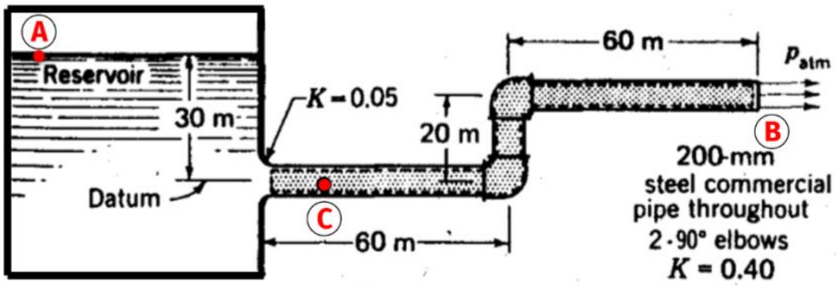
Final Exam

Question 1 (12 points)

A pipe system carries water (ρ= 1000 kg/m^3; μ= 8.9 x 10^{-4} Pa·s) from a reservoir and discharges it as a free jet in the atmosphere, as shown in the following figure.

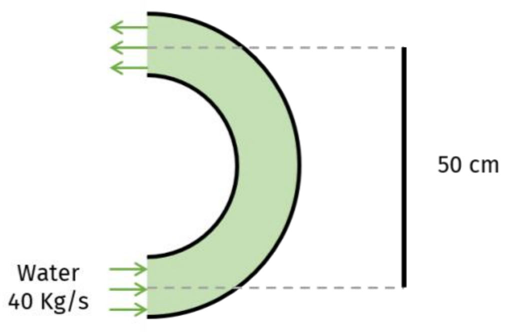
a) Ignoring the frictional losses inside the system and assuming a closed and pressurized reservoir, if the pressure inside the reservoir (P_A) is 7 x 10^5 Pa, what is the flow of the water coming out from the pipe at point B? Consider the inner diameter of the pipe to be 200 mm.
b) What pressure in the reservoir is needed to provide a flow rate in the pipe system of 12 m^3/min of water? Assume a smooth pipe. In this part all of the losses in the system should be considered.
c) If the frictional loss is supposed to be compensated with a pump located at point C, what will be the required power of the pump?



Note: K=0.40 includes both 90 degrees elbows (i.e. no need to multiply by 2)

Question 2 (10 points)

A 180° elbow is used to direct water flow upward at a rate of 40 Kg/s. The diameter of the elbow pipe is 10 cm. The elbow discharges water in the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 50 cm. The weight of the elbow is 2 kg. Determine the gauge pressure at the inlet of the elbow and the anchoring force needed to hold the elbow in place.



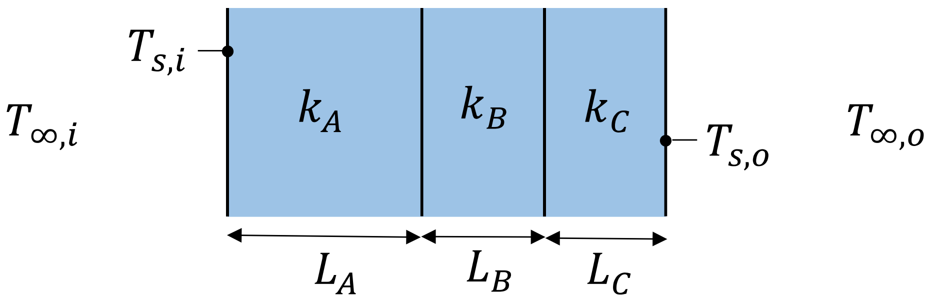
<u>Question 3</u> (10 points)

A cake is baked in a convection oven.

a) Calculate the heat flux at the cake surface when it has just been inserted in the oven if the initial temperature of the cake is 24 °C, while the oven air and wall are at the same temperature $T_{\infty,i} = T_{s,i} = 180°C$ and the forced convection transfer coefficient between the oven and the cake is 25 W/m^2·K.

