Final Exam

Note: answer each question in a separate paper sheet.

Question 1 (10 points)

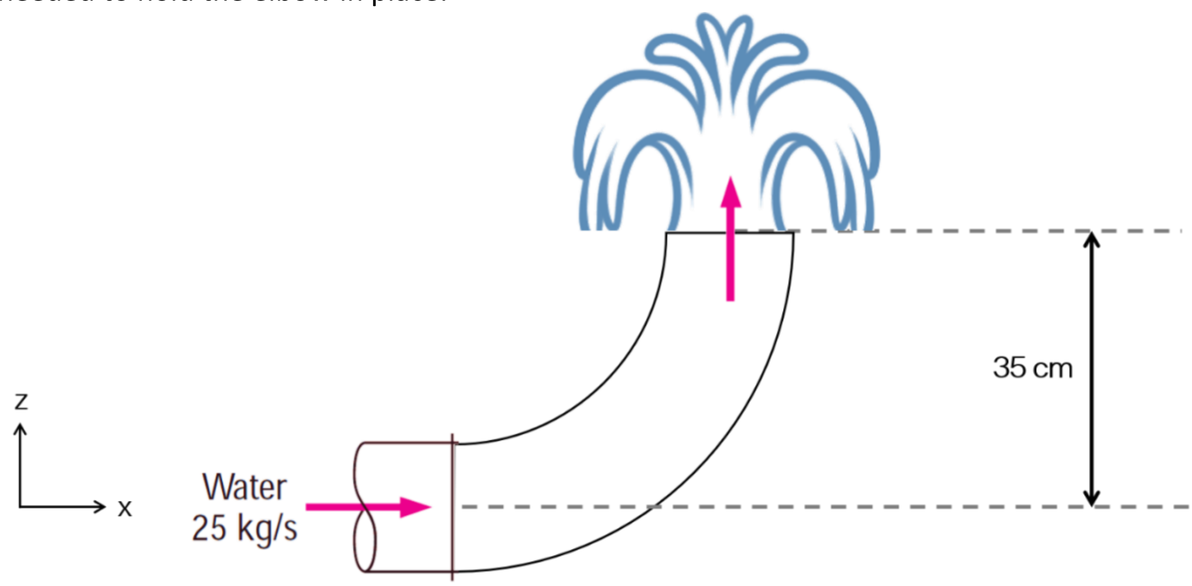
Water (ρ = 1000 kg/m^3 and μ = 1.307 · 10^{-3} kg/m·s) is flowing steadily in a 15 m long and 0.2 cm diameter PVC horizontal pipe at an average velocity of 1.2 m/s. Determine (a) the pressure drop and (b) the pumping power requirement to overcome this pressure drop, expressed in Watts and in pump head (H_p).

After some reconsideration, you decide to change the pipe for an 8 cm diameter stainless steel. Which is the (c) pressure drop and (d) pumping power required to overcome the pressure drop of the new piping system?

Surface	**ε (*roughness*) (·10^{-3} m)**
Copper, Lead, Brass, Aluminum	0.001-0.002
PVC and Plastic Pipes	0.0015-0.007
Epoxy, Vinyl Ester and Isophthalic pipe	0.005
Stainless steel	0.015

