ChE 204 – Introduction to Transport Phenomena

Spring 2020

Mid-term Examination Simulation

Question 1:
A pipe system in stainless steel carries water (ρ = 1000 kg/m^3; μ= 8.9 x 10^{-4} Pa·s) and discharges it as a free jet in the atmosphere, as shown in the following figure.

a) Ignoring all the losses, if the pressure P_3, which is right at the entrance of the pipe system, is 7 x 10^5 Pa, what will be the volumetric and mass flow rate of the water coming out from the pipe? Consider the inner diameter of the pipe to be 200 mm.

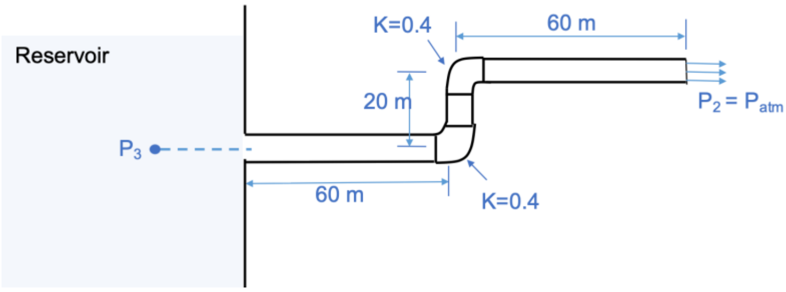


Figure for Question 1a)

For Questions 1b) and 1c) we remove the reservoir and we focus only on the pipe system as depicted here:

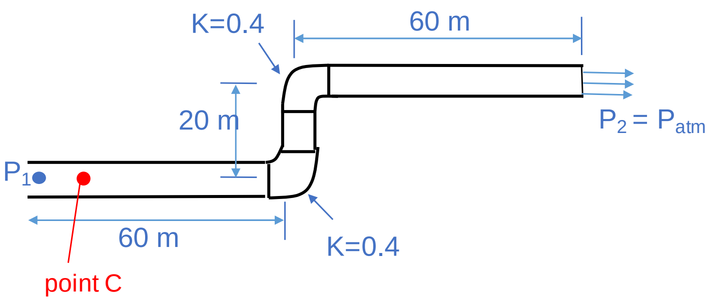


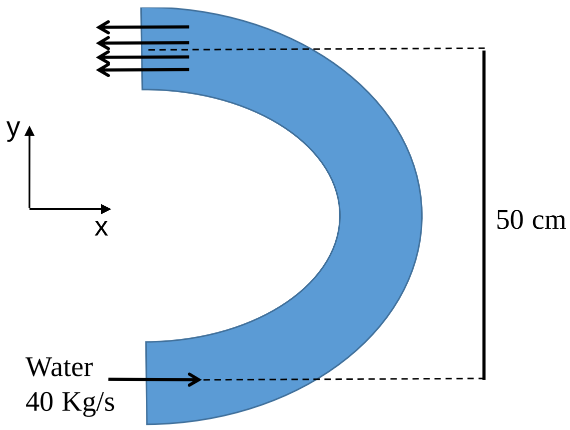
Figure for Question 1b) and 1c)

b) What gauge pressure P_1 would be needed to provide a volumetric flow rate of 12 m^3/min of water? Assume a smooth pipe. In this part all of the losses in the system should be considered.

c) If the frictional losses inside the system were supposed to be compensated with a pump located at point C, what will be the required power of the pump?

