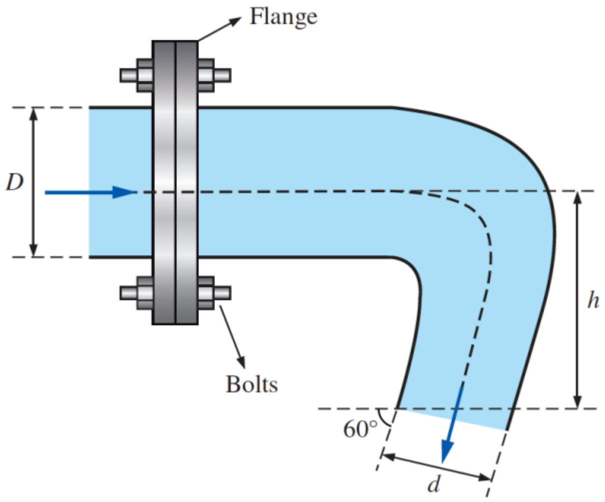
ChE 204 – Introduction to Transport Phenomena

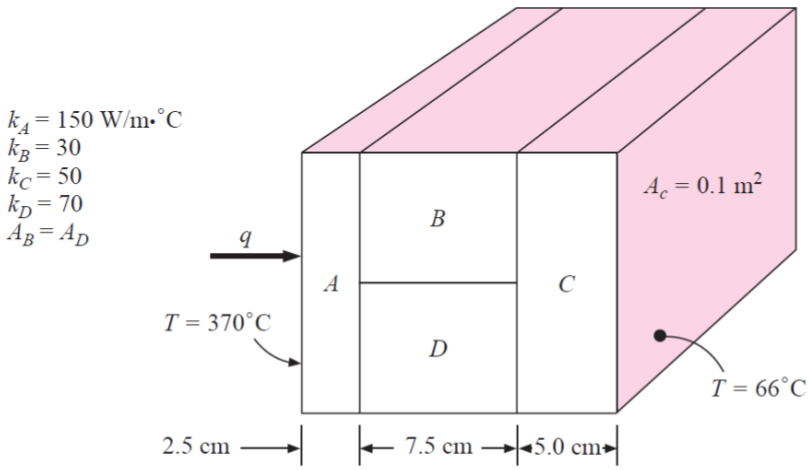
Mid-term Simulation

Question 1:
Water flowing steadily at a volumetric rate of 0.16 m^3/s is deflected downward by an angled elbow as shown in the figure below. The water is discharged to atmospheric pressure. For D = 30 cm, d = 10 cm, and h = 50 cm, determine the force acting on the flanges of the elbow and the angle its line of action makes with the horizontal (x-component only). Ignore gravity and friction.

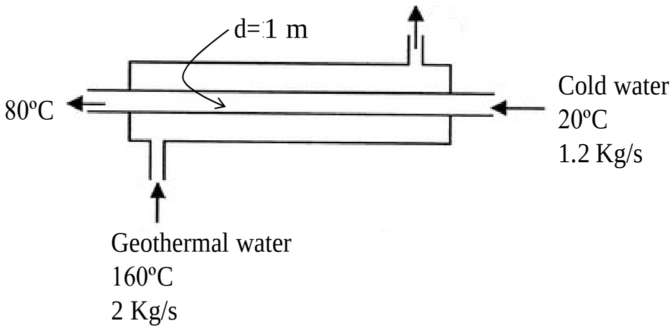


Note. This elbow is not laying flat on the ground (see h infact)

Question 2:
Find the heat transfer flow rate (Q) through the composite wall in the following figure. Assume one-dimensional heat flow.



Question 3:



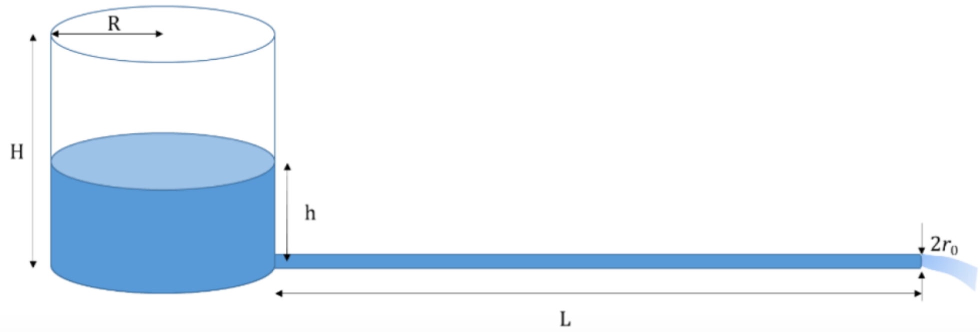
A counter-flow double pipe heat exchanger has to heat water from 20°C to 80°C which is flowing at a mass flow rate of 1.2 Kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin walled (means that the thermal resistance is negligible) and has a diameter of 1 m.

a) If the overall heat transfer coefficient of the heat exchanger is 640W/m^{2}C determine the length of the heat exchanger required to achieve the desired heating. ($C_{p,water}$ = 4.187 kJ/ kg K).

b) Calculate the new overall heat transfer coefficient in the presence of a fouling layer in the inside tube with thickness 5 cm, assuming h_i = 1280 W/m^2.K and K_{foul}= 10 W/m^2.K.