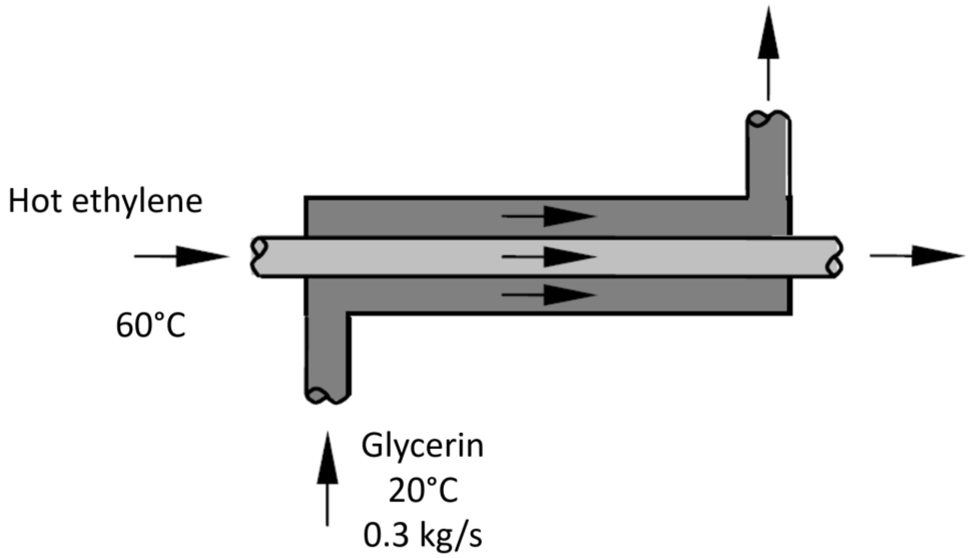
Question 2 (9 points)

A 90 degrees elbow is used to direct water flow at a rate of 25 kg/s in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be negligible. Determine (a) the gauge pressure at the inlet of the elbow and (b) the total anchoring force needed to hold the elbow in place.



Question 3 (9 points)

Glycerin ($C_p = 2400\ J/kg \cdot °C$) at $20°C$ and 0.3 kg/s must be heated by ethylene glycol ($C_p = 2500\ J/kg \cdot °C$) at $60°C$ in a thin-walled double pipe parallel-flow heat exchanger. The temperature difference between the two fluids is $15°C$ at the outlet of the heat exchanger. If the overall heat transfer coefficient is $240\ W/m^2 \cdot °C$ and the heat transfer surface area is $3.2\ m^2$, determine (a) the rate of heat transfer, (b) the outlet temperature of the glycerin, and (c) the mass flow rate of the ethylene glycol.



Question 4 (10 points)

You've probably noticed that balloons inflated with helium rise in the air the first day but begin to fall down the next day and act like ordinary balloons filled with air. This is because helium slowly leaks out through the wall of the balloon while air diffuses in.

Consider a balloon that is made of 0.1 mm thick, soft rubber and has a diameter of 15 cm when inflated (assume that the balloon is a sphere and that the volume does not change). The pressure and temperature inside the balloon are initially 110 kPa and 25°C. The diffusion coefficients of helium, oxygen, and nitrogen in rubber at 25 °C are $2.33 * 10^{-11}$, $1.75 * 10^{-11}$, and $6.44 * 10^{-12}\ m^2/s$, respectively. The molar mass of helium is 4 g/mol and the gas constant $R = 8.314\ \left(\frac{m^3 \cdot Pa}{K \cdot mol}\right)$.

Determine the initial rates of diffusion of helium, oxygen, and nitrogen through the balloon wall in molar flow rates and the mass fraction of helium that escapes the balloon during the first 5 hours assuming the helium pressure inside the balloon remains constant. Assume air to be 21 mole percent oxygen and 79 mole percent nitrogen and assume room conditions of 100 kPa and 25 °C.

Was it a good assumption that the pressure of helium inside the balloon remains constant over 5 hours? Explain with words *and* calculation.

<u>Question 5</u> (12 points)

We are considering the transport of SO_2 from a gas (air) to a liquid phase (ethylene glycol) at a temperature of 40°C. The initial partial pressure of SO_2 in the air is 0.1 bar. The local mass transfer coefficient of SO_2 in air is $4.0 \cdot 10^{-3}\,\frac{cm}{s}$ and the one in ethylene glycol is $2.5 \cdot 10^{-4}\,\frac{cm}{s}$ Consider the Henry's Law relation $P_{SO2}=H^*C_{SO2}$ where H=1.05atm/M.

a) Calculate the **interfacial concentrations** of SO_2 in the gas and in the liquid.