******

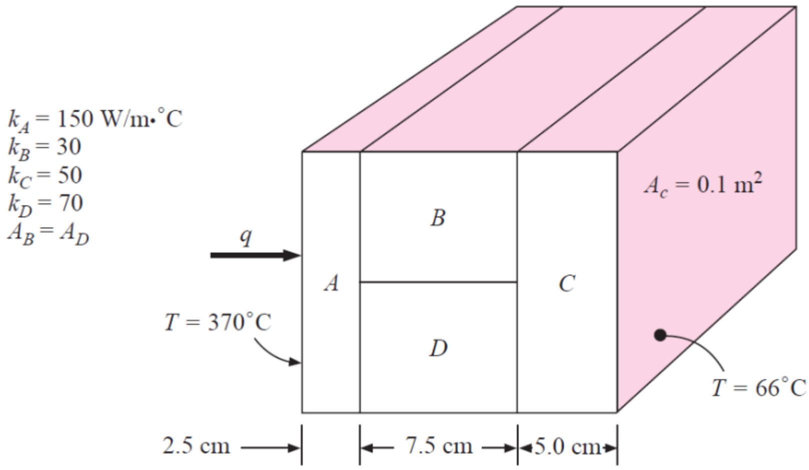
Question 2:



A 180° elbow is used to direct water (ρ= 1000 kg/m^3) flow upward at a mass flow rate of 40 Kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow, including the water within it, is 2 kg. Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.

******

Question 3:
Find the heat transfer flow rate (Q) through the composite wall in the following figure. Assume one-dimensional heat flow.



Question 04:

A concentric tube heat exchanger is used to thermally process a pharmaceutical product flowing at a mean velocity of $u_{m,c}$ =3.5 m/s with an inlet temperature of $T_{c,i}$= 20 °C. The inner tube of diameter d_i=10mm, the thickness of the inner tube is 2 mm with the thermal conductivity of k_t = 40 W/m^{2}K, and the exterior of the outer tube (d_o= 20 mm) is well insulated. Water flows in the outside annular region at a mean velocity of $u_{m,h}$ = 5 m/s with an inlet temperature of $T_{h,i}$ = 60 °C. Properties of the pharmaceutical product are μ=1.1*10^{-2} (N·s)/m^2, k= 0.25 W/mK, ρ =1100 kg/m^3, and c_p= 2460 J/kgK. Properties of the water may be approximated as μ =9.60*10^{-4} (N·s)/m^2, k=0.6. The specific heat of the water is 4180 J/kgK. Consider the surface roughness of the tube to be 0.1 mm.

(a) If the convective heat transfer coefficient for water corresponding to the outer surface area of inner tube is h_o=1200 W/m^{2}K, determine the value of the overall heat transfer coefficient U.

Note: The heat transfer coefficient inside the tubes, h_i, over a large range of the Reynolds number can be obtained by Gnielinski equation:

$$Nu_D = \frac{\left(\frac{f}{8}\right) * (Re_D - 1000) * Pr}{1 + 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}$$

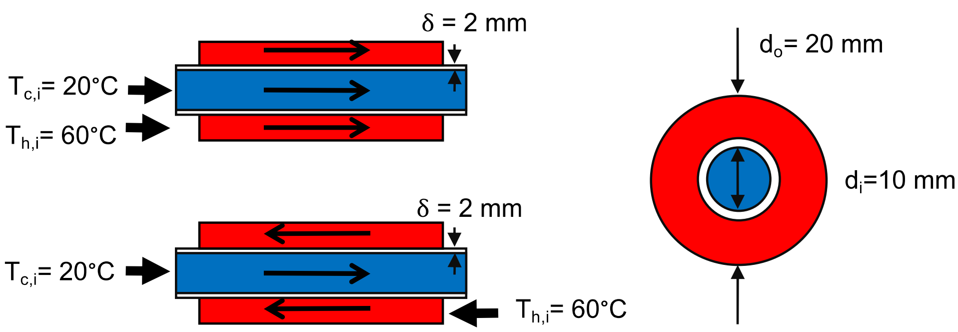
Where

$$\text{Pr (Prandtl number)} = \frac{C_p\mu}{k} \quad and\ Nu\ (Nusselt\ number) = \frac{h_i d_i}{k}$$

where the friction factor may be obtained from the Moody diagram. The correlation is valid for 0.5 =< Pr=< 2000 and 3000 =< Re_D =< 5*10^6.

