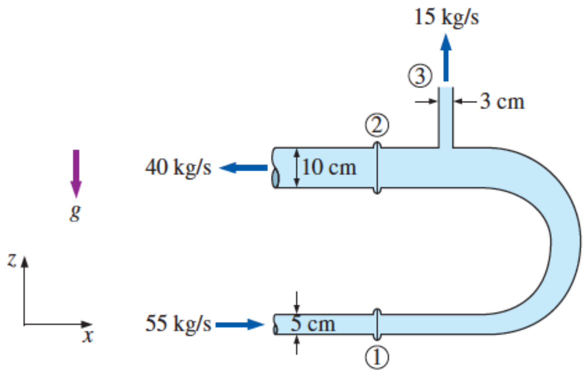
Questions

Question 1

Water is flowing into and discharging from a pipe U-section as shown in the figure.



At flange (1), the total absolute pressure is 200 kPa, and 55 kg/s of water flows into the pipe. At flange (2), the total absolute pressure is 150 kPa. At location (3), 15 kg/s of water discharges to the atmosphere, which is at 100 kPa. **Determine the total x-force and z-force at the two flanges connecting the pipe**. Consider frictionless flow.

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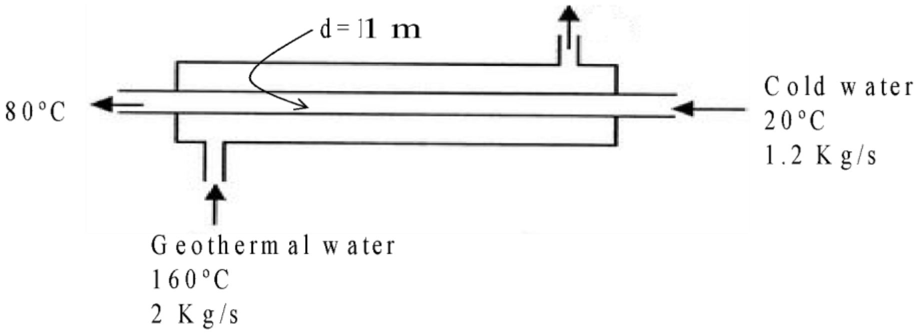
Question 2

A counter-flow double pipe heat exchanger has to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating has to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thinned walled (it means you can neglect the conduction resistance across the wall) and has a diameter of 1 meter.



a) If the overall heat transfer coefficient of the heat exchanger is $640 W/m^2K$, determine the length of the heat exchanger required to achieve the desired heating. ($C_{p,water}$ = 4.187 kJ/ kg K).

b) Assuming h_c = 1280 W/m^2K in the inside tube, calculate h_h in the outside tube.

c) Calculate a new overall heat transfer coefficient in the presence of a fouling layer with thickness 5 cm and with k_{foul}= 10 W/m^2K (tip: you need to use the expression for the overall heat transfer coefficient in a cylindrical geometry)

d) Assuming that the overall heat transfer coefficient and that all temperatures stay the same, do you expect a parallel flow or a counter flow heat exchanger to be more efficient? Comment on why.

e) When heat is transferred by conduction across a series of walls perpendicular to the flux, at steady state and in the absence of losses, how does the heat flux change at each interface? Comment on why.

Question 3

Let us consider the transport of SO_2 from gas (air) to liquid phase (ethylene glycol) at a temperature of 40 ºC. The initial partial pressure of SO_2 in air is 0.1 bar. The local mass transfer coefficients of SO_2 in air and ethylene glycol are $4.0 \cdot 10^{-3}\,\frac{cm}{s}$ and $2.5 \cdot 10^{-4}\,\frac{cm}{s}$ respectively.