b) Calculate the **mass flux.**

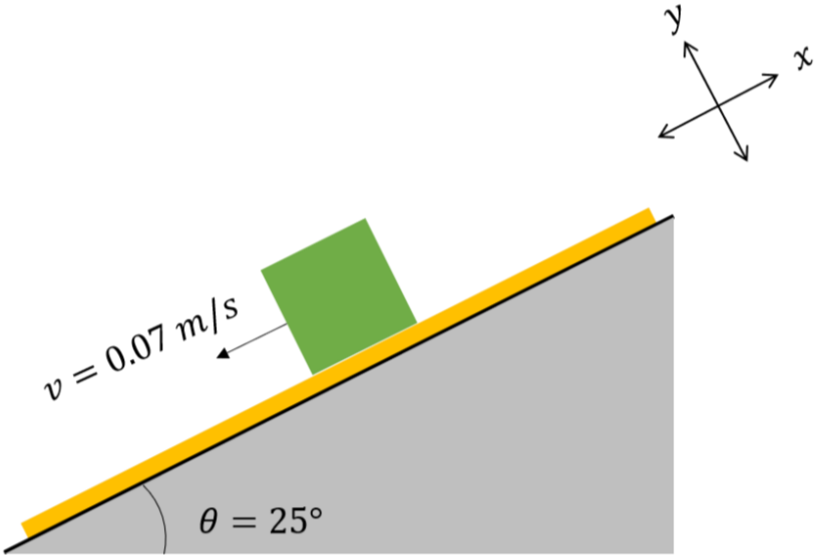
c) Determine if one side is limiting the mass transport.

d) Draw a *quantitative* gas/liquid **concentration diagram**, i.e., identify the concentration values in the bulk and at the interface, along with the direction of the molar flux.

e) Qualitatively identify bulk, equilibrium and interface pressure and concentration in the **equilibrium diagram**.

<u>Question 6</u> (5 points)

A cube (50 cm x 50 cm x 50 cm) weighing 50 N is placed on an inclined ramp with a thin oil film between them (film thickness h=0.8 mm). The plate slides down the ramp at a constant speed of 0.07 m/s. Calculate the viscosity of the oil assuming a linear velocity profile. Draw a free body diagram of the cube indicating the forces acting on it.



QC1. How does the dynamic viscosity of liquids and gases vary with temperature? **(3 points)**

QC2. How does advective mass transport differ from diffusive mass transport? **(3 points)**

Solutions

Solution 1 (10 points)

(a) First, we need to determine the flow regime. The Reynolds number of the flow is:

$$Re = \frac{\rho v_{avg} D}{\mu} = \frac{(1000\ {}^{kg}/_{m^3}) \cdot (1.2\ m/s) \cdot (2 \cdot 10^{-3} m)}{1.307\ \cdot\ 10^{-3}\ \mathrm{kg/m \cdot s}} = 1836$$

The flow is laminar. Therefore, the friction factor is in function of the Reynolds number only and is independent of the roughness of the pipe surface:

$$f_f = \frac{16}{Re} = \frac{16}{1836} = 0.008715$$

$$\Delta P_f = 4 f_f \frac{L}{D} \frac{\rho v_{avg}^2}{2} = 4 \cdot (0.008715) \cdot \frac{15\ m}{0.002\ m} \cdot \frac{(1000\ {}^{kg}/_{m^3}) \cdot (1.2\ m/s)^2}{2} = 188\ kPa$$

(b) The head pressure of the pump must be equal to the head loss due to pressure drop, for the pump to compensate it:

$$H_f = \frac{\Delta P_f}{\rho g} = \frac{188000\ Pa}{\left(1000\ {}^{kg}/_{m^3}\right) \cdot \left(9.8\ {}^{m}/_{s^2}\right)} = 19.2\ m = H_p$$

To calculate the pump power, we require the volume flow rate:

$$Q = A \cdot v_{avg} = [\pi (0.001 m)^2] \cdot \left(1.2 \frac{m}{s}\right) = 3.77 \cdot 10^{-6}\ {}^{m^3}/_{s}$$

$$Power_{pump} = Q \rho g H_p = \left(3.77 \cdot 10^{-6}\ {}^{m^3}/_{s}\right) \cdot \left(1000\ {}^{kg}/_{m^3}\right) \cdot \left(9.8\ {}^{m}/_{s^2}\right) \cdot 19.2\ m = 0.71\ W$$