(b) If there is a fouling material inside the inner tube with the thickness of 2 mm and the thermal conductivity of k_{foul}= 0.5 W/mK, determine the overall heat transfer coefficient. For the sake of simplicity, assume that the velocities remain unchanged.

(c) If the desired outlet temperature of the pharmaceutical product is 40 °C, obtain the required heat transfer area (A) in both parallel and countercurrent situation considering the overall heat transfer coefficient obtained in the section a.



Introduction to Transport Phenomena: <u>Solutions for Mid-Term Simulation</u>

Question 1

a)

The velocity of water in the tank, before it enters the pipe, can be approximated to 0 due to the large area at the top of the tank in comparison to the small area of the pipe.

Thus the Bernoulli equation between point 3 and point 2 becomes:

$$P_3 + \cancel{\rho g h_3} + \cancel{\frac{1}{2}\rho v_3^2} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \cancel{\Delta P_f}$$

Solve equation for v_2:

$$v_2 = \left(2 * \frac{P_3 - P_2 - \rho g h_2}{\rho}\right)^{\frac{1}{2}}$$

Numerical solution:

$$v_2 = \left(2 * \frac{(7-1) * 10^5 Pa - 1000\frac{kg}{m^3} * 9.81\frac{m}{s^2} * 20m}{1000\frac{kg}{m^3}}\right)^{\frac{1}{2}} = 28.41\frac{m}{s}$$

Volumetric flow rate:

$$Q = A_2 * v_2$$

$$Q = \left(\frac{d_i}{2}\right)^2 * \pi * v_2$$

$$Q = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\frac{m}{s} = 0.892\frac{m^3}{s}$$

Mass flow rate:

$$\dot{m} = A_2 * v_2 * \rho$$

$$\dot{m} = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\frac{m}{s} * 1000\frac{kg}{m^3} = 892\frac{kg}{s}$$

b)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Applying mass conservation at point 1 and point 2,

$$\rho v_1 A_1 = \rho v_2 A_2 = \dot{m}$$

Since $A_1 = A_2$, we get $v_1 = v_2$

$$P_1 + \rho g h_1 + \cancel{\frac{1}{2}\rho v_1^2} = P_2 + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \Delta P_f$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v_1 * D}{\mu}$$

The volumetric flow rate, $\dot{v} = \frac{\dot{m}}{\rho}$

$$v_1 = \frac{\dot{v}}{\rho A_1} = \frac{\frac{12}{60}\ (\frac{m^3}{s})}{\frac{\pi(0.2)^2\ (m^2)}{4}} = 6.36\ m/s$$

$$Re = \frac{1000\frac{kg}{m^3} * 6.366\frac{m}{s} * 0.2m}{8.9 * 10^{-4} Pas} = 1.431\ * 10^6$$

Friction factor from Moody diagram (consider the “hydraulically smooth” curve):

$$f_f = 0.0028$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2 \left(\frac{4 f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

Loss coefficients:

$$\sum_i L_i = 60m + 20m + 60m = 140m$$

$$\sum_j K_j = 0.4 + 0.4 = 0.8$$

$$\Delta P_f = \frac{1}{2} * 1000\frac{kg}{m^3} * \left(6.366\frac{m}{s}\right)^2 * \left(4 * \frac{0.0028}{0.2m} * (140m + 0.80)\right) = 1.59 * 10^5 Pa = 1.59 bar$$

$$P_1 = P_2 + \rho g(h_2 - h_1) + \Delta P_f = 1 * 10^5 + (1000 * 9.81 * 20) + 1.59 * 10^5 = 4.55 * 10^5 Pa = 4.55 bar$$

c)
The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f * Q = 1.59 * 10^5 Pa * 0.2 \frac{m^3}{s} = 31.8kW$$

