ChE 204 – Introduction to Transport Phenomena

Mid Term Exam Simulation

Question 1:

A pipe system in stainless steel carries water (ρ = 1000 kg/m^3; μ= 8.9 x 10^{-4} Pa·s) and discharges it as a free jet in the atmosphere, as shown in the following figure.

a) Ignoring all the losses, if the pressure P_3, which is right at the entrance of the pipe system, is 7 x 10^5 Pa, what will be the volumetric and mass flow rate of the water coming out from the pipe? Consider the inner diameter of the pipe to be 200 mm.

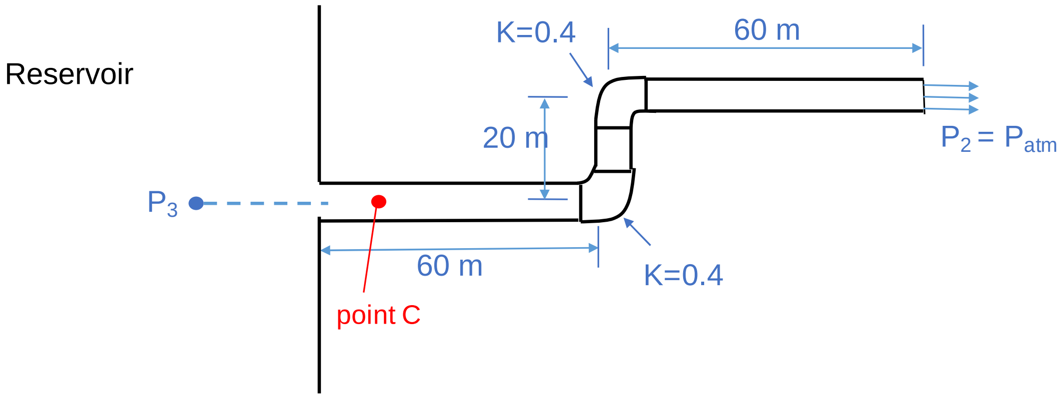


Figure for Question 1a)

For Questions 1b) and 1c) we remove the reservoir and we focus only on the pipe system as depicted here:

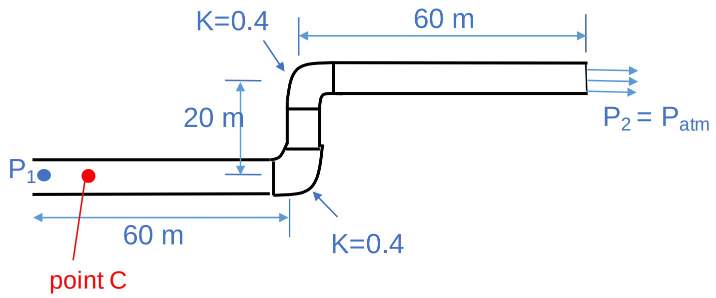
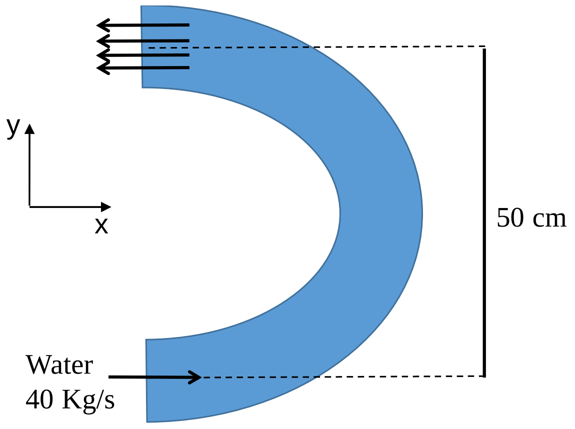


Figure for Question 1b) and 1c)

b) What gauge pressure P_1 would be needed to provide a volumetric flow rate of 12 m^3/min of water? Assume a smooth pipe. In this part all of the losses in the system should be considered.