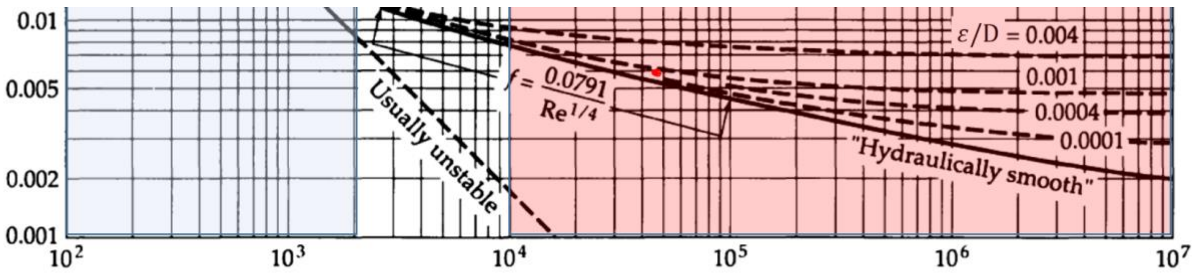
(c) With the new pipe diameter, we have to determine again which flow regime are we in:

$$Re = \frac{\rho v_{avg} D}{\mu} = \frac{(1000\ {}^{kg}/_{m^3}) \cdot (1.2\ m/s) \cdot (0.04\ m)}{1.307\ \cdot\ 10^{-3}\ \mathrm{kg/m \cdot s}} = 36725$$

The flow is turbulent. The relative roughness of the pipe is:

$$\frac{\varepsilon}{D} = \frac{0.015 \cdot 10^{-3}\ m}{0.04\ m} = 0.000375 \cong 0.0004$$

From the Moody diagram:



$$f_f = 0.004$$

$$\Delta P_f = 4 f_f \frac{L}{D}\frac{\rho v_{avg}^2}{2} = 4 \cdot (0.004) \cdot \frac{15\ m}{0.04\ m} \cdot \frac{(1000\ ^{kg}/_{m^3}) \cdot (1.2\ m/s)^2}{2} = 4.32\ kPa$$

(d) The head pressure of the pump must be equal to the head loss due to pressure drop, for the pump to compensate it:

$$H_f = \frac{\Delta P_f}{\rho g} = \frac{4320\ Pa}{\left(1000\ ^{kg}/_{m^3}\right) \cdot \left(9.8\ ^{m}/_{s^2}\right)} = 0.44\ m = H_p$$

$$Power_{pump} = Q\rho g H_p = \left(3.77 \cdot 10^{-6}\ ^{m^3}/_{s}\right) \cdot \left(1000\ ^{kg}/_{m^3}\right) \cdot \left(9.8\ ^{m}/_{s^2}\right) \cdot 0.44\ m = 16.3\ mW$$

Solution 2 (9 points)

(a) The Bernoulli equation applied between the inlet and the outlet allows us to determine the gauge pressure at the inlet of the elbow

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = v_2 = \frac{Q}{\rho A} = \frac{25\ kg/s}{(1000\ {}^{kg}/_{m^3}) \cdot [\pi(0.05m)^2]} = 3.18\ m/s$$

$$P_{2,gauge} = 0\ , v_1 = v_2$$

$$P_1 = \rho g(h_2 - h_1) = \left(1000\ {}^{kg}/_{m^3}\right) \cdot \left(9.8\ {}^{m}/_{s^2}\right) \cdot (0.35\ m - 0\ m) = 3.43\ kPa$$

(b) Using the momentum balance equation to obtain the reaction force:

$$\sum F_{surface} + \sum F_{volume} = \sum_i^N \int_{A_i} \rho\bar{v}(\bar{v}\cdot\hat{n})dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