Consider Henry's Law $P_{SO_2} = H * C_{SO_2}$ where $H = 1.05\ atm/M$.

a) Calculate the **interfacial concentrations** of SO_2 in the gas and liquid phases.

b) Calculate the **mass flux** of SO_2 from air to ethylene glycol.

c) Determine if one phase is controlling the mass transport.

d) Draw a *quantitative* gas/liquid **concentration diagram**. More specifically, identify the concentration values in the bulk liquid and gas, and at the interface, along with the direction of the molar flux of SO_2.

e) *Qualitatively* identify bulk, equilibrium, and interface concentrations on a **vapor-liquid equilibrium diagram.**

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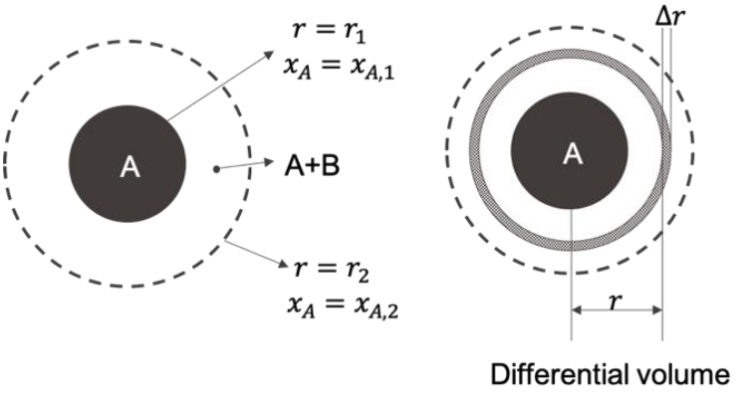
Question 4

a) A sphere of iodine (A), 0.5 cm in radius (r_1), is placed in still air (B) at 40°C and a pressure of 747 mm Hg as shown in the figure below. At this temperature the vapor pressure of iodine is about 1.03 mm Hg. Estimate the **rate of sublimation in mol/s** basing calculations on the net diffusive rate given by:

$$N_A = \frac{4\pi c D_{AB}}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)} ln\left(\frac{1-x_{A,2}}{1-x_{A,1}}\right)$$

Equation 1

where c is the concentration of I_2 in the air at the interface of the air and spherical pellet.
The diffusion coefficient of I_2 in air is 0.088 cm^2/s.



Hint: Assume r_2 to be very large (*i.e.* ∞) and that I_2 vapor exhibits ideal gas behaviour.

b) For the system shown in the figure above, **derive the expression for the net diffusion rate of A through a spherical shell of stagnant gas B at steady-state conditions** that is given in Equation 1.
Make the following assumptions:

- Gas B is not soluble in A.
- The net diffusive flux in radial coordinate system is given as:

$$n_{A,r} = x_A\left(n_{A,r} + n_{B_r}\right) + j_{A,r}$$

- The diffusive flux in radial coordinate system is given as:

$$j_{A,r} = -cD_{AB}\frac{dx_A}{dr}$$

- Assume that the change in r_1 is negligible with respect to time.
- Hint: Solve the equation for rate of net diffusion at steady state, *i.e.* $\frac{d(n_{A,r}W)}{dr} = 0$ where W is the differential area. If needed, $\int_{\square}^{\square} \square \frac{dx}{1-x} = -ln\,(1-x)$.

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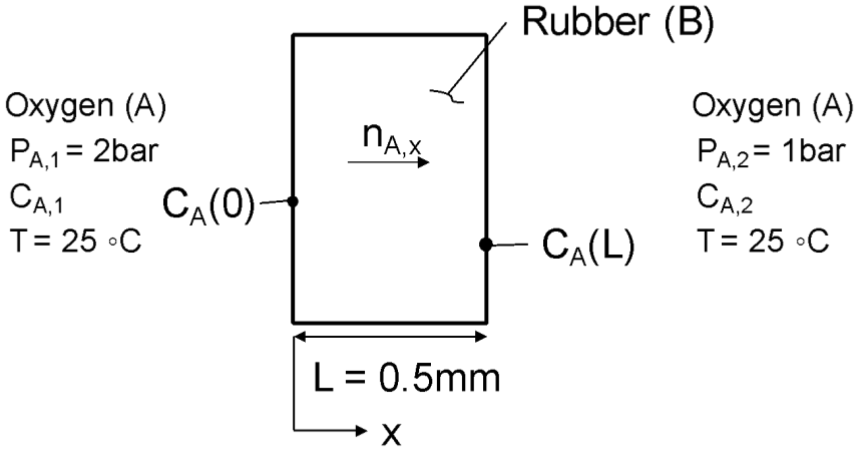
Question 5