Question 4:

The cylindrical tank (R = 1 m) and the pipe shown in the figure below are initially filled with a fluid (μ = 0.001 Pa·s, ρ =1000 kg m^{-3}). Initially the tank is filled all the way to the full height H = 2m. Have in mind that both the tank and the exit of the pipe are at atmospheric pressure. Considering friction in the pipe (r0 = 0.0005 m, L = 10 m, ε = 0.000002 m), estimate the time needed to drain the tank.



Note. While solving the exercize, you will need to make a first approximation for the velocity. You can consider the maximum possible velocity, which is the one you get assuming h=H and the friction coefficient being the minimum possible considering your ε/D in the Moody diagram.

Solutions

Question 1:

We write the momentum balance:

$$\overrightarrow{F_{rxn}} - p_1 A_1 \overrightarrow{n_1} - p_2 A_2 \overrightarrow{n_2} = \int_{A_1} \rho \overrightarrow{v_1} (\overrightarrow{v_1} \cdot \overrightarrow{n_1})\, dA_1 + \int_{A_2} \rho \overrightarrow{v_2} (\overrightarrow{v_2} \cdot \overrightarrow{n_2})\, dA_2$$

Immediediately, we realize that we don't have pressures, BUT we can use the Bernoulli's equation for them between point 1 (elbow entrance) and point 2 (exit)

$$p_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

We can use gauge pressures, so $p_2 = 0$. We can also reference h_1 to h_2, so set h_2=0

Then we can get $v_1 = \frac{Q}{A_1} = \frac{0.16}{\pi^{0.3^2}/_4} = 2.26\frac{m}{s}$, $v_2 = \frac{Q}{A_2} = 20.37\frac{m}{s}$

Thus $p_1 = -\rho g\, h_1 + \frac{1}{2}\rho(v_2^2 - v_1^2) = -9810\times 0.5 + \frac{1}{2}\, 1000\, (20.37^2 - 2.26^2) = 20010\, Pa \cong 200\, kPa$

Going back to the momentum balance we can now write momentum balance for x-component:

$$F_{rxn} + p_1 A_1 = \rho v_1(-v_1)A_1 + \rho v_2(-v_2 cos\theta)A_2$$

$F_{rxn} = -200010 \times 0.070 - 1000\times\, (0.070\times 2.26^2 + 0.0078\times 20.385^2 \times cos60°)$
$F_{rxn} = -14000 - 1977$

$F_{rxn} = -15.9$ kN

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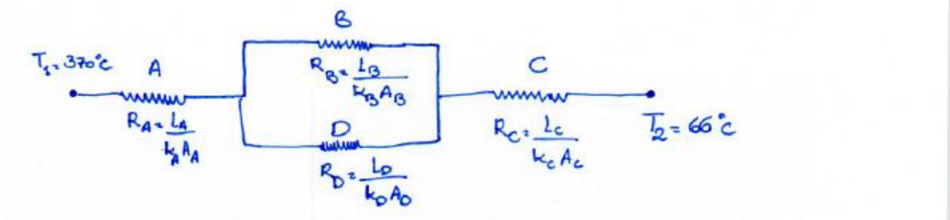
Question 2:

The first thing we do for this heat transport problems is to draw the thermal circuit:



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

$$R_{tot} = \frac{L_A}{k_A A_A} + \frac{1}{\left(\frac{k_B A_B}{L_B} + \frac{k_D A_D}{L_D}\right)} + \frac{L_C}{k_C A_C}$$

$$R_{tot} = \frac{0.025}{150 * 0.1} + \frac{1}{\left(\frac{30 * 0.05}{0.074} + \frac{70 * 0.05}{0.075}\right)} + \frac{0.05}{50 * 0.1}$$

$$R_{tot} = 0.001667 + 0.015 + +0.01 = 0.026667\ K/W$$

$$q_{1\to 2} = \frac{-\Delta T}{R_{tot}} = \frac{-(66 - 370)}{0.02667}$$

$$q_{1\to 2} = 11399.8\ W \approx 12\ kW$$

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Question 3:

a) Overall heat transfer coefficient, U = 640 $\frac{W}{m^2\,°C}$

$$\dot{Q} = \dot{m}_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right) = \dot{m}_{hot}c_{p_{hot}}\left(T_{hot_{in}} - T_{hot_{out}}\right)$$

$$T_{hot_{out}} = T_{hot_{in}} - \frac{\dot{m}_{cold}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{\dot{m}_{hot}}$$

$$T_{hot_{out}} = 160 - \left(1.2 * \frac{60}{2}\right) = 124°C$$

$$\Delta T_{LM} = \frac{(160-80)-(124-20)}{\ln\left(\frac{160-80}{124-20}\right)} = 91.5°C$$

$$\text{Heat transfer} = \text{UA}(\Delta T_{LM}) = \dot{m}_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)$$

$$\text{Area, } A = 2 * \pi * r * L$$

$$L = \frac{\dot{m}_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{U * T_{LM} * 2 * \pi * r}$$

$$\boldsymbol{L = \frac{1.2*4187*60}{640*91.5*2*3.14*0.5} = 1.639\ m}$$

b) Because the tube is thin walled, the resistance to heat transfer from the walls of the tube is neglected. From U given at point a) in the absence of fouling, we can calculate the heat transfer coefficient outside the tube

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$h_o = \frac{1}{\left(\frac{1}{U} - \frac{1}{h_i}\right)} = 1280\ \text{W}\ m^{-2}K^{-1}$$

Please note that the value of h_o is calculated to be equal to h_i in this problem. However, this is not always the case despite both hot and cold fluids happen to be water here.

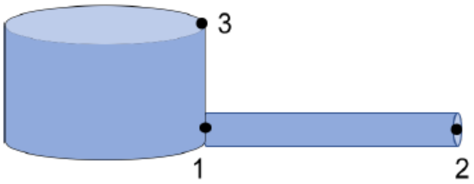
So, the new U (from slide 52, Module 3)

$$\frac{1}{r_o U_{new}} = \frac{1}{(r_o - \delta_{foul})h_i} + \frac{1}{h_o r_o} + \frac{\ln\left(\frac{r_o}{(r_o - \delta_{foul})}\right)}{k_{foul}}$$

$$\frac{1}{U_{new}} = \left(\frac{1}{(0.5-0.05)*1280} + \frac{1}{1280*0.5} + \frac{\ln\left(\frac{0.5}{(0.5-0.05)}\right)}{10}\right) * 0.5$$

$$\boldsymbol{U_{new} = 144.59\ Wm^{-2}K^{-1}}$$

Question 4:



Point 1 is at the inlet of the pipe and point 2 is just the outlet. Point 3 indicates the top of the water level in the tank.

Applying Bernoulli's equation between point 1 and point 2:

$$p_1 + \frac{1}{2}\rho v_{avg}^2 = p_2 + \frac{4fL}{4r_o}\rho v_{avg}^2 + \frac{1}{2}\rho v_{avg}^2$$

The velocity is the same because the diameter of the pipe does not change (continuity equation).

Then we apply the Bernoulli equation, between point 1 and point 3. Keep in mind that for the same assumption made in the Torricelli's theorem $v_3 \ll v_{avg}$. So we write:

$$p_1 + \frac{1}{2}\rho v_{avg}^2 = p_3 + \rho g h(t)$$

Thus, comparing the two equations and considering that $p_3 = p_2 = p_{atm}$

$$\rho g h(t) = \frac{4fL}{4r_0}\rho v_{avg}^2 + \frac{1}{2}\rho v_{avg}^2$$

$$v_{avg} = \sqrt{\frac{gh(t)}{\frac{fL}{r_o} + \frac{1}{2}}}$$

Here we may note that as the height of the water in the tank changes, so do the v_{avg} and f.

There is no analytical solution to the resulting differential equation and numerical solutions are out of the scope of this course. Hence, we will be simplifying the problem as follows:

We will consider the maximum possible velocity within the tank, which we obtain by taking h=H and the minimum friction value for $\varepsilon/D = 0.002$, which is at 0.005 from the Moody chart

Substituting the numbers we get $v_{max} = 0.44\, ms^{-1}$

We will then calculate the amount of time required for the tank to drain with this value of velocity. Mass balance between the water in the tank and water exiting the pipe gives us

$$\rho AH = \rho a v_{max} t$$

here *A* indicates the cross section area of the tank and *a* indicates the cross section area of the pipe. Putting in the numbers we get

t ≈ 210 days

This is the minimum amount of time that it will take to drain the tank.