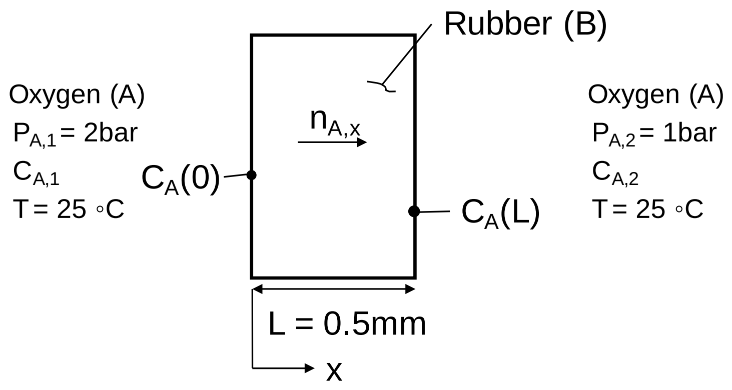
b) Consider that the walls of the oven consist of a 30-mm-**thick** layer of insulation characterized by k= 0.03W/m·K that is sandwiched between two **very thin** layers of sheet **metal**. The exterior surface of the oven is exposed to air at $T_{\infty,o} = 24°C$ with h_O = 2W/m^2·K. The interior oven air temperature is $T_{\infty,i} = T_{s,i} = 180°C$. Neglecting radiation heat transfer, calculate the steady-state heat flux through the oven walls (**Tip:** use your engineering intuition to reasonably ignore what you don't have enough information about. Express your reasoning why you think it can be ignored).

c) Consider now a composite wall of the oven consisting of three materials, two of which are of known thermal conductivity, k_A=20 W/m·K and k_C=50 W/m·K, and known thickness, L_A= 0.30 m and L_C= 0.15 m. The third material, B, which is sandwiched between materials A and C, is of known thickness, L_B= 0.15 m, but unknown thermal conductivity k_B. Under steady-state operating conditions, the outer temperature $T_{\infty,O}$=24°C, the inner surface temperature $T_{s,i} = 768°C$ and the oven air temperature $T_{\infty,i} = 800°C$. The outside convection coefficient h_O is 2W/m^2·K while the internal convection coefficient must be calculated. Find the value of k_B corresponding to overall heat loss of 800 W/m^2.



<u>Question 4</u> (10 points)

Consider the sketch and the data provided:



$D_{AB} = 0.21 \cdot 10^{-9} \frac{m^2}{s}$; $S_{AB} = 3.12 \cdot 10^{-3} \frac{kmol}{m^3 bar}$; $M_A = 32\ g/mol$; $R = 0.082 \frac{L \cdot atm}{K \cdot mol}$

Find:

a) The molar diffusion flux of oxygen

b) The mass diffusion flux of oxygen

c) The molar concentrations of oxygen outside the rubber ($C_{A,1}$ and $C_{A,2}$)assuming perfect gas behavior

d) Can you briefly explain why the molar concentrations outside the membrane differ from those within the membrane?

<u>Question 5</u> (10 points)

A waste water stream is introduced to the top of a mass transfer tower where it flows counter current to an air stream. At one point in the tower, the waste water stream contains 10^{-3} mol/m^3 ($C_{A,L,bulk}$) and the air is essentially free of any waste (A). At any operating conditions within the tower, the local mass-transfer coefficients are $k_{L,loc} = 5 \cdot 10^{-4} \frac{m}{s}$ and $k_{G,loc} = 0.01 \frac{kmol}{m^2 \cdot s \cdot atm}$.

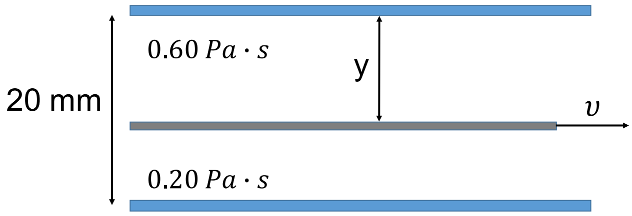
The concentrations are in the Henry's law range where $P_{A \cdot i} = H \cdot C_{A \cdot i}$ with H = $10 \frac{atm \cdot m^3}{kmol}$

Determine:

a) The overall flux of A
b) The overall mass transfer coefficients K_L and K_G

<u>Question 6</u> (8 points)

A thin plate is placed between two flat surfaces 20 mm apart such that the viscosity of liquids on the top and bottom of the plate are 0.60 Pa·s and 0.20 Pa·s, respectively. Determine the position of the thin plate such that the viscous resistance to uniform motion of the thin plate is minimum.