Consider the sketch and the data below:



$$D_{AB} = 0.21 * 10^{-9}\,\frac{m^2}{s},\ S_{AB} = 3.12 * 10^{-3}\,\frac{kmol}{m^3 \cdot bar},\ M_A = 32\,\frac{g}{mol},\ \text{and } R = 0.08206\,\frac{L \cdot atm}{K \cdot mol}$$

a) Find the molar diffusive flux of oxygen.

b) Find the mass diffusive flux of oxygen.

c) Find the molar concentration of oxygen outside the rubber assuming ideal gas behaviour.

d) Briefly explain why the molar concentrations outside the membrane differ from those within the membrane.

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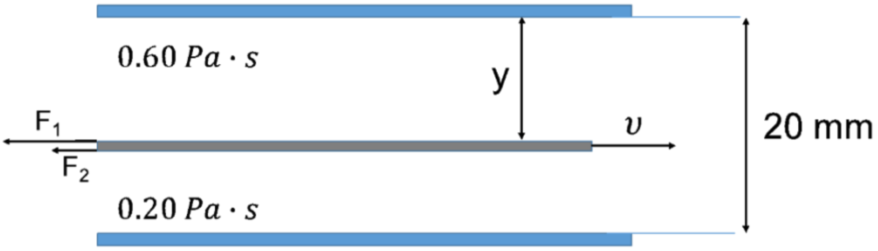
Question 6

A thin plate with an area of 1 m^2 is placed between two fixed surfaces 20 mm apart and the viscosity of the liquids on the top and bottom of the plate are 0.60 Pa · s and 0.20 Pa · s, respectively.



a) If y=10 mm and the plate is being pulled with a velocity of v=3m/s, **which resistance force is being experimented by the plate**?
b) Determine **the position** of the thin plate such that the total viscous resistance force to uniform motion of the thin plate is **minimum**. (*Impose dF/dy=0 to find the minimum*)
c) Would momentum be transported in the absence of viscosity? Comment on why.

Solutions

Question 1 (10 points)

In order to determine the total x-force and y-force acting on the control volume, which is represented by the U-section of the pipe and the water, we have to apply the equation for conservation of momentum considering the velocities and pressures. at the various inlets and outlets (gauge pressure might be use as atmospheric pressure acts on all the surfaces).

We can easily calculate the velocities first:

$$v_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{55\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.05^2}{4}\ m^2} = 28.01\ m^2$$

$$v_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{40\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.1^2}{4}\ m^2} = 5.093\ m^2$$

$$v_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{15\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.03^2}{4}\ m^2} = 21.22\ m^2$$

Equation for momentum conservation:

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . \widehat{n_i}$$

Resolving for X:

$$\sum_{\square}^{\square} \square F_{volume} = 0$$

$$F_{surface-x} = \sum_{i=1}^{N} \square \int_{A_i}^{\square} \square \rho v(v \cdot \hat{n}) dA_i = \rho v_1(v_1 \cdot \widehat{n_1})A_1 + \rho v_2(v_2 \cdot \widehat{n_2})A_2$$

$$\rho v_1(v_1 \cdot \widehat{n_1})A_1 + \rho v_2(v_2 \cdot \widehat{n_2})A_2 = F_{reaction-x} - P_1.A_1.\widehat{n_1} - P_2.A_2.\widehat{n_2}$$

$$-\rho v_1^2 A_1 - \rho v_2^2 A_2 = F_{reaction-x} + P_1.A_1 + P_2.A_2$$

$$F_{reaction-x} = -P_1.A_1 - P_2.A_2 - \rho v_1^2 A_1 - \rho v_2^2 A_2$$

$$F_{reaction-x} = -\left[(200-100)\times 10^3 \times \pi \times \frac{0.05^2}{4}\right] - [(150-100)\times 10^3 \times \pi \times \frac{0.1^2}{4}] - 1000[\left\{28.01^2 \times \pi \times \frac{0.05^2}{4}\right\} + \{5.09^2 \times \pi \times \frac{0.1^2}{4}\}]$$