Volume and friction are neglected:

$$\sum F_{volume} = 0, \sum F_{friction} = 0$$

$$\sum_i^N F_{pressure} = -P_i . A_i . \hat{n_i}$$

Solving for x:

$$F_{reaction-x} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho\bar{v}_1(\bar{v}_1 \cdot \widehat{n_1})A_1 + \rho\bar{v}_2(\bar{v}_2 \cdot \widehat{n_2})A_2$$

$$F_{reaction-x} + P_1 A_1 = -\rho v_1^2 A_1$$

$$F_{reaction-x} + 3430 \cdot \pi \cdot (0.05)^2 = -1000 \cdot (3.18^2) \cdot \pi \cdot (0.05)^2$$

$$F_{reaction-x} = -109\ N$$

Solving for z:

$$F_{reaction-Z} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho\bar{v}_1(\bar{v}_1 \cdot \widehat{n_1})A_1 + \rho\bar{v}_2(\bar{v}_2 \cdot \widehat{n_2})A_2$$

$$F_{reaction-Z} = \rho v_2^2 A_2$$

$$F_{reaction-Z} = -1000 \cdot (3.18^2) \cdot \pi \cdot (0.05)^2$$

$$F_{reaction-Z} = 81.9\ \text{N}$$

$$F_r = \sqrt{F_{r-x}^2 + F_{r-Z}^2} = 136\ N$$

<u>Solution 3</u> (9 points)

(a) For the heat transfer rate, the ΔT_{lm} is needed: $\dot{Q} = UA_S\Delta T_{lm}$

The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,in} = 60°C - 20°C = 40°C$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = T_{h,out} - (T_{h,out} - 15°C) = 15°C$$

And

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{40 - 15}{\ln\left(\frac{40}{15}\right)} = 25.5°C$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_S\Delta T_{lm} = \left(240\frac{W}{m^2 \cdot °C}\right)(3.2m^2)(25.5°C) = 19{,}584\ W = 19.58\ kW$$

(b) The outlet temperature of the glycerin is determined from

$$\dot{Q} = \left[\dot{m}C_p(T_{out} - T_{in})\right]_{glycerin} \rightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}C_p} = 20°C + \frac{19{,}584W}{\left(0.3\frac{kg}{s}\right)\left(\frac{2.4kJ}{kg \cdot °C}\right)}$$
$$= 47.2°C$$

(c) Then the mass flow rate of ethylene glycol becomes

$$Q = \left[\dot{m}C_p(T_{in} - \dot{T}_{out})\right]_{ethylene\ glycol}$$

$$\dot{m}_{ethylene\ glycol} = \frac{\dot{Q}}{C_p(T_{in} - T_{out})} = \frac{19{,}584\frac{J}{s}}{\left(2.5\left(\frac{kJ}{kg \cdot °C}\right)\right)[(47.2 + 15)°C - 60°C]} = \frac{3.56kg}{s}$$

<u>Solution 4</u> (10 points)

<u>In air</u>, the partial pressures of oxygen and nitrogen are

$$P_{N_2} = y_{N_2}P = (0.79)(100\ kPa) = 79\ kPa = 0.79\ bar = 79{,}000\ Pa$$

$$P_{O_2} = y_{O_2}P = (0.21)(100\ kPa) = 21\ kPa = 0.21\ bar = 21{,}000\ Pa$$

Partial pressure of helium in the air is negligible.

<u>Inside the balloon</u>, the initial partial pressure of helium in the balloon is 110 kPa while the partial pressure of nitrogen and oxygen are zero.

The molar flow rate is given by

$$J_A = A * D_{AB} * \left(\frac{dc_A}{dy}\right) \rightarrow A * D_{AB} * \frac{c_A}{Y}$$

For a system where the concentration on one side of the wall is negligible.

When substituting pressure for concentration we obtain

$$J_A = \frac{AD_{AB}}{RT} * \frac{p_{A_1} - p_{A_2}}{Y}$$

The balloon can be treated as an infinitesimally thin sphere.