Introduction to Transport Phenomena: Final Exam Solutions

Question 1 – Solutions (12 points)

a)

To calculate the velocity at the exit of the pipe system we apply the Bernoulli equation:

$$P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2 + \Delta P_f$$

Ignore frictional losses: $\Delta P_f = 0$

$$P_A + \rho g h_A + \cancel{\frac{1}{2}\rho v_A^2} = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2 + \cancel{\Delta P_f}$$

Simplified Bernoulli equation according to the question:

Solve equation for v_2:

$$v_B = \left(2 * \frac{P_A - P_B + \rho g(h_A - h_B)}{\rho}\right)^{\frac{1}{2}}$$

Numerical solution:

$$v_B = \left(2 \cdot \frac{(7 - 1{,}01325) \cdot 10^5 Pa + 1000\frac{kg}{m^3} \cdot 9.81\frac{m}{s^2} \cdot 10m}{1000\frac{kg}{m^3}}\right)^{\frac{1}{2}} = 31.64\frac{m}{s}$$

Volumetric flow:

$$Q = A_B \cdot v_B$$

$$Q = \left(\frac{d_B}{2}\right)^2 \cdot \pi \cdot \mathrm{v_B}$$

$$Q = \left(\frac{0.2m}{2}\right)^2 \cdot \pi \cdot 31.64\frac{m}{s} = 0.994\frac{m^3}{s}$$

Mass flow:

$$\dot{m} = Q \cdot \rho$$

$$\dot{m} = 0.994\frac{m^3}{s} \cdot 1000\frac{kg}{m^3} = 994\frac{kg}{s}$$

b)

Bernoulli equation:

$$P_A + \rho g h_A + \cancel{\frac{1}{2}\rho v_A^2} = P_B + \rho g h_B + \frac{1}{2}\rho v_B^2 + \Delta P_f$$

$$v_{avg} = v_B = \frac{Q_2}{A_2} = \frac{12\frac{m^3}{min}\cdot\frac{1\,min}{60\,s}}{\left(\frac{0.2m}{2}\right)^2\cdot\pi} = 6.366\frac{m}{s}$$

$$Re = \frac{\rho\cdot v_{avg}\cdot D}{\mu}$$

$$Re = \frac{1000\frac{kg}{m^3}\cdot 6.366\frac{m}{s}\cdot 0.2m}{8.9\cdot 10^{-4}Pa\cdot s} = 1.431\cdot 10^6$$

Friction factor from Moody diagram:

$$f_f = 0.0025$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2\left(\frac{4f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

Loss coefficients:

$$\sum_i L_i = 60m + 20m + 60m = 140m$$

$$\sum_j K_j = 0.05 + 0.4 = 0.45$$

The pressure losses are:

$$\Delta P_f = \frac{1}{2}\cdot 1000\frac{kg}{m^3}\cdot\left(6.366\frac{m}{s}\right)^2\cdot\left(4\cdot\frac{0.0025}{0.2m}\cdot 140m + 0.45\right) = 1.51\cdot 10^5 Pa$$

Finally:

$$\begin{aligned} P_A &= P_B + \rho g(h_B - h_A) + \frac{1}{2}\rho v_B^2 + \Delta P_f \\ &= 101325\ Pa + 1000\frac{kg}{m^3}\cdot 9.81\frac{m}{s^2}\cdot(20 - 30\ m) + \frac{1}{2}\cdot 1000\frac{kg}{m^3}\cdot\left(6.366\frac{m}{s}\right)^2 \\ &+ 1.51\cdot 10^5 Pa = 1.74\cdot 10^5 Pa \end{aligned}$$

c)

The pump has to overcome the pressure induced by friction. Therefore:

$$P_{pump} = \Delta P_f\cdot Q = 1.51\cdot 10^5 Pa\cdot 0.2\frac{m^3}{s} = 30.2\ kW$$

Question 2 – Solutions (10 points)