$$F_{reaction-x} = -196.34N - 392.69N - 1540.48N - 203.48N = -2333N$$

Resolving for Z:

$$\sum_{\square}^{\square} \square F_{volume} = 0$$

$$\sum_{\square}^{\square} \square F_{surface-z} = F_{reaction-z} - P_3.A_3.\widehat{n_3}$$

$$+\rho v_3^2 A_3 = F_{reaction-z} - P_3.A_3.n_3$$

$$+\rho v_3^2 A_3 = F_{reaction-z} + 0 \ (Working\ with\ gauge\ pressure)$$

$$F_{reaction-z} = 1000 \times 21.22^2 \times \pi \times \frac{0.03^2}{4} = 318N$$

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Question 2 (9 points)

a)

$$m_{cold}c_{p_{cold}}\left(T_{cold_{out}}-T_{cold_{in}}\right)=\ m_{hot}c_{p_{hot}}(T_{hot_{in}}-T_{hot_{out}})$$

$$T_{hot_{out}}=T_{hot_{in}}-\frac{m_{cold}(T_{cold_{out}}-T_{cold_{in}})}{m_{hot}}$$

$$T_{hot_{out}}=160-\left(1.2*\frac{60}{2}\right)=124℃$$

$$\Delta T_{lm}=\frac{(160-80)-(124-20)}{ln\ (\frac{160-80}{124-20})}=91.5℃$$

$\dot{Q}$= UAΔT_{lm} = $m_{cold}c_{p_{cold}}\left(T_{cold_{out}}-T_{cold_{in}}\right)$

Area, $A\ =2\pi rL$

$$L=\ \frac{m_{cold}c_{p_{cold}}\left(T_{cold_{out}}-T_{cold_{in}}\right)}{U\Delta T_{lm}2\pi r}$$

$$L=\ \frac{1.2*4187*60}{640*91.5*2*\pi*0.5}=1.639\ m$$

b) The tube is thin walled and hence the resistance to heat transfer from the walls of the tube is neglected.

$$\frac{1}{U}=\frac{1}{h_i}+\frac{1}{h_o}$$

$h_o\ =\ \frac{1}{(\frac{1}{U}-\frac{1}{h_i})}$=1280 W m^{-2}K^{-1}

c)

$$\frac{1}{r_o U_{new}} = \frac{1}{(r_o - \delta_{foul})h_i} + \frac{1}{h_o r_o} + \frac{ln\ (\frac{r_o}{(r_o - \delta_{foul})})}{k_{foul}}$$

$$\frac{1}{U_{new}} = \left(\frac{1}{(0.5 - 0.05) * 1280} + \frac{1}{1280 * 0.5} + \frac{ln\ ln\ \left(\frac{0.5}{(0.5 - 0.05)}\right)}{10} \right) * 0.5$$

$$U_{new} = 144.56\ Wm^{-2}K^{-1}$$

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Question 3 (12 points)

We refer to SO_2 as A, air as G and ethylene glycol as L

a) If we equate the fluxes of SO_2 in the two phases, we obtain:

$$n_A = k_G^{loc}\left(C_{A,G,bulk} - C_{A,G,o}\right) = k_L^{loc}\left(C_{A,L,o} - C_{A,L,bulk}\right) \qquad (Eq. 1)$$

It is given that the interface concentrations are in equilibrium according to the following relation:

$$P_{A,G,o} = H * C_{A,L,o}$$

We consider SO_2 as an ideal gas, which means that:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} \text{ and } C_{A,G,bulk} = \frac{P_{A,G,bulk}}{RT}$$

Also, we can assume that $C_{A,L,bulk} = 0$

To solve for $P_{A,G,o}$:

$$k_G^{loc}\frac{1}{RT}\left(P_{A,G,bulk} - P_{A,G,o}\right) = k_L^{loc}\frac{P_{A,G,o}}{H}$$

Thus, we obtain:

$$P_{A,G,o} = \frac{\frac{k_G^{loc}P_{A,G,bulk}}{RT}}{\left(\frac{k_L^{loc}}{H} + \frac{k_G^{loc}}{RT}\right)}$$

