Mid Term Exam Simulation

Question 01:
Draining a tank:

The cylindrical tank and pipe shown in the figure below are initially filled with a Newtonian liquid of viscosity μ _and density ρ _(the tank is initially filled all of the way to the full height H). Taking pipe friction to be the only resistance to flow, and ignoring exit kinetic-energy effects and entrance effects, estimate the time needed to drain just the tank with the following data:

R = 1 m
H = 2 m
L = 10 m
r_0 = 0.0005 m
μ = 0.001 Pa s
ρ =1000 kg m^{-3}
ε = 0.000002 m (Tube roughness)

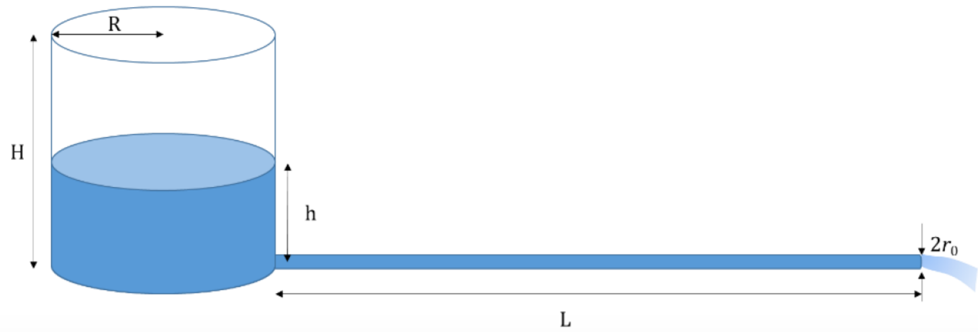

Figure 1 Draining tank

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Question 02:

Water flowing steadily at a rate of 0.16 m3/s is deflected downward by an angled elbow as shown in Figure 02.
For D = 30 cm, d = 10 cm, and h = 50 cm, determine the force acting on the flanges of the elbow and the angle its line of action makes with the horizontal. Take the internal volume of the elbow to be 0.03 m3 and disregard the weight of
the elbow material and the frictional effects.