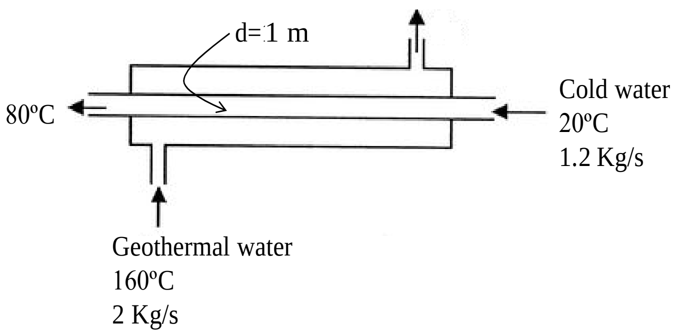
c) If the frictional losses inside the system were supposed to be compensated with a pump located at point C, what will be the required power of the pump?

Question 2:



A 180° elbow is used to direct water (ρ = 1000 kg/m^3) flow upward at a mass flow rate of 40 Kg/s. The diameter of the entire elbow is 10 cm. The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow, including the water within it, is 2 kg. Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.

Question 3:

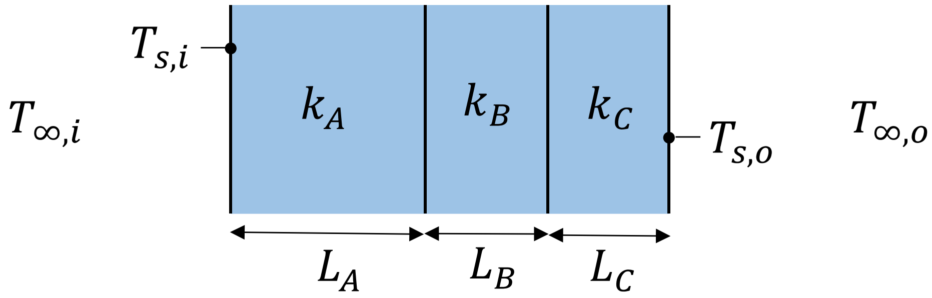


A counter-flow double pipe heat exchanger has to heat water from 20°C to 80°C at a rate of 1.2 Kg/s. The heating has to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thinned walled (you can neglect the conduction resistance across the wall) and has a diameter of 1 m.

a) If the overall heat transfer coefficient of the heat exchanger is 640W/m^2 C determine the length of the heat exchanger required to achieve the desired heating. ($C_{p,water}$ = 4.187 kJ/ kg K).
b) Assuming h_c = 1280 W/m^{2}K in the inside tube, calculate h_h in the outside tube.
c) Calculate a new overall heat transfer coefficient in the presence of a fouling layer with thickness 5 cm and with k_{foul}= 10 W/m^{2}K (tip: you need to use the expression for the overall heat transfer coefficient in a cylindrical geometry)

Question 4:

Consider a composite wall of a convection oven consisting of three materials, two of which are of known thermal conductivity, k_A=20 W/m^{2}K and k_C=50 W/m^{2}K, and known thickness, L_A= 0.30 m and L_C= 0.15 m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B= 0.15 m, but unknown thermal conductivity k_B. Under steady-state operating conditions, the outer temperature $T_{\infty,o}$= 24 °C, the inner surface temperature $T_{s,i}$=768 °C and the oven air temperature $T_{\infty,i}$=800 °C. The outside convection coefficient h_o is 2W/m^{2}K while the internal convection coefficient must be calculated. Find the value of k_B corresponding to overall heat loss of 800 W/m^2.



Solutions

Question 1

a)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Ignore frictional losses: $\Delta P_f = 0$

The velocity of water in the tank, before it enters the pipe, can be approximated to 0 due to the large area at the top of the tank in comparison to the small area of the pipe.

Thus the equation becomes

$$P_3 + \cancel{\rho g h_3} + \cancel{\frac{1}{2}\rho v_3^2} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \cancel{\Delta P_f}$$

Solve equation for v_2:

$$v_2 = \left(2 * \frac{P_3 - P_2 - \rho g h_2}{\rho}\right)^{\frac{1}{2}}$$

Numerical solution:

$$v_2 = \left(2 * \frac{(7-1) * 10^5 Pa - 1000\frac{kg}{m^3} * 9.81\frac{m}{s^2} * 20m}{1000\frac{kg}{m^3}}\right)^{\frac{1}{2}} = 28.41\frac{m}{s}$$

Volumetric flow rate:

$$Q = A_2 * v_2$$

$$Q = \left(\frac{d_i}{2}\right)^2 * \pi * v_2$$

$$Q = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\frac{m}{s} = 0.892\frac{m^3}{s}$$