****

Question 2

$$\sum_{\square}^{\square} \square F_{surface} + \sum_{\square}^{\square} \square F_{volume} = \sum_{i=1}^{N} \square \int_{A_i}^{\square} \square \rho v(v \cdot n) dA_i$$

$$\sum_{\square}^{\square} \square F_{surface} = \sum_{\square}^{\square} \square F_{friction} + \sum_{\square}^{\square} \square F_{pressure} + \sum_{\square}^{\square} \square F_{reaction}$$

$$\sum_{\square}^{\square} \square F_{friction} = 0$$

$$\sum_{\square}^{\square} \square F_{pressure} = -P_i . A_i . n_i$$

$$\sum_{\square}^{\square} \square F_{volume} = m_{elbow} . g$$

We need to calculate the gauge pressure so we apply Bernoulli's principle between the inlet point (1) and the discharge point (0)

$$P_1 + \frac{1}{2}\rho v^2 = P_0 + \frac{1}{2}\rho v^2 + \rho g h_0$$

We know that mass flow rate, $\dot{m} = \rho A v$

Thus, velocity of water in the pipe inlet , $v_1 = \frac{\dot{m}}{\rho A} = \frac{40}{1000 \times (\pi \times (0.1)^2)/4} = 5.09\ m/s$

Applying conservation of mass between the inlet point (1) and the discharge point (0)

$$A_1 v_1 = A_0 v_0$$

Since the area of pipe is not changing,

$$A_1 = A_0 = A$$

Hence, $v_1 = v_0 = v = 5.09\ m/s$

By replacing the values in the Bernoulli's equation:

$$P_1 + 0.5 \times 1000 \times (5.09)^2 = P_o + 0.5 \times 1000 \times (5.09)^2 + 1000 \times 9.81 \times 0.5$$

$$P_1 - P_o = 4.9\ kPa$$

We consider the pressures as gauge pressures, thus

$$P_{0gauge} = 0 \qquad P_{1gauge} = 4.9\ KPa$$

In order to determine the anchoring force, we need to resolve the forces along x and y directions.

$$\sum F_{surface} + \sum F_{pressure} = \sum F_{reaction}$$

Resolving the forces along the +x direction using gauge pressure:

$$\sum F_{pressure,x} = -P_{1gauge}.A_{1}.n_1 - \cancel{P_{0gauge}.A_0.n_0} = -P_{1gauge}.A_{1}.(-1) = +P_{1gauge}A$$

$$\sum F_{pressure,x} = \left(\pi \times \frac{(0.1)^2}{4}\right)(4.9 \times 10^3) = 38.48\ N$$

$$\sum F_{volume,x} = 0$$

Therefore,

$$\sum F_{surface,x} = \rho v_1 A_1 (v_1.n) + \rho v_0 A_0 (v_0.n) = \rho v_1 A_1 (+v_1)(-1) + \rho v_0 A_0 (-v_0)(+1)$$

$\rho v_1 A_1 = \rho v_0 A_0 = \dot{m}$ ⍰

$$\sum F_{surface,x} = \dot{m}(-v_1) + \dot{m}(-v_0) = \dot{m}(-v_1 - v_0)$$

$$\sum F_{surface,x} = 40 \times (-5.09 - 5.09) = -407.2\ N$$

$$\sum F_{reaction,x} = \sum F_{surface,x} - \sum F_{pressure,x} = -407.2 - 38.48 = -445.68\ N$$