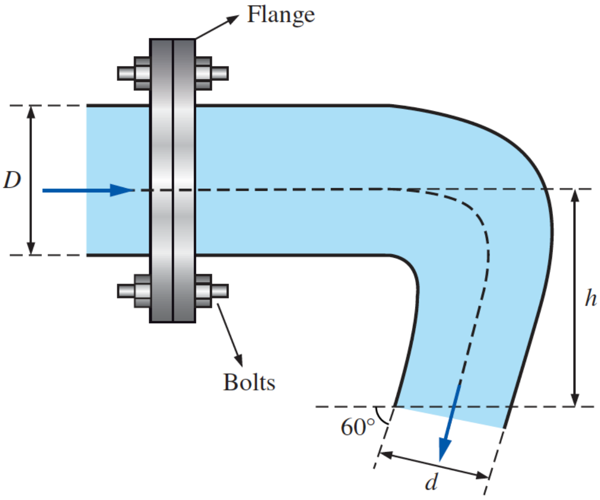

Figure 2 Angled elbow

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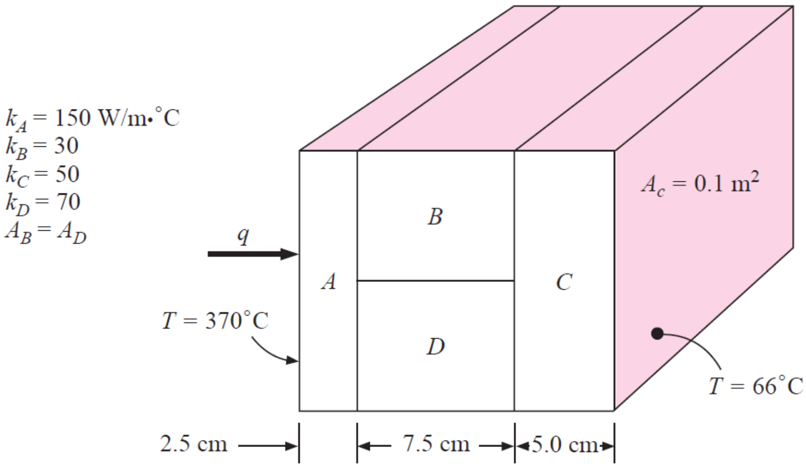
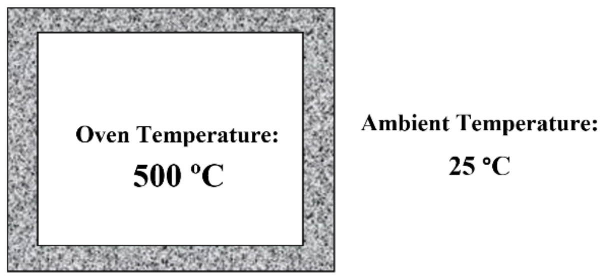
Question 03:

A) Find the heat transfer flow rate (Q) through the composite wall in the following figure. Assume one-dimensional heat flow.



B) Consider the object that has been shown above is actually a wall of the furnace, the temperature of which is 500 °C. The furnace has been put in a room with a temperature of 25 °C. The internal and external heat transfer coefficient are 15 and 5 W/m^2.K, respectively. Determine the heat loss rate from the furnace and the inner and outer temperatures.

A scheme of oven located in an environment:



C) Considering that heat transfer flux per unit area by radiation can be obtained through the following equation:

$$q_{rad} = \varepsilon\sigma(T^4_{object} - T^4_{surrounding})$$

Where ε is emissivity factor and has a value between 0 and 1. The value of the Stefan–Boltzmann constant is given in SI units by **σ= 5.670367(13)×10^{-8} W·m^{-2}·K^{-4}.** Assuming ε to be 0.25 for this furnace, conclude if radiation heat transfer plays a role in whole heat loss from the surface.

Question 04:

A concentric tube heat exchanger is used to thermally process a pharmaceutical product flowing at a mean velocity of $u_{m,c}$=3.5 m/s with an inlet temperature of $T_{c,i}$= 20 °C. The inner tube of diameter D_i=10mm, the thickness of the inner tube is 2 mm with the thermal conductivity of k_t = 40 W/m^2.K, and the exterior of the outer tube (D_o= 20 mm) is well insulated. Water flows in the annular region between the tubes at a mean velocity of $u_{m,h}$ = 5 m/s with an inlet temperature of $T_{h,i}$ = 60 °C. Properties of the pharmaceutical product are μ=1.1*10^{-2} (N·s)/m^2, k= 0.25 W/m.K, ρ =1100 kg/m^3, and c_p= 2460 J/kg.K. Properties of the water may be approximated as μ =9.60*10^{-4} (N·s)/m^2, k=0.6. The specific heat of the water is 4180 J/kg. K. Consider the surface rough ness of the tube to be 0.1 mm.

(a) If the convective heat transfer coefficient for water correspond to the outer surface area of inner tube, h_o=1200 W/m^2 .K, Determine the value of the overall heat transfer coefficient U.
Tip: The heat transfer coefficient inside the tubes over a large range of the Reynolds number can be obtained by Gnielinski equation:

$$Nu_D = \frac{\left(\frac{f}{8}\right) * (Re_D - 1000) * Pr}{1 + 12.7\left(\frac{f}{8}\right)^{\frac{1}{2}}\left(Pr^{\frac{2}{3}} - 1\right)}$$