Mass flow rate:

$$\dot{m} = A_2 * v_2 * \rho$$

$$\dot{m} = \left(\frac{0.2m}{2}\right)^2 * 3.141 * 28.41\frac{m}{s} * 1000\frac{kg}{m^3} = 892\frac{kg}{s}$$

b)

Bernoulli equation: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$

Applying mass conservation at point 1 and point 2,

$$\rho v_1 A_1 = \rho v_2 A_2 = \dot{m}$$

Since $A_1 = A_2$, we get $v_1 = v_2$

$$P_1 + \rho g h_1 + \cancel{\frac{1}{2}\rho v_1^2} = P_2 + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \Delta P_f$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v_1 * D}{\mu}$$

The volumetric flowrate, $\dot{v} = \frac{\dot{m}}{\rho}$

$$v_1 = \frac{\dot{v}}{\rho A_1} = \frac{\frac{12}{60}\,(\frac{m^3}{s})}{\frac{\pi(0.2)^2\ (m^2)}{4}} = 6.36\ m/s$$

$$Re = \frac{1000\frac{kg}{m^3} * 6.366\frac{m}{s} * 0.2m}{8.9 * 10^{-4}Pas} = 1.431\ * 10^6$$

Friction factor from Moody diagram:

$$f_f = 0.0028$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

Loss coefficients:

$$\sum_i L_i = 60m + 20m + 60m = 140m$$

$$\sum_j K_j = 0.4 + 0.4 = 0.8$$

$$\Delta P_f = \frac{1}{2} * 1000\frac{kg}{m^3} * \left(6.366\frac{m}{s}\right)^2 * \left(4 * \frac{0.0028}{0.2m} * (140m + 0.80)\right) = 1.59 * 10^5 Pa = 1.59 bar$$

$$P_1 = P_2 + \rho g(h_2 - h_1) + \Delta P_f = 1 * 10^5 + (1000 * 9.81 * 20) + 1.59 * 10^5 = 4.55 * 10^5 Pa = 4.55 bar$$

c)

The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f * Q = 1.59 * 10^5 Pa * 0.2\frac{m^3}{s} = 31.8 kW$$

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Question 2

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v \cdot n) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . n_i$$

$$\sum F_{volume} = m_{elbow} . g$$

We need to calculate the gauge pressure so we apply Bernoulli's principle between the inlet point (1) and the discharge point (0)

$$P_1 + \frac{1}{2}\rho v^2 = P_0 + \frac{1}{2}\rho v^2 + \rho g h_0$$

We know that mass flow rate, $\dot{m} = \rho A v$

Thus, velocity of water in the pipe inlet , $v_1 = \frac{\dot{m}}{\rho A} = \frac{40}{1000 \times (\pi \times (0.1)^2)/4} = 5.09\ m/s$

Applying conservation of mass between the inlet point (1) and the discharge point (0)

$$A_1 v_1 = A_0 v_0$$

Since the area of pipe is not changing,

$$A_1 = A_0 = A$$

Hence, $v_1 = v_0 = v = 5.09\ m/s$

By replacing the values in the Bernoulli's equation:

$$P_1 + 0.5 \times 1000 \times (5.09)^2 = P_o + 0.5 \times 1000 \times (5.09)^2 + 1000 \times 9.81 \times 0.5$$

$$\boldsymbol{P_1 - P_o = 4.9\ kPa}$$

We consider the pressures as gauge pressures, thus

$$P_{0gauge} = 0 \qquad P_{1gauge} = 4.9\ KPa$$

In order to determine the anchoring force, we need to resolve the forces along x and y directions.

$$\sum F_{surface} + \sum F_{pressure} = \sum F_{reaction}$$

Resolving the forces along the +x direction using gauge pressure:

$$\sum F_{pressure,x} = -P_{1gauge}.A_{1.}.n_1 - \cancel{P_{0gauge}.A_{0.}.n_0} = -P_{1gauge}.A_{1.}.(-1) = +P_{1gauge}A$$

$$\sum F_{pressure,x} = (+1) \times \left(\pi \times \frac{(0.1)^2}{4}\right)(4.9 \times 10^3) = 38.48\ N$$

$$\sum F_{volume,x} = 0$$

Therefore,

$$\sum F_{surface,x} = \rho v_1 A_1 (v_1.n) + \rho v_0 A_0 (v_0.n) = \rho v_1 A_1 (+v_1)(-1) + \rho v_0 A_0 (-v_0)(+1)$$

Since the direction of v_0 is in the negative x direction and we are writing the forces acting in the positive x direction.