To calculate the surface area of the balloon

$$A = \pi D^2 = \pi * (0.15m)^2 = 0.07069m^2$$

To calculate the initial rates of diffusion for the three gases we have

$$J_{He} = \frac{\left(0.07069m^2 * 2.33 * 10^{-11}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{110{,}000Pa}{0.1 * 10^{-3}m}\right) = 7.31 * 10^{-7}mol/s$$

$$J_{O_2} = \frac{\left(0.07069m^2 * 1.75 * 10^{-11}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{-21{,}000Pa}{0.1 * 10^{-3}m}\right) = -1.05 * 10^{-7}mol/s$$

$$J_{N_2} = \frac{\left(0.07069m^2 * 6.44 * 10^{-12}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{-79{,}000Pa}{0.1 * 10^{-3}m}\right) = -1.15 * 10^{-7}mol/s$$

The initial mass of helium that escapes during the first 5 hours is

$$m_{diff,He} = M * J_{He} * \Delta t = \frac{4kg}{kmol} * 7.31 * \frac{10^{-7}mol}{s} * 5hr * \frac{3600s}{hr} = 5.26 * 10^{-5}kg$$
$$= 52.6\ mg$$

Initial mass of helium in balloon is

$$m_{initial} = \frac{PV}{RT} * M = \frac{110{,}000Pa * 4 * \pi * (0.075m)^3 * 4}{8.314 * 298 * 3} = 0.315\ g = 315\ mg$$

Fraction of helium that has escaped is therefore 16.7% so our assumption of constant helium pressure in the balloon is not great.

<u>Solution 5</u> (12 points)

We refer to SO_2 as A, air as G and ethylene glycol as L.

a) If we equate the fluxes of SO_2 in the two phases

$$n_A = k_G^{loc}\left(C_{A,G,bulk} - C_{A,G,o}\right) = k_L^{loc}\left(C_{A,L,o} - C_{A,L,bulk}\right)\ (Eq.1)$$

Also, it is given that the interface concentrations are in equilibrium according to the following relation:

$$P_{A,G,o} = H * C_{A,L,o}$$

We consider SO_2 as an ideal gas, which means that:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} \text{ and } C_{A,G,bulk} = \frac{P_{A,G,bulk}}{RT}$$

Also, we can assume that $C_{A,L,bulk} = 0$

To solve for $P_{A,G,o}$:

$$k_G^{loc}\frac{1}{RT}\left(P_{A,G,bulk} - P_{A,G,o}\right) = k_L^{loc}\frac{P_{A,G,0}}{H}$$

Thus, we get

$$P_{A,G,o} = \frac{\frac{k_G^{loc}P_{A,G,bulk}}{RT}}{\left(\frac{k_L^{loc}}{H} + \frac{k_G^{loc}}{RT}\right)}$$

If we put the values from the question,

$$P_{A,G,o} = \frac{\frac{4\times10^{-5}\times0.1}{0.082\times313}}{\left(\frac{2.5\times10^{-6}}{1.05} + \frac{4\times10^{-5}}{0.082\times313}\right)}$$

$$P_{A,G,o} = 4\times10^{-2}atm$$

Thus:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} = 1.6\times10^{-3}M$$

$$C_{A,L,o} = \frac{P_{A,G,o}}{H} = 3.8\times10^{-2}M$$

b) In order to solve for the mass flux, we need first the molar flux. We can use the $(Eq.1)$

$$n_A = k_L^{loc}\times C_{A,L,o} =$$

$$= 2.5\times10^{-6}m/s\ \times 0.038\times10^3 mol/m^3$$

$$n_A = 0.095\times10^{-3}mol/m^2s$$

thus

$$\dot{m}_A = M_A \times n_A = \frac{64g}{mol} \times 0.095 \times \frac{10^{-3}mol}{m^2 s} = 6 \times 10^{-3} g/m^2 s$$

c) To determine if mass transfer is limited on one side, we have to calculate m_G, m_L and m_{avg} and also the C^{eq}values

$$m_G = \frac{c_{A,G,0} - c_{A,G}^{eq}}{c_{A,L,0} - c_{A,L,bulk}} = \frac{1.6 \times 10^{-3} - 0}{3.8 \times 10^{-2} - 0} = 0.042$$

$$m_L = \frac{c_{A,G,bulk} - c_{A,G,0}}{c_{A,L}^{eq} - c_{A,L,0}} = \frac{4 \times 10^{-3} - 1.6 \times 10^{-3}}{0.095 - 3.8 \times 10^{-2}} = 0.042$$