Where

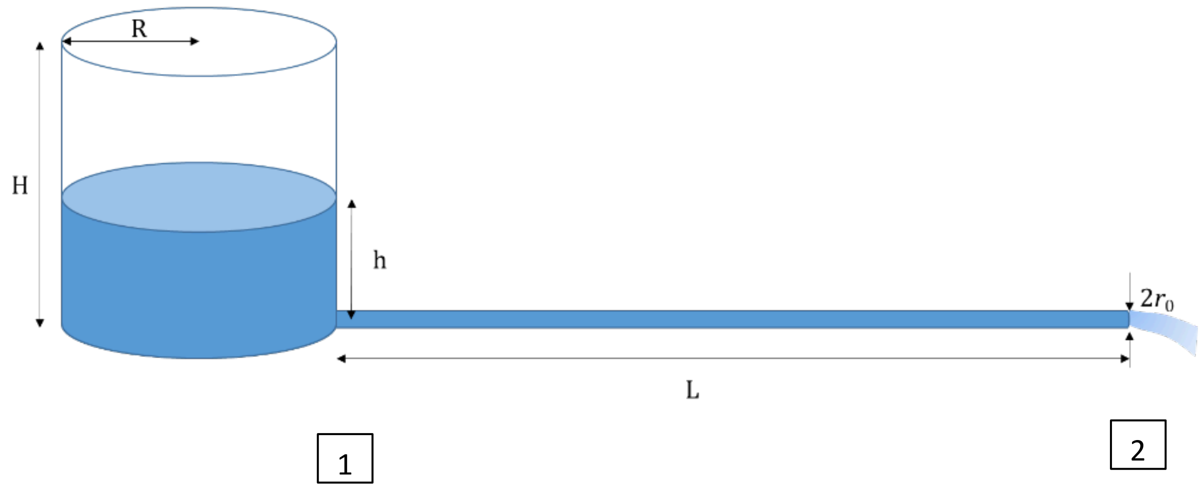
$$\text{Pr (Prandtl number)} = \frac{C_p\mu}{k} \quad and\ Nu\ (Nusselt\ number) = \frac{h_i d_i}{k}$$

where the friction factor may be obtained from the Moody diagram. The correlation is valid for 0.5 =< Pr=< 2000 and 3000 =< Re_D =< 5*10^6.

(b) If there is a fouling material inside the inner tube with the thickness of 2 mm and the thermal conductivity of k_{foul}= 0.5 W/m.K, determine the overall heat transfer coefficient. For the sake of simplicity, assume that the velocity remains unchanged.

(c) If the desired outlet temperature of the pharmaceutical product is 40 °C, obtain the required heat transfer area (A) in both concurrent and countercurrent situation considering the overall heat transfer coefficient obtained in the section a.

Solutions



Applying Bernoulli's theorem at 1 and 2,

$$p_1 = p_2 + \frac{4fL}{4r_o}\rho v_{avg}^2$$

The pressure at 1 due to the water in the tank is given below:

$$p_1 = p_{atm} + \rho g h(t)$$

So,

$$\rho g h(t) = \frac{4fL}{4r_0}\rho v_{avg}^2$$

$$v_{avg} = \sqrt{\frac{r_o g h(t)}{fL}}$$

To get the maximum possible velocity, we take the maximum height of the tank and Lowest value of f=0.005 (From moody's chart),

We get $v_{avg} = 0.44\, ms^{-1}$

The Reynold number corresponding to this velocity is $Re = \frac{2r_o v_{avg}\rho}{\mu} = 440$

So, this is Laminar region and the friction factor in this region is given by $f = \frac{16}{Re}$

Substitutting in the equation above,

$$\rho g h(t) = \frac{16L}{Re\, r_0}\rho v_{avg}^2$$

we get

$$v_{avg}(t) = \frac{\rho g h(t) r_o^2}{8\mu L}$$

The mass balance at the top of the tank and at the end of the pipe gives:

$$\frac{\rho A dh(t)}{dt} = -\rho a v_{avg}$$

$$\frac{dh(t)}{dt} = -\frac{\left(\frac{r_o}{R}\right)^2 \rho g r_o^2}{8\mu L} h(t)$$

