force a support must withstand. In this case, a 180° elbow is used to direct water (ρ= 1000 kg/m^3) flow upward at a mass flow rate of 40 kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow, including the water within it, is 2 kg. Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.



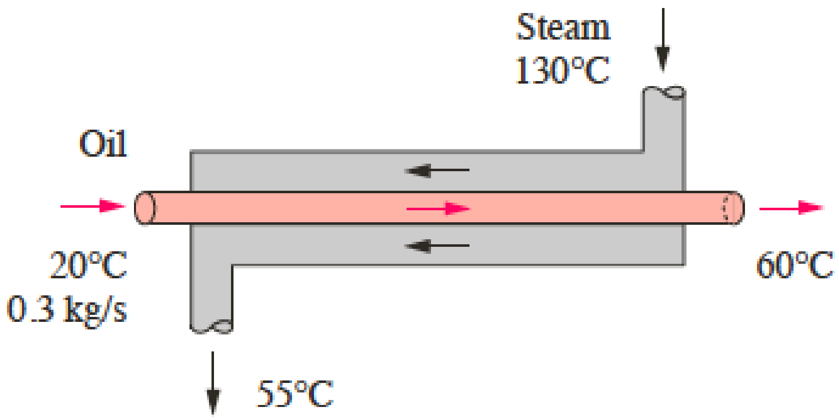
Question 3 (Thermal resistance)

Find the heat transfer flow rate (q) through the composite wall in the following figure. Assume one-dimensional heat flow in the direction of the bold arrow.



Question 4 (Heat exchanger)

Engine oil (c_p=2100 J/ kg °C) is to be heated from 20°C to 60°C at a rate of 0.3 kg/s in a 2-cm-diameter thin walled copper tube by condensing steam outside at a temperature of 130°C. For an overall heat transfer coefficient of 650 W/m^2°C, determine a) the rate of heat transfer and b) the length of the tube required to achieve it.



Introduction to Transport Phenomena: mid-term exam simulation solutions

Question 1

a) We apply the Bernoulli's equation between point 1 and point 3. Keep in mind that for the same assumption made in the Torricelli's theorem $v_1 \ll v_3$. So, we write:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho \overset{0}{v_1^2} = P_3 + \rho g \overset{0}{h_3} + \frac{1}{2}\rho v_3^2$$

Then we apply the Bernoulli equation between point 2 and point 3, ($h_2 = h_3 = 0$).

$$P_3 + \frac{1}{2}\rho v_3^2 + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

Comparing the two equations, we can write:

$$P_1 + \rho g h_1 + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

$P_{1,gauge} = P_{2,gauge} = 0$

$$v_2 = \frac{Q}{A_2} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.04\,m)^2} = 31.84\,\frac{m}{s}$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v_{pipe} * D}{\mu}$$

$$v_{pipe} = \frac{Q}{A_{pipe}} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2} = 14.15\,\frac{m}{s}$$

Consider $v_{pipe}\ is\ v_3$

$$Re = \frac{1000\,\frac{kg}{m^3} * 14.15\,\frac{m}{s} * 0.06m}{1.69 * 10^{-3} Pas} = 5 * 10^5$$

$$\frac{\varepsilon}{D_{pipe}} = \frac{6 * 10^{-6}\,m}{0.06\,m} = 0.0001$$

Friction factor from Moody diagram:

$$f_f = 0.004$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{pipe}^2 \left((\frac{4f_f}{D_{pipe}} \sum_i L_i) + \sum_j K_j \right)$$

$$\Delta P_f = \frac{1}{2} * 1000 \frac{kg}{m^3} * \left(14.15 \frac{m}{s}\right)^2 * \left(4 * \frac{0.004}{0.06m} * 30 + 0\right) = 8 * 10^5\ Pa$$

$$P_{pump} = \frac{power}{Q} = \frac{25 \times 10^3 W}{0.04 \frac{m^3}{s}} = 625000\ Pa$$

$$h_1 \rho g = \frac{\rho v_2^2}{2} + \Delta P_f - P_{pump} = \frac{1000 \frac{kg}{m^3} * \left(31.8 \frac{m}{s}\right)^2}{2} + 8 * 10^5 - 625000 = 680620\ Pa$$

$$\boldsymbol{h_1} = \frac{680620 Pa}{1000 \frac{kg}{m^3} * 9.81 \frac{m}{s^2}} = \boldsymbol{69.4\ m}$$

b)

$$Q = A_{pipe} * v_{pipe} = A_2 * v_2$$

$$v_2 = \frac{A_{pipe} * v_{pipe}}{A_2} = (\frac{D_{pipe}}{D_2})^2 * v_{pipe} = (\frac{0.06}{0.04})^2 * v_{pipe} = 2.25 * \ v_{pipe}$$