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v.n) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . n_i$$

$$\sum F_{volume} = m_{water} . g + m_{elbow} . g$$

Velocity of the water in the pipe:

$$\dot{m} = A_2 \cdot v_2 \cdot \rho$$

$$v_2 = \frac{\dot{m}}{A_2 \cdot \rho} = \frac{40\ kg/s}{\left(\frac{0.1m}{2}\right)^2 \cdot \pi \cdot 1000 kg/m^3} = 5.09 \frac{m}{s}$$

Applying Bernoulli's principle between the inlet (1) and the discharge points (O),

$$P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_O + \rho g h_O + \frac{1}{2}\rho v_O^2$$

$$P_1 + \frac{1}{2} \cdot 1000 \frac{kg}{m^3} \cdot \left(5.09 \frac{m}{s}\right)^2 = P_o + \left(1000 \frac{kg}{m^3} \cdot 9.81 \frac{m}{s^2} \cdot 0.5m\right) + \frac{1}{2} \cdot 1000 \frac{kg}{m^3} \cdot \left(5.09 \frac{m}{s}\right)^2$$

$$P_1 - P_o = 4.9\ kPa$$

Solving for x using gauge pressures:

$$F_{reaction,x} - P_1 . A_1 . \widehat{n_1} - P_O . A_O . \widehat{n_O} = \rho \bar{v}_1 (\bar{v}_1 \cdot \widehat{n_1}) \cdot A_1 + \rho \bar{v}_O (\bar{v}_O \cdot \widehat{n_O}) \cdot A_O$$

$$F_{reaction,x} + P_1 A_1 = -\rho v_1^2 A_1 - \rho v_O^2 A_O$$

$$F_{reaction,x} + 4.9 \cdot 10^3 Pa \cdot \left(\frac{d_1}{2}\right)^2 \cdot \pi = -2 \cdot \left[1000 \frac{kg}{m^3} \cdot \left(\frac{0.1m}{2}\right)^2 \cdot \pi \cdot \left(5.09 \frac{m}{s}\right)^2\right]$$

$$F_{reaction,x} = -407 - 38.5 = -445.5\ N$$

Solving for y using gauge pressures:

Now resolving in the y direction,

$$\sum F_{pressure.y} = 0$$

$$\sum F_{reaction.y} = -F_{volume.y} = \rho g V = 1000\,\frac{kg}{m^3}\cdot 9.81\,\frac{m}{s^2}\cdot\left(\frac{0.1m}{2}\right)^2\cdot\,\pi\cdot\frac{0.5m}{2}\cdot\pi = 60.45\ N$$

Weight of the bend is 2 kg. This additional weight must also be supported by the anchoring force.

Therefore,

$$F_{reaction.y.total} = 60.45 + (2\times 9.8) = 80.05N$$

$$F_{net} = ((445.44)^2 + (80.05)^2)^{\frac{1}{2}} = 452.57N$$

<u>Question 3 – Solutions (10 points)</u>

a) *Heat flux (x points)*

$$\dot{Q} = hA\Delta T$$

$$\dot{q} = \frac{\dot{Q}}{A} = h\Delta T = 25\left[\frac{W}{m^2K}\right]\cdot(180-24)[K] = 3900\ \left[\frac{W}{m^2}\right]$$

b) *Steady-state heat flux through the oven walls (x points)*

$$\dot{Q} = \frac{\Delta T}{R_{total}}$$

Here, we have resistances in series so:

$$R_{total} = R_1 + R_2 + R_3 + R_4 + R_5 = \frac{1}{Ah_i} + \frac{L_{metal}}{Ak_{metal}} + \frac{L_{insulation}}{Ak_{insulation}} + \frac{L_{metal}}{Ak_{metal}} + \frac{1}{Ah_o}$$

Reminder:

$$R_{cond} = \frac{L}{Ak} \quad and \quad R_{conv} = \frac{1}{Ah}$$

Simplification :

1) We can neglect the conductive heat transfer resistance of the thin layers of metal because of their high conductivity and very small thickness.
$$\frac{L_{metal}}{Ak_{metal}} \cong 0\ [W^{-1}]$$
2) We can neglect the internal heat transfer coefficient because $T_{\infty,i} = T_{s,i}$
$$\frac{1}{Ah_i} = 0\ [W^{-1}]$$