Integrating the equations as below:

$$\int_H^{2r_0} \frac{dh}{h} == -\left(\frac{r_0}{R}\right)^2 \frac{\rho g r_0^2}{8\mu L} \int_0^{t_f} dt$$

$$\ln\left(\frac{2r_0}{H}\right) = -\frac{r_0^4 \rho g}{8\mu R^2 L} t_f$$

The expression for time required to empty the tank is given as follows:

$$\boxed{t_f = \frac{8\mu R^2 L}{\rho g r_0^4} \ln\left(\frac{H}{2r_0}\right)}$$

Putting the numerical values to the above expressions, $t_f = 32\ years$

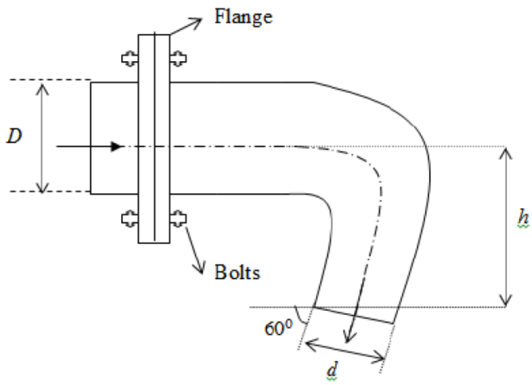
2.

Solution Water is deflected by an elbow. The force acting on the flanges of the elbow and the angle its line of action makes with the horizontal are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Frictional effects are negligible (so that the Bernoulli equation can be used). **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero.

Properties We take the density of water to be 1000 kg/m^3.

Analysis



Writing Bernoulli equation between 1 (elbow entrance)-2 (exit) ;

$$P_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2 + negligible\ losses$$

$$V_1 = \frac{0.16}{\pi 0.3^2/4} = 2.26\ m/s\ ,\ V_2 = 20.37\ \ m/s\ ,\ z_1=0.5\text{m}\ ,\ z_2=0\text{m}\ ,P_2=0$$

$$P_1 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) - \gamma z_1 = \frac{1}{2}1000\left(20.37^2 - 2.26^2\right) - 9810\times 0.5$$

$$P_1 = 20010\ Pa \cong 200\ kPa$$

Linear momentum equation for the CV gives;

$$\frac{\partial}{\partial t}\int_{cv}\vec{V}\rho dV + \int_{cs}\vec{V}\rho\vec{V}\vec{n}dA = \sum F_x$$

<u>x component</u>

$$V_1\rho(-V_1)A_1 + (-V_2\cos\theta)\ \rho(V_2)A_2 = P_1A_1 + R_x$$

$$-\rho Q V_1^2 - \rho Q V_2^2\cos\theta - P_1A_1 = R_x$$

$$-\rho Q(V_1^2 - V_2^2\cos\theta) - P_1A_1 = R_x$$

$$-1000\times 0.16\left(2.26^2 + 20.37^2\cos 60\right) - 200010\pi\frac{0.3^2}{4} = R_x$$

To include elbow weight we must modify y-momentum equation as follows:

y component:

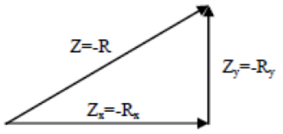
$$0+(-V_2 \sin\theta)\rho V_2 A_2 = -W_{water,cv} - W_{elbow} + R_y$$

$$-\rho Q V_2^2 \sin\theta + W_{water,cv} + W_{elbow} = R_y$$

$$-1000\times 20.37^2 \sin 60 + 9810\times 0.03 + 5\times 9.81 = R_y$$

$$R_y \approx -359003 \ N\ (down)$$

These forces are exerted by elbow on water confined by CV. The force exerted by water on elbow is therefore;



$$Z = -R = \sqrt{48150^2 + 359052^2} = 362266 \text{ N} \cong \mathbf{362kN}$$

and

$$\tan\beta = \frac{R_y}{R_x} = \frac{359{,}003 \text{ N}}{48{,}150 \text{ N}} = 7.457 \rightarrow \beta = \mathbf{82.4°}$$

Therefore we could neglect the weight of the elbow.