Without the pump: $P_{pump} = 0$ and we already calculated $h_1 = 69.4\ m$. So, we can write:

$$h_1 = 69.4\ m = \frac{v_2^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}} \sum_i L_i\right) = \frac{(2.25\ \ v_{pipe})^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}} \sum_i L_i\right)$$

$$69.4\ m = \frac{{v_{pipe}}^2}{2 * 9.81 \frac{m}{s^2}} \left(5.06 + \frac{4 * 0.004\ \ * 30\ m}{0.06\ m}\right)$$

$$\boldsymbol{v_{pipe}} = \sqrt{\frac{69.4 * 2 * 9.81\ m^2}{13.06\ s^2}} = \boldsymbol{10.21 \frac{m}{s}}$$

$$Q = v_{pipe} * A_{pipe} = 10.21\,\frac{m}{s} * \frac{\pi}{4}(0.06\ m)^2 = 0.0288\,\frac{m^3}{s}$$

Question 2

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v \cdot n) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . n_i$$

$$\sum F_{volume} = m_{elbow} . g$$

We need to calculate the gauge pressure so we apply Bernoulli's principle between the inlet point (1) and the discharge point (0)

$$P_1 + \frac{1}{2}\rho v^2 = P_0 + \frac{1}{2}\rho v^2 + \rho g h_0$$

We know that mass flow rate,$\dot{m} = \rho A v$

Thus, velocity of water in the pipe inlet ,$v_1 = \frac{\dot{m}}{\rho A} = \frac{40}{1000 \times (\pi \times (0.1)^2)/4} = 5.09\ m/s$

Applying conservation of mass between the inlet point (1) and the discharge point (0)

$$A_1 v_1 = A_0 v_0$$

Since the area of pipe is not changing,

$$A_1 = A_0 = A$$

Hence, $v_1 = v_0 = v = 5.09\ m/s$

By replacing the values in the Bernoulli's equation:

$$P_1 + 0.5 \times 1000 \times (5.09)^2 = P_o + 0.5 \times 1000 \times (5.09)^2 + 1000 \times 9.81 \times 0.5$$

$$\boldsymbol{P_1 - P_o = 4.9\ kPa}$$

We consider the pressures as gauge pressures, thus

$$P_{0gauge} = 0 \qquad P_{1gauge} = 4.9\ KPa$$

In order to determine the anchoring force, we need to resolve the forces along x and y directions.

$$\sum F_{surface} + \sum F_{pressure} = \sum F_{reaction}$$

Resolving the forces along the +x direction using gauge pressure:

$$\sum F_{pressure,x} = -P_{1gauge}.A_{1.}.n_1 - \cancel{P_{0gauge}.A_{0.}.n_0} = -P_{1gauge}.A_{1.}.(-1) = +P_{1gauge}A$$

$$\sum F_{pressure,x} = \left(\pi \times \frac{(0.1)^2}{4}\right)(4.9 \times 10^3) = 38.48\ N$$

$$\sum F_{volume,x} = 0$$

Therefore,

$$\sum F_{surface,x} = \rho v_1 A_1 (v_1.n) + \rho v_0 A_0 (v_0.n) = \rho v_1 A_1 (+v_1)(-1) + \rho v_0 A_0 (-v_0)(+1)$$

$\rho v_1 A_1 = \rho v_0 A_0 = \dot{m}$ ➔

$$\sum F_{surface,x} = \dot{m}(-v_1) + \dot{m}(-v_0) = \dot{m}(-v_1 - v_0)$$

$$\sum F_{surface,x} = 40 \times (-5.09 - 5.09) = -407.2\ N$$

$$\sum F_{reaction,x} = \sum F_{surface,x} - \sum F_{pressure,x} = -407.2 - 38.48 = -445.68\ N$$