Therefore:

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta T}{AR_{total}} = \frac{\Delta T}{AR_{total}} = \frac{\Delta T}{A\left(\frac{L_{insulation}}{Ak_{insulation}} + \frac{1}{Ah_o}\right)} = \frac{\Delta T}{\frac{L_{insulation}}{k_{insulation}} + \frac{1}{h_o}}$$

$$\dot{q} = \frac{(180-24)[K]}{\frac{0.03[m]}{0.03\left[\frac{W}{mK}\right]} + \frac{1}{2\left[\frac{W}{m^2K}\right]}} = 104\left[\frac{W}{m^2}\right]$$

c) *Calculation of k_b (x points)*

From b), we know:

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta T}{AR_{total}}$$

$$R_{total} = R_1 + R_2 + R_3 + R_4 + R_5 = \frac{1}{Ah_i} + \frac{L_A}{Ak_A} + \frac{L_B}{Ak_B} + \frac{L_C}{Ak_C} + \frac{1}{Ah_o}$$

Because we know the temperature of the wall, we can neglect the internal heat transfer coefficient.

$$\frac{1}{Ah_i} = 0\ [W^{-1}]$$

Therefore:

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta T}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_o}}$$

Doing some math:

$$\frac{L_B}{k_B} = \frac{\Delta T}{\dot{q}} - \frac{L_A}{k_A} - \frac{L_C}{k_C} - \frac{1}{h_o}$$

$$k_B = \frac{L_B}{\frac{\Delta T}{\dot{q}} - \frac{L_A}{k_A} - \frac{L_C}{k_C} - \frac{1}{h_o}}$$

So:

$$k_B = \frac{0.15[m]}{\frac{(768-24)[K]}{800\left[\frac{W}{m^2}\right]} - \frac{0.3[m]}{20\left[\frac{W}{mK}\right]} - \frac{0.15[m]}{20\left[\frac{W}{mK}\right]} - \frac{1}{2\left[\frac{W}{m^2K}\right]}} = 0.368\left[\frac{W}{mK}\right]$$

Question 4 – Solutions (10 points)

a) *Molar diffusion flux of oxygen*

$$C_{A,0} = S_{AB} \cdot P_{A,1} = 3.12 \cdot 2 = 6.24\ mol/m^3$$

$$C_{A,L} = S_{AB} \cdot P_{A,2} = 3.12 \cdot 1 = 3.12\ mol/m^3$$

$$n_A = -\frac{D_{AB}\left(C_{A,L} - C_{A,0}\right)}{L} = \frac{0.21 \cdot 10^{-9} \cdot 3.12}{5 \cdot 10^{-4}} = 1.31 \cdot 10^{-6}\ mol/(m^2 \cdot s)$$

b) *Molar diffusion flux of oxygen*

$$\dot{m}_A = n_A \cdot M_A = 1.31 \cdot 10^{-6} \cdot 32 = 41.93 \cdot 10^{-6}\ \frac{g}{m^2 \cdot s}$$

c) *Molar concentration of oxygen outside the rubber*

Concentrations outside the rubber on either side can be found using the ideal gas law

$$C_{A,1} = \frac{P_{A,1}}{R \cdot T}$$

$$C_1 = \frac{2}{0.082 \cdot 298} = 0.08\frac{mol}{L}$$

Similarly,

$$C_{A,2} = \frac{P_{A,2}}{R \cdot T}$$

$$C_2 = \frac{1}{0.082 \cdot 298} = 0.04\frac{mol}{L}$$

d) The concentration of the oxygen outside the membrane is determined directly by the partial pressure of oxygen, whereas in the membrane one needs to consider the solubility of the oxygen

Question 5 – Solutions (10 points)

Overall flux of A: n_a

We know that:

$$n_A = k_L\left(C_{A,L,bulk} - C_{A,L,0}\right) = k_G(P_{A,G,0} - P_{A,G,bulk})$$