Upon substitution of the values from the question,

$$P_{A,G,o} = \frac{\frac{4 \times 10^{-5} \times 0.1}{0.082 \times 313}}{\left(\frac{2.5 \times 10^{-6}}{1.05} + \frac{4 \times 10^{-5}}{0.082 \times 313}\right)}$$

$$P_{A,G,o} = 4 \times 10^{-2} atm$$

Thus:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} = 1.6 \times 10^{-3} M$$

$$C_{A,L,o} = \frac{P_{A,G,o}}{H} = 3.8 \times 10^{-2} M$$

b) In order to solve for the mass flux, we need first the molar flux. We can use the $(Eq. 1)$

$$n_A = k_L^{loc} \times C_{A,L,o} = 2.5 \times 10^{-6} m/s \ \times 0.038 \times 10^3 mol/m^3$$

$$n_A = 0.095 \times 10^{-3} mol/m^2 s$$

Thus:

$$\dot{m}_A = M_A \times n_A = \frac{64g}{mol} \times 0.095 \times \frac{10^{-3} mol}{m^2 s} = 6 \times 10^{-3} g/m^2 s$$

c) To determine if mass transfer is limited by one phase, we must calculate m_G, m_L and m_{avg} as well as the c^{eq} values

$$m_G = \frac{c_{A,G,0} - c_{A,G}^{eq}}{c_{A,L,0} - c_{A,L,bulk}} = \frac{1.6 \times 10^{-3} - 0}{3.8 \times 10^{-2} - 0} = 0.042$$

$$m_L = \frac{c_{A,G,bulk} - c_{A,G,0}}{c_{A,L}^{eq} - c_{A,L,0}} = \frac{4 \times 10^{-3} - 1.6 \times 10^{-3}}{0.095 - 3.8 \times 10^{-2}} = 0.042$$

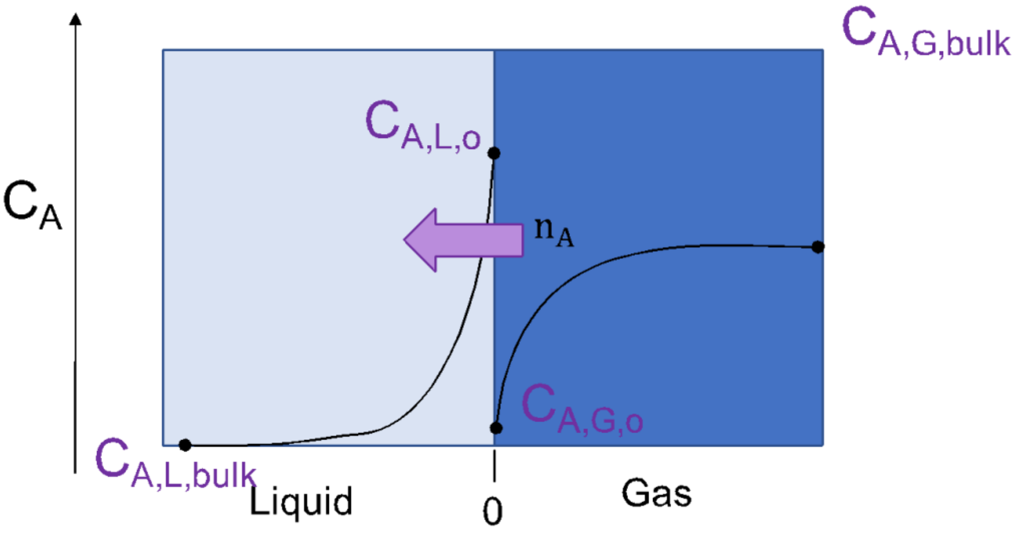
where $c_{A,L}^{eq} = \frac{P_{A,G,bulk}}{H}$.

$$m_{avg} = \frac{1}{2}(m_L + m_G) = 0.042$$

$$\frac{k_L^{loc}}{m_{avg} k_G^{loc}} = \frac{2.5 \times 10^{-6} \frac{m}{s}}{0.042 \times 4 \times 10^{-5} \frac{m}{s}} = 1.5$$

Since $\frac{k_W^{loc}}{m_{avg} k_B^{loc}} > 1$, the mass transfer is mildly gas phase controlled.