Now resolving in the y direction,

$$\sum F_{pressure,y} = 0$$

The weight of the elbow, including the water within it, is 2 kg. Therefore,

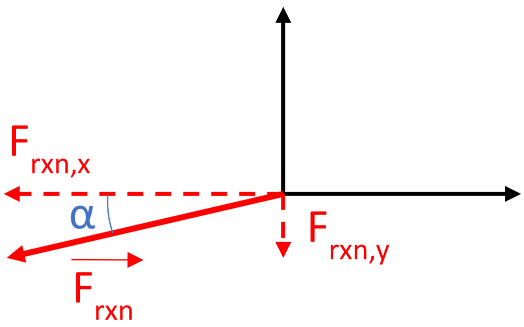
$$\sum F_{reaction,y.total} = -F_{volume,y} = -m_{elbow}.g = -2 \times 9.8 = -19.6\ N$$

$$F_{net} = ((445.68)^2 + (19.6)^2)^{\frac{1}{2}} = 446.11\ N$$

And it is directed along angle α:

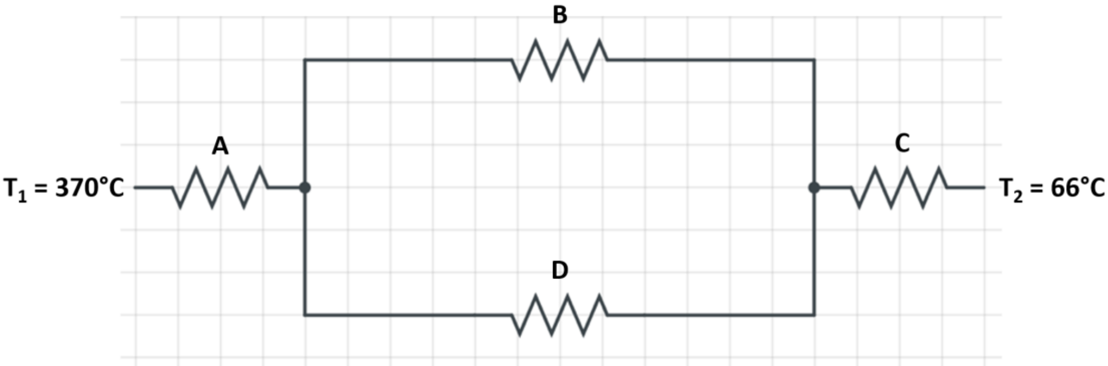
$$tan\ \alpha = \frac{F_{rxn,y}}{F_{rxn,x}}$$

$$\alpha = arctg\ \frac{19.6}{445.68} = 2.51°$$



Question 3

The first thing we do for these heat transfer problems is to draw the thermal circuit



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

$$R_{tot} = \frac{L_A}{k_A A_A} + \frac{1}{\left(\frac{k_B A_B}{L_B}\right) + \left(\frac{k_D A_D}{L_D}\right)} + \frac{L_C}{k_C A_C}$$

$$R_{tot} = \frac{0.025}{150 * 0.1} + \frac{1}{\left(\frac{30 * 0.05}{0.0075}\right) + \left(\frac{70 * 0.05}{0.0075}\right)} + \frac{0.05}{50 * 0.1} = 0.026667\ K \cdot W^{-1}$$

$$q = -\frac{\Delta T}{R_{tot}} = -\frac{66 - 370}{0.026667} = 11399.8\ W \ \approx 11.4\ kW$$

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Question 4

a) All the parameters required to find the rate of heat transfer to the cold fluid have been given.

$$\dot{Q}_c = \dot{m}_c c_p \left(T_{c_{out}} - T_{c_{in}}\right) = 0.3 * 2100 * (60 - 20) = 25.2\ kW$$

b) The inlet and outlet temperatures of the hot fluid have also been given. As we have all four temperatures to calculate the ΔTm, we will do so using the formula for counter current heat exchangers:

$$\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{(130 - 60) - (55 - 20)}{\ln\left(\frac{130 - 60}{55 - 20}\right)} = 50.5\ K$$

$$Q = UA\Delta T_{LM} \ \rightarrow A = \frac{Q}{U\Delta T_{LM}}$$

$$A = \frac{25200}{650 * 50.5} = 0.77\ m^2$$

$$A = 2\pi r l \ \rightarrow l = \frac{A}{2\pi r} = \frac{0.77}{2 * \pi * 0.01} = 12.22\ m$$