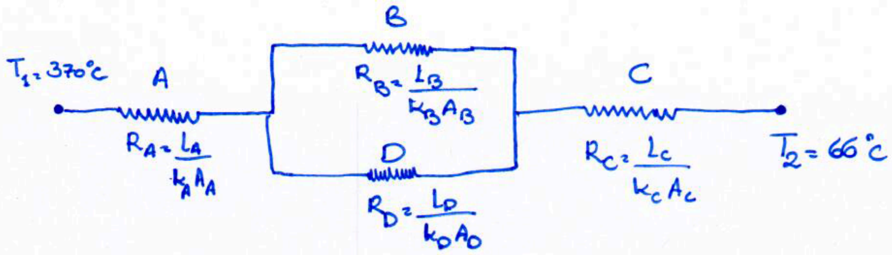
3.

$q = -kA\frac{\Delta T}{\Delta x}$ ⇒ analogy with electrical resistance

$\Delta T \sim \Delta V$

$q \sim I$ ⇒ $-\frac{\Delta T}{q} = \frac{L}{kA}$ ⇒ $R_{\text{conductive heat transfer in cartesion coordinate}} = \frac{L}{kA}$

Whole heat transfer path can be modeled with the analogy to electrical circuit.



$$R_{tot} = R_A + \{R_B, R_D\} + R_C$$

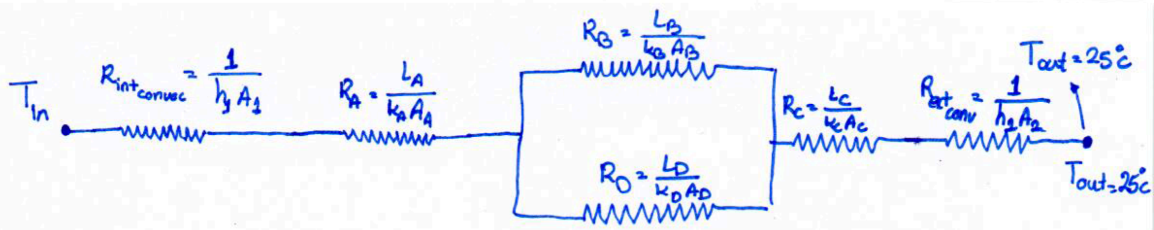
$$R_{tot} = \frac{L_A}{k_A A_A} + \frac{1}{\frac{k_B A_B}{L_B} + \frac{k_D A_D}{L_D}} + \frac{L_C}{k_C A_C} \Rightarrow$$

$$R_{tot} = \frac{0.025}{150*0.1} + \frac{1}{\frac{30*0.05}{0.075} + \frac{70*0.05}{0.075}} + \frac{0.05}{50*0.1} =$$

$$0.001667 + 0.015 + 0.01 = 0.026667 \quad K/W \text{ or } °C/W$$

$$q_{1\to 2} = \frac{-\Delta T}{R_{tot}} = -\frac{T_2 - T_1}{R_{tot}} = -\frac{66-370}{0.026667} = \boxed{11399.8\ W \simeq 12\ kW}$$

b) In this section, the heat transfer circuit will change a little bit and two additional convective heat transfer resistance will be added to the overall heat transfer resistances. The convective heat transfer resistance can be obtained from ⇒ $q = hA(T_{hot} - T_{cold}) \Rightarrow \frac{\overbrace{(T_{hot} - T_{cold})}^{\Delta T}}{q} = \frac{1}{hA} = R_{convection}$