d)



where

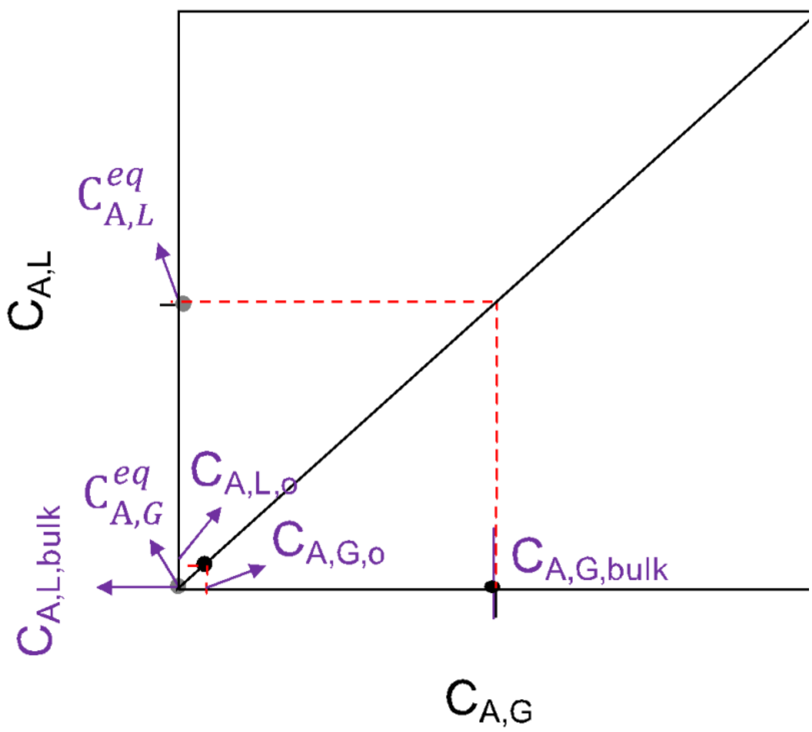
$$C_{A,L,bulk} = 4 \times 10^{-3} M$$

$$C_{A,G,o} = 1.6 \times 10^{-3} M$$

$$C_{A,L,bulk} = 0$$

$$C_{A,L,o} = 3.8 \times 10^{-2} M$$

e) The graph below is quantitative, but a qualitative version is also accepted considering the challenge in positioning $c_{A,L,bulk}$ and $c_{A,G,eq}$ at the origin.



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Question 4 (12 points)

a) The rate of sublimation is given by

$$N_A = \frac{4\pi c D_{AB}}{\left(\frac{1}{r_1} - 0\right)} ln\left(\frac{1 - x_{A,2}}{1 - x_{A,1}}\right)$$

$$c = \frac{P}{RT} = \frac{\frac{747}{760} \times 101.3 \times 10^3\ (Pa)}{8.314\ (J\ mol^{-1}K^{-1}) \times 313(K)} = 38.26\ mol/m^3$$

$$x_{A,1} = \frac{p_A}{P} = \frac{1.03}{747} = 0.0013$$

$$N_A = \frac{4\pi \times 38.26\ (\frac{mol}{m^3}) \times 0.088 \times 10^{-4} (\frac{m^2}{s})\ ln\left(\frac{(1-0)}{(1-0.0013)}\right)}{\left(\frac{1}{0.005} - 0\right)}$$

$$N_A = 2.7\ \times 10^{-8}\ (\frac{mol}{s})$$

b) The differential area, $W = 4\pi r^2$

$$\frac{d(n_{A,r}W)}{dr} = \frac{d(n_{A,r}4\pi r^2)}{dr} = \frac{d(n_{A,r}r^2)}{dr} = 0$$

$$n_{A,r}\, r^2 = C_1 \rightarrow n_{A,r} = \frac{C_1}{r^2}$$

Since B is not diffusing into A, we can assume $n_{B,r} = 0$

$$n_{A,r} = x_A n_{A,r} + j_{A,r}$$

$$n_{A,r} = -\frac{cD_{AB}}{(1 - x_A)}\frac{dx_A}{dr}$$

So, by equating our two expressions for $n_{A,r}$ we obtain

$$-\frac{cD_{AB}}{(1 - x_A)}\frac{dx_A}{dr} = \frac{C_1}{r^2}$$

$$\int_{\square}^{\square} \square \frac{-cD_{AB}}{(1 - x_A)}dx_A = \int_{\square}^{\square} \square \frac{C_1}{r^2}dr$$