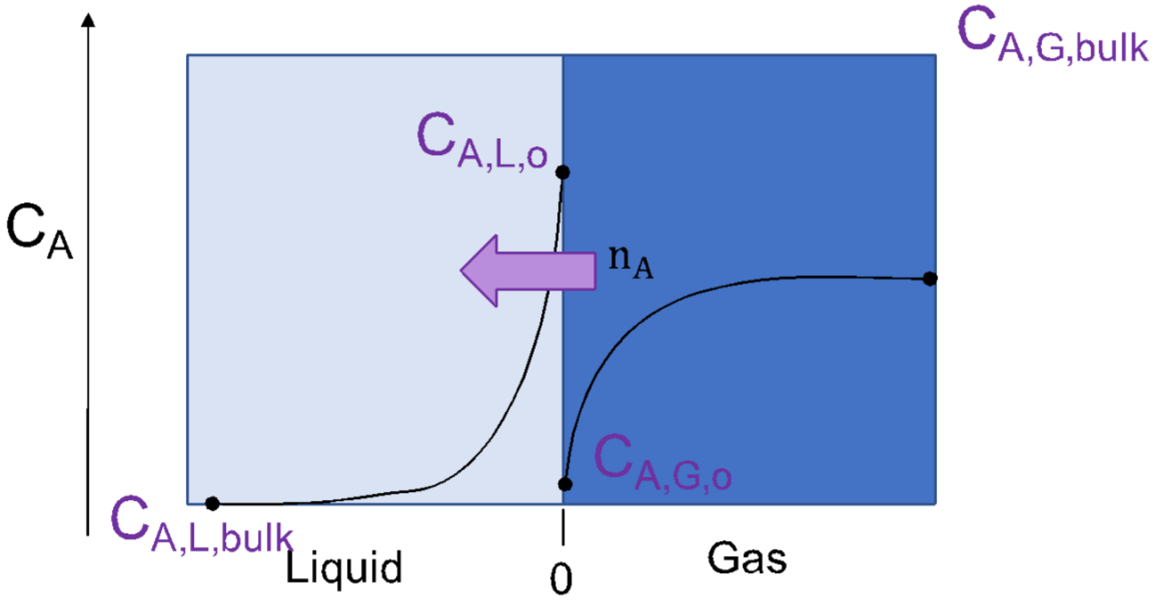
where $c_{A,L}^{eq} = \frac{P_{A,G,bulk}}{H}$

$$m_{avg} = \frac{1}{2}(m_L + m_G) = 0.042$$

$$\frac{k_L^{loc}}{m_{avg} k_G^{loc}} = \frac{2.5 \times 10^{-6} \frac{m}{s}}{0.042 \times 4 \times 10^{-5} \frac{m}{s}} = 1.5$$

Since $\frac{k_W^{loc}}{m_{avg} k_B^{loc}} > 1$, the mass transfer is mildly gas phase controlled.

d)