Therefore, with these new sets of resistances, we can write:

$$R_{tot} = \frac{1}{h_1 A_1} + R_{tot_{conduction}} + \frac{1}{h_2 A_2} = \frac{1}{15 * 0.1} + 0.026667 + \frac{1}{5 * 0.1} =$$

$$0.6667 + 2 + 0.0267 = 2.6934 \ K/w \text{ or } °c/w$$

$$q = \frac{\Delta T}{R_{tot}} \Rightarrow$$

$$q = \frac{500 - 25}{2.6934} = \boxed{176.357 \ w}$$

In the question, the temperature of interior part of the furnace wall has been asked. As we have a heat transfer circuit in series, we have

$$q_{convection_{in}} = q_{tot} = h_{in} A_{in} \Delta T_{inside} \Rightarrow \Delta T_{inside} = \frac{q_{tot}}{h_{in} A_{in}}$$

$$= \Delta T_{inside} = \frac{176.357}{15 * 0.1} \simeq 117.6 °c$$

$$T_{wall\ inside} = 500 - 117.6 = \boxed{382.4 °c}$$

$$q = q_{tot} = h_{out} A_{out} \Delta T_{out} \Rightarrow \Delta T_{out} = \frac{q}{h A_{out}} \Rightarrow \Delta T_{out} = \frac{176.357}{5 * 0.1} \simeq 352.7 °c$$

$$\Rightarrow T_{wall\ outside} = 25 + 352.7 = \boxed{377.7 °c}$$

C) The stephane boltzman coefficient $\sigma = 5.67 * 10^{-8}\ W.m^{-2}.K^{-4}$

Therefore, for the heat transferred by radiation we have:

$$q = 0.25 * 5.67 * 10^{-8} * \left(T_{obj}^{4} - T_{surr}^{4}\right) * A$$

$$\Rightarrow q = 0.25 * 5.67 * 10^{-8} * (773^4 - 298^4) * 0.1 = 494.9\ W$$

Considering the value obtained in sec b, radiation plays an important role in heat transfer. Considering the result obtained in sec a., it is not so considerable.

4.

Ans 2:

$d_i = 10$ mm $\delta = 2$ mm $d_o = 14$ mm

$$\frac{1}{d_o U_o} = \frac{1}{d_i h_i} + \frac{\ln(d_o/d_i)}{k_w} + \frac{1}{d_o h_o}$$

For calculation of h_i, we should calculate Re and then Nu:

$$Re_i = \frac{\rho_i u_i d_i}{\mu_i} = \frac{1100 * 3.5 * 0.01}{1.1 * 10^{-2}} = 3500$$

$$\varepsilon/d = \frac{0.1}{10} = 0.01$$

From the moody diagram ($\varepsilon/d = 0.01$ $Re = 3500$)

$\Rightarrow f = 0.052$

$$Pr = \frac{c_p \cdot \mu}{k} = \frac{2460 * 1.1 * 10^{-2}}{0.25} = 108.4$$

$$Nu_D = \frac{\left(\frac{0.052}{8}\right) * (3500 - 1000) * 108.24}{1 + 12.7 * \left(\frac{0.052}{8}\right)^{1/2} * (108.24^{2/3} - 1)}$$

$$\Rightarrow Nu_D = \frac{1758.9}{1 + 0.08 * 12.7 * 21.91} = 76.28$$

$$Nu_D = \frac{hd}{k} \Rightarrow h = \frac{Nu_D * k}{d} \Rightarrow h = \frac{76.28 * 0.25}{0.01} =$$

$$1907 \ W/m^2 \, ^\circ C$$

For the calculation of U_o

$$\frac{1}{r_o U_o} = \frac{1}{r_i h_i} + \frac{\ln (r_o/r_i)}{k} + \frac{1}{r_o h_o} \Rightarrow$$

$$\frac{1}{r_o U_o} = \frac{1}{0.05 * 1907} + \frac{\ln \left(\frac{0.07}{0.05}\right)}{40} + \frac{1}{0.07 * 1200} \Rightarrow$$

$$\frac{1}{0.07 * U_o} = 0.0105 + 0.0084 + 0.0119$$

$$\Rightarrow U_o = 463.82 \ W/m^2 . K$$

b) We should re-calculate velocities

$$Q = \frac{V_1 \pi d_1^2}{4} = \frac{V_2 \pi d_2^2}{4}$$

$d_1 = 10$ mm

$d_2 = 10 - 2 * \delta_{foul} = 6$ mm

$$\Rightarrow V_2 = V_1 * \left(\frac{D_1}{D_2}\right)^2 \Rightarrow V_2 = 3.5 * \left(\frac{10}{6}\right)^2 = 9.7 \ m/s$$