Now resolving in the y direction,

$$\sum F_{pressure,y} = 0$$

The weight of the elbow, including the water within it, is 2 kg. Therefore,

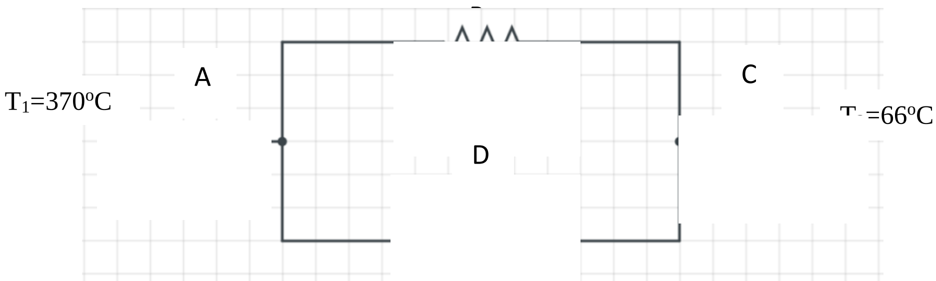
$$\sum F_{reaction,y.total} = -F_{volume,y} = -m_{elbow}.g = -2 \times 9.8 = -19.6\ N$$

$$F_{net} = ((445.68)^2 + (19.6)^2)^{\frac{1}{2}} = 446.11\ N$$

****

Question 3

The first thing we do for these heat transfer problems is to draw the thermal circuit



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

$$R_{tot} = \frac{L_A}{k_A.A_A} + \frac{1}{\frac{(k_B.A_B\,)}{L_B} + \frac{(k_D.A_D\,)}{L_D}} + \frac{L_C}{k_C.A_C}$$

$$R_{tot} = \frac{0.025}{150*0.1} + \frac{1}{\frac{(30*0.05)}{0.074} + \frac{(70.0.05)}{0.075}} + \frac{0.05}{50.0.1}$$

$$R_{tot} = 0.001667 + 0.015 + 0.01 = 0.026667\; K.W^{-1}$$

$$q_{1\rightarrow 2} = \frac{-\Delta T}{R_{tot}} = -\frac{(66-370)}{0.02667}$$

$$q_{1\rightarrow 2} = 11399.8\; W = 12\; kW$$

****

Question 4

$d_{i,i} = 10\ mm$ $\delta = 2\ mm$ $d_{o,i} = 14\ mm$ h_o=1200 W/m^{2}K

$$\frac{1}{r_{o,i}\ U_0} = \frac{1}{r_{i,i}\ h_i} + \frac{ln\ ln\ \left(\frac{r_{o,i}}{r_{i,i}}\right)}{k} + \frac{1}{r_{o,i}\ h_o}$$

For calculation of h_i, we should calculate Re and then Nu:

$$Re_i = \frac{\rho_i\ u_i d_{i,i}}{\mu_i} = \frac{1100 * 3.5 * 0.01}{1.1 * 10^{-2}} = 3500$$

$$\frac{\varepsilon}{d} = \frac{0.1}{10} = 0.01$$

From the Moody diagram and the values of Re and $\frac{\varepsilon}{d}$ $\longrightarrow$ $f = 0.013$

$$Pr = \frac{C_p.\mu}{k} = \frac{2460 * 1.1 * 10^{-2}}{0.25} = 108.24$$

$$Nu_D = \frac{\left(\frac{0.013}{8}\right) * (3500 - 1000) * 108.24}{1 + 12.7 * (\frac{0.013}{8})^{\frac{1}{2}} * (108.24^{\frac{2}{3}} - 1)} = 36.26$$

$Nu_D = \frac{hd}{k}$ $\longrightarrow$ $h = \frac{Nu_D * k}{d}$ $\longrightarrow$ $h = \frac{36.26*0.25}{0.01} = 906.5\ w/m^2°C$

For the calculation of U_0:

$$\frac{1}{r_{o,i}\ U_0} = \frac{1}{r_{i,i}\ h_i} + \frac{ln\ ln\ \left(\frac{r_{o,i}}{r_{i,i}}\right)}{k} + \frac{1}{r_{o,i}\ h_0}$$

$$\frac{1}{r_{o,i}\ U_o} = \frac{1}{0.005 * 906.5} + \frac{ln\ ln\ \left(\frac{0.007}{0.005}\right)}{40} + \frac{1}{0.007 * 1200}$$

$$\frac{1}{0.007 * U_0} = 0.22 + 0.008 + 0.119$$

$$U_0 = 409.83\ w/m^2K$$

b) The velocity remains the same

$$u_{m,c} = 3.5\ m/s$$

$$Re_i = \frac{\rho_i\ u_i d_{i,i}}{\mu_i} = \frac{1100 * 3.5 * 6 * 10^{-3}}{1.1 * 10^{-2}} = 2100$$