where

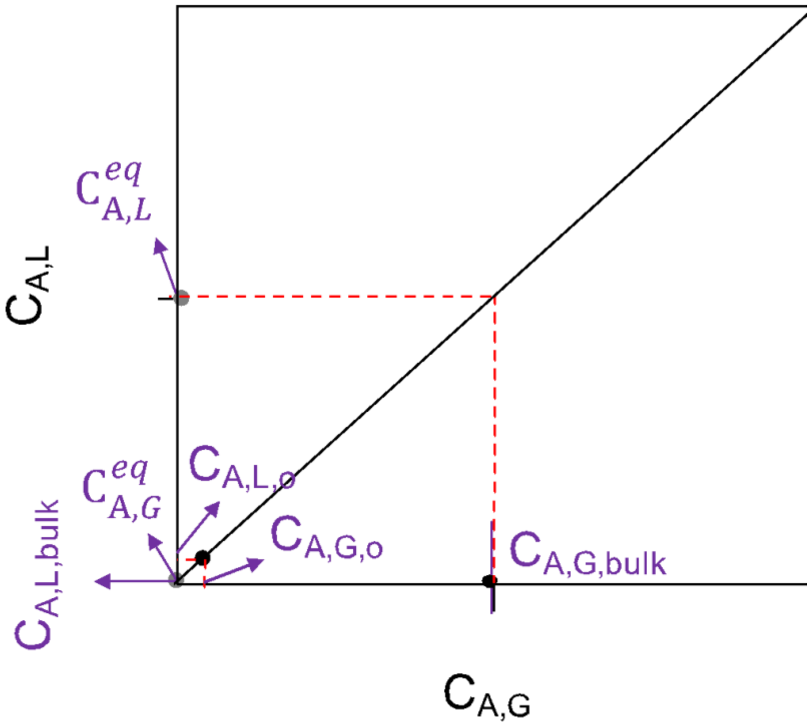
$$C_{A,L,bulk} = 4 \times 10^{-3} M$$

$$C_{A,G,o} = 1.6 \times 10^{-3} M$$

$$C_{A,L,bulk} = 0$$

$$C_{A,L,o} = 3.8 \times 10^{-2} M$$

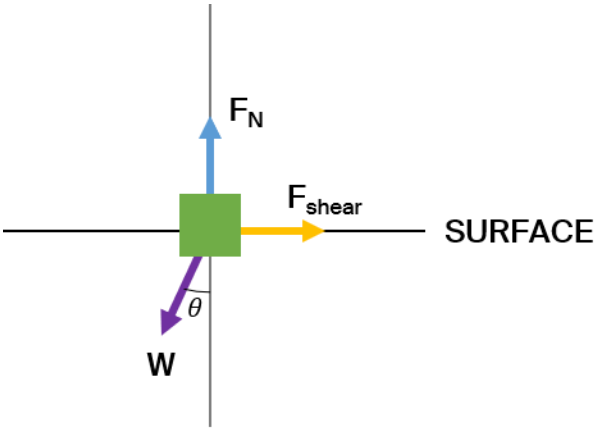
e) The graph below is quantitative, a qualitative version is also accepted considering the challenge in positioning $C_{A,L,bulk}$ and $C_{A,G,eq}$ at the origin.



****

Solution 6 (5 points)

As the velocity of the block is constant, the acceleration and the net force acting on it must be zero. A free body diagram of the block looks like:



The force due to viscosity is by definition:

$$F_{shear} = \tau A = \mu \frac{dv}{dy} A = \mu \frac{v}{h} A$$

The shear force can be equated to the component of weight along the ramp plane:

$$F_{shear} = W \cdot sin\theta$$

Equating the two forces expressions:

$$\mu \frac{v}{h} A = W \cdot sin\theta$$

$$\mu = \frac{W \cdot h \cdot sin\theta}{v \cdot A} = \frac{50\ N \cdot 0.0008\ m \cdot \sin(25°)}{0.07 \frac{m}{s} \cdot (0.5\ m)^2} = 0.879\ Pa \cdot s$$

QC1. How does the dynamic viscosity of liquids and gases vary with temperature? **(3 points)**

The viscosity of gases increases with temperature. According to the kinetic theory of gases, viscosity should be proportional to the square root of the absolute temperature. The higher the temperature, the faster the gas molecules move and the more efficient momentum transfer is.

The viscosity of liquids decreases with temperature. Indeed, as temperature increases the intermolecular forces which are responsible for viscosity weaken, which causes such a decrease.

QC2. How does advective mass transport differ from diffusive mass transport? **(3 points)**

Advective mass transport refers to the *transportation of a fluid on a macroscopic level* from one location to another, wherein the transport is induced by an external force, such as a fun or a pump or gravity. Instead, diffusive mass transport requires the presence of *two regions at different concentration* as it refers to the movement of molecules from a *high concentration region towards a lower concentration*.