Note: In the last term of above equation "$\rho v_0 A_0(-v_0)(+1)$", the first v_0 is a scalar value and doesn't need minus sign.

$\rho v_1 A_1 = \rho v_0 A_0 = \dot{m}$ ➔

$$\sum F_{surface,x} = \dot{m}(-v_1) + \dot{m}(-v_0) = \dot{m}(-v_1 - v_0)$$

$$\sum F_{surface,x} = 40 \times (-5.09 - 5.09) = -407.2\ N$$

$$\sum F_{reaction,x} = \sum F_{surface,x} - \sum F_{pressure,x} = -407.2 - 38.48 = -445.68\ N$$

Now resolving in the y direction,

$$\sum F_{pressure,y} = 0$$

The weight of the elbow, including the water within it, is 2 kg. Therefore,

$$\sum F_{reaction,y.total} = -F_{volume,y} = -m_{elbow} . g = -2 \times 9.8 = -19.6\ N$$

(2 pts)

$$F_{net} = ((445.68)^2 + (19.6)^2)^{\frac{1}{2}} = 446.11\ N$$

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Question 3

a) Overall heat transfer coefficient, U = 640 $\frac{W}{m^2}$

$$m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right) = m_{hot}c_{p_{hot}}\left(T_{hot_{in}} - T_{hot_{out}}\right)$$

$$T_{hot_{out}} = T_{hot_{in}} - \frac{m_{cold}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{m_{hot}}$$

$$T_{hot_{out}} = 160 - \left(1.2 * \frac{60}{2}\right) = 124°C$$

$$\Delta T_{lm} = \frac{(160 - 80) - (124 - 20)}{\ln\left(\frac{160 - 80}{124 - 20}\right)} = 91.5°C$$

$\dot{Q}$= UAΔT_{lm} = $m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)$

Area, $A \ = 2\pi rL$

$$L = \frac{m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{U\Delta T_{lm}2\pi rL}$$

$$\boldsymbol{L = \frac{1.2 * 4187 * 60}{640 * 91.5 * 2 * \pi * 0.5} = 1.639\ m}$$

b) The tube is thin walled and hence the resistance to heat transfer from the walls of the tube is neglected.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$h_o = \frac{1}{\left(\frac{1}{U} - \frac{1}{h_i}\right)}$=1280 W m^{-2}K

c)

$$\frac{1}{r_o U_{new}} = \frac{1}{(r_o - \delta_{foul}) h_i} + \frac{1}{h_o r_o} + \frac{\ln\left(\frac{r_o}{(r_o - \delta_{foul})}\right)}{k_{foul}}$$

$$\frac{1}{U_{new}} = \left(\frac{1}{(0.5 - 0.05) * 1280} + \frac{1}{1280 * 0.5} + \frac{\ln\left(\frac{0.5}{(0.5 - 0.05)}\right)}{10}\right) * 0.5$$

$$\boldsymbol{U_{new} = 145.62\ Wm^{-2}K}$$

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<u>Question 4</u>

Heat loss $(\frac{Q}{A}) = 800\,\frac{W}{m^2}$

Heat transfer, $Q = h_i A(T_{\infty,i} - T_{s,i})$

The internal heat transfer co-efficient, $h_i = \frac{Q}{A(T_{\infty,i} - T_{s,i})} = \frac{800}{(800-768)} = 25\ W/m^2{}^\circ\mathrm{C}$

The heat transfer rate, $Q = UA(T_{s,i} - T_{\infty,o})$

$$\frac{1}{U} = \frac{A(T_{s,i} - T_{\infty,o})}{Q} = \frac{(768-24)}{800} = 0.93{}^\circ\mathrm{C}/W$$

The overall heat transfer resistance between the inner wall and the outside atmosphere, (1/U) can be related to each of the series resistances as follows:

$$\frac{1}{U} = \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_o}$$

$$0.93 = \frac{0.3}{20} + \frac{0.15}{k_B} + \frac{0.15}{50} + \frac{1}{2}$$

Thus, $\frac{0.15}{k_B} = 0.93 - 0.015 - 0.003 - 0.5$

$$k_B = 0.364\ W/m{}^\circ\mathrm{C}$$

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