Furthermore, the Henry's law gives us a relation between $C_{A,0}^{L}$ and $P_{A,0}^{G}$:

$$P_{A,G,0} = HC_{A,L,0}$$

By replacing in the first equation, we get that:

$$k_L\left(C_{A,L,b} - C_{A,L,0}\right) = k_G(HC_{A,L,0} - P_{A,G,bulk})$$

Doing some math:

$$k_L C_{A,L,b} + k_G P_{A,G,b} = C_{A,L,0}(k_L + k_G H)$$

$$C_{A,L,0} = \frac{k_L C_{A,L,bulk} + k_G P_{A,G,bulk}}{k_L + k_G H}$$

$$C_{A,L,0} = \frac{5 \cdot 10^{-4}\left[\frac{m}{s}\right] 10^{-6}\left[\frac{kmol}{m^3}\right] + 0.01\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right] 0[atm]}{5 \cdot 10^{-4}\left[\frac{m}{s}\right] + 0.01\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right] 10\left[\frac{atm \cdot m^3}{kmol}\right]} = 4.97 \cdot 10^{-9}\left[\frac{kmol}{m^3}\right]$$

By using back Henry's law:

$$P_{A,G,0} = HC_{A,L,0} = 10\left[\frac{atm \cdot m^3}{kmol}\right] \cdot 4.97 \cdot 10^{-9}\left[\frac{kmol}{m^3}\right] = 4.97 \cdot 10^{-8}[atm]$$

Therefore:

$$n_A = k_G\left(P_{A,G,0} - P_{A,G,bulk}\right) = 0.01\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right](4.97 \cdot 10^{-8} - 0)[atm] = 4.97 \cdot 10^{-10}\left[\frac{kmol}{m^2 \cdot s}\right]$$

a) *Steady-state heat flux through the oven walls*

We have that:

$$n_A = K_{OL}\left(C_{A,L,bulk} - C_{A,eq,0}\right) = K_{OG}(P_{A,eq,0} - P_{A,G,bulk})$$

We can calculate $C_{A,eq,0}$ and $P_{A,eq,0}$ using the Henry's law:

$$P_{A,eq,0} = HC_{A,L,bulk} = 10\left[\frac{atm \cdot m^3}{kmol}\right] 10^{-6}\left[\frac{kmol}{m^3}\right] = 10^{-5}[atm]$$

$$C_{A,eq,0} = \frac{P_{A,G,bulk}}{H} = \frac{0\ [atm]}{10\left[\frac{atm \cdot m^3}{kmol}\right]} = 0\left[\frac{kmol}{m^3}\right]$$

Therefore:

$$K_{OL} = \frac{n_A}{C_{A,L,bulk} - C_{A,eq,0}} = \frac{4.97 \cdot 10^{-10}\left[\frac{kmol}{m^2 \cdot s}\right]}{(10^{-6}\left[\frac{kmol}{m^3}\right] - 0)} = 4.97 \cdot 10^{-4}\left[\frac{m}{s}\right]$$

$$K_{OG} = \frac{n_A}{P_{A,eq,0} - P_{A,G,bulk}} = \frac{4.97 \cdot 10^{-10}\left[\frac{kmol}{m^2 \cdot s}\right]}{(10^{-5} - 0)[atm]} = 4.97 \cdot 10^{-5}\left[\frac{kmol}{m^2 \cdot s \cdot atm}\right]$$

Question 6 – Solutions (10 points)

We can write the shear stress on the top of the plate:

$$\tau_1 = -\mu_1 \frac{dv}{dy} = \mu_1 \frac{v}{y}$$

and on the bottom of the plate:

$$\tau_2 = -\mu_2 \frac{dv}{dy} = \mu_2 \frac{v}{20 - y}$$

The viscous force (viscous resistance) $F = A * \tau$, thus:

$$F = A * \tau = A * (\tau_1 + \tau_2) = Av \left(\frac{\mu_1}{y} + \frac{\mu_2}{20 - y}\right)$$

For F to be minimum:

$$\frac{dF}{dy} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(20 - y)^2} = 0$$

$$\frac{y^2}{(20 - y)^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{y}{20 - y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$y = 12.6\, mm$$

The solution with shear stress is accepted too.

$$\tau = (\tau_1 + \tau_2) = v \left(\frac{\mu_1}{y} + \frac{\mu_2}{20 - y}\right)$$

$$\frac{d\tau}{dy} = 0$$