$$Re = \frac{\rho_i u_i d_i}{\mu} = \frac{1100 * 9.7 * 6 * 10^{-3}}{1.1 * 10^{-2}} \Rightarrow Re = 5820$$

calculation of h_i again we have

$$\left(\varepsilon/d = \frac{0.1}{0.6} = 0.01666 \simeq 0.015 \text{ and } Re = 5820\right)$$

$$\Rightarrow f = 0.053$$

Therefore, for the calculation of Nu, we have

$$Nu_D = \frac{\left(\frac{0.053}{8}\right) * (5820 - 1000) * 108.27}{1 + 12.7 * \left(\frac{0.053}{8}\right)^{1/2} * \left(108.27^{2/3} - 1\right)} =$$

$$\frac{3456.4}{1 + 12.7 * 0.0811394 * 21.71} = 147.445$$

$$Nu_D = \frac{hd_i}{k} \Rightarrow h = \frac{Nu_D * k}{d_i} \Rightarrow h = \frac{147.445 * 0.25}{6 * 10^{-3}} \Rightarrow$$

$$h = 6143.54 \ W/m^2 K$$

$$\frac{1}{d_o * U_o} = \frac{1}{((d_m - 2\delta_{foul}) * h_i)} + \frac{\ln(d_o/d_m)}{k_w} + \frac{\ln\left(d_m/(d_m - 2\delta_{foul})\right)}{k_{foul}} + \frac{1}{d_o h_o} \Rightarrow$$

$$\frac{1}{d_o * U_o} = \frac{1}{(0.1-0.04) * 6143.37} + \frac{\ln\left(0.14/0.1\right)}{40} + \frac{\ln\left(0.1/0.06\right)}{0.5} + \frac{1}{0.14 * 1200} =$$

$$0.0027 + 0.0084 + 1.02 + 0.00595 \rightarrow U = 13.77\ W/m^2 k$$

For Calculating $T_{out\ water}$, firstly we calculate thermal load:

$$Q = U * A * \rho * c_p * \Delta T = \frac{0.01^2 * \pi}{4} * 3.5 * 1100 * 2460 * 20$$

$$= 14876.6\ W$$

$$q = \dot{m}_{w_h} * c_{p_h} * \Delta T_h = \frac{(0.02^2 - 0.014^2)}{4} * \pi * 5 * 1000 * 4180 * \Delta T \rightarrow \Delta T = 4.4\ °c$$

$$T_{out\ hot} = T_{exit\ water} = 60 - 4.4 = 55.6\ °c$$

Now, we should calculate LMTD

$$\text{Cocurrent} \begin{cases} \Delta T_1 = 60 - 20 = 40 \\ \Delta T_2 = 55.6 - 40 = 15.6 \end{cases} \Rightarrow \Delta T = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = 25.9\ °c$$

Counter current $\left\{ \begin{array}{l} \Delta T_1 = 60 - 40 = 20 \ ^\circ C \\ \Delta T_2 = 55.6 - 20 = 35.6 \ ^\circ C \end{array} \right.$

$\Rightarrow \Delta T_{LMTD\ counter\ current} = \dfrac{35.6 - 20}{\ln \dfrac{35.6}{20}} = 27.05$

$q = UA\,LMTD$

$\Rightarrow$ for cocurrent flow:

$A = \dfrac{q}{U \Delta T_{LMTD}} = \dfrac{14876.6}{463.8 * 25.09} = 1.28 \ m^2$

For Counter current flow:

$A = \dfrac{q}{U \Delta T_{LMTD}} = \dfrac{14876.6}{463.8 * 27.05} = \boxed{1.186 \ m^2}$