Please note that the internal diameter is now 10mm-4mm because of the presence of the 2mm thick fouling layer

$$\frac{\varepsilon}{d} = \frac{0.1}{6} = 0.01666 \cong 0.015\ and\ Re = 2100\ \ \rightarrow f = 0.015$$

Therefore, for the calculation of Nu, we have:

$$Nu_D = \frac{\left(\frac{0.015}{8}\right) * (2100 - 1000) * 108.24}{1 + 12.7 * (\frac{0.015}{8})^{\frac{1}{2}} * (108.24^{\frac{2}{3}} - 1)} = \frac{1738.9}{1 + 12.7 * 0.08 * 81.71} = 17.25$$

$$Nu_D = \frac{hd}{k} \longrightarrow h = \frac{Nu_D * k}{d} \longrightarrow h = \frac{17.25 * 0.25}{6 * 10^{-3}} = 718.84\, w/m^2K$$

$$\frac{1}{r_{o,i}\, U_o} = \frac{1}{(r_{i,i} - \delta_{foul})h_i} + \frac{ln\, ln\, \left(\frac{r_{o,i}}{r_{i,i}}\right)}{k_w} + \frac{ln\, ln\, \left(\frac{r_{i,i}}{(r_{i,i} - \delta_{foul})}\right)}{k_{foul}} + \frac{1}{r_{o,i}\, h_o}$$

$$= \frac{1}{(0.005 - 0.002) * 718.84} + \frac{ln\, ln\, \left(\frac{0.007}{0.005}\right)}{40} + \frac{ln\, ln\, \left(\frac{0.005}{0.003}\right)}{0.5} + \frac{1}{0.007 * 1200}$$

$$= 0.46 + 0.0084 + 1.021 + 0.119$$

$$U_0 = 88.82\, w/m^2K$$

c)

For calculating T_{outlet}, first we calculate the thermal load:

$$Q_c = = \dot{m}_c \times C_{p_c} \times \Delta T_c = (A \times u \times \rho) \times C_{p_c} \times \Delta T_c$$

$$= \frac{0.01^2 * \pi}{4} * 3.5 * 1100 * 2460 * (40 - 20) = 14876\, w$$

$$Q_h = \dot{m}_h \times C_{p_h} \times \Delta T_h = \frac{(0.02^2 - 0.014^2)}{4} * \pi * 5 * 1000 * 4180 * \Delta T$$

$$\Delta T = 4.4℃$$

$$T_{out_{hot}} = T_{exit_{water}} = 60 - 4.4 = 55.6℃$$

Now, we should calculate LMTD:

For parallel :

$$\Delta T_1 = 60 - 20 = 40\ ℃$$

$$\Delta T_2 = 55.6 - 40 = 15.6\ ℃$$

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{ln\ ln\ \frac{\Delta T_2}{\Delta T_1}} = 25.9\ ℃$$

For counter current :

$$\Delta T_1 = 60 - 40 = 20\ ℃$$

$$\Delta T_2 = 55.6 - 20 = 35.6\ ℃$$

$$\Delta T_{LMTD} = \frac{35.6 - 20}{ln\ ln\ \frac{35.6}{20}} = 27.05\ ℃$$

$$q = U \times A \times \Delta T_{LMTD}$$

For parallel flow

$$A = \frac{q}{U.\Delta T_{LMTD}} = \frac{14876.6}{409.83 * 25.9} = 1.4\ m^2$$

For counter current flow

$$A = \frac{q}{U.\Delta T_{LMTD}} = \frac{14876.6}{409.83 * 27.05} = 1.34\ m^2$$