Solving the integral gives:

$$cD_{AB}\ ln\,(1-x_A)\ =\frac{-C_1}{r}+C_2$$

We know the following boundary conditions:

$$r=r_1,\quad x_A=x_{A,1}\qquad\qquad r=r_2,\quad x_A=x_{A,2}$$

$$cD_{AB}\ ln\left(1-x_{A,1}\right)\ =\frac{-C_1}{r_1}+C_2$$

$$cD_{AB}\ ln\left(1-x_{A,2}\right)\ =\frac{-C_1}{r_2}+C_2$$

Solving the above equation gives:

$$C_1\ =\ \frac{-cD_{AB}\ ln\left(\frac{\left(1-x_{A,1}\right)}{\left(1-x_{A,2}\right)}\right)}{(\frac{1}{r_1}-\frac{1}{r_2})}$$

$$C_2\ =cD_{AB}\ ln\left(1-x_{A,1}\right)\ -\frac{cD_{AB}\ \ ln\left(\frac{\left(1-x_{A,1}\right)}{\left(1-x_{A,2}\right)}\right)}{r_1(\frac{1}{r_1}-\frac{1}{r_2})}$$

The net diffusion rate (N_A) is given by

$$N_A\ =n_{A,r}\times 4\pi r^2$$

$$n_{A,r}\ =\frac{C_1}{r^2}=\ \frac{-cD_{AB}\ ln\left(\frac{\left(1-x_{A,1}\right)}{\left(1-x_{A,2}\right)}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)r^2}$$

$$N_A = \frac{-cD_{AB}\ ln\left(\frac{(1 - x_{A,1})}{(1 - x_{A,2})}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r^2} \times 4\pi r^2$$

$$N_A = \frac{4\pi cD_{AB}\ ln\left(\frac{(1 - x_{A,2})}{(1 - x_{A,1})}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

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Question 5 (9 points)

a)

$$C_{A,0} = S_{AB} \cdot P_{A,1} = 3.12 \cdot 2 = 6.24\, mol/m^3$$

$$C_{A,L} = S_{AB} \cdot P_{A,2} = 3.12 \cdot 1 = 3.12\, mol/m^3$$

$$n_A = -\frac{D_{AB}\left(C_{A,L} - C_{A,0}\right)}{L} = \frac{0.21 \cdot 10^{-9} \cdot 3.12}{5 \cdot 10^{-4}} = 1.31 \cdot 10^{-6}\, mol/(m^2 \cdot s)$$

b)

$$\dot{m}_A = n_A \cdot M_A = 1.31 \cdot 10^{-6} \cdot 32 = 41.93 \cdot 10^{-6}\, \frac{g}{m^2 \cdot s}$$

c) Concentrations outside the rubber on either side can be found using the ideal gas law

$$C_{A,1} = \frac{P_{A,1}}{R \cdot T}$$

$$C_1 = \frac{2}{0.082 \cdot 298} = 0.08 \frac{mol}{L}$$

Similarly,

$$C_{A,2} = \frac{P_{A,2}}{R \cdot T}$$

$$C_2 = \frac{1}{0.082 \cdot 298} = 0.04 \frac{mol}{L}$$

d) The concentration of the oxygen outside the membrane is determined directly by the partial pressure of oxygen, whereas in the membrane one needs to consider the solubility of the oxygen in the membrane material.

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Question 6 (8 points)

a)

$$\tau_n = -\mu_n \frac{dv}{dy} = \mu_n \frac{v}{y}; \qquad F_{total} = A * \sum_{n=1}^{2} \square\, \tau_n$$

$$\tau_1 = 0.6\, Pa \cdot s\, \frac{3\, m/s}{0.01\, m} = 180\, Pa$$

$$\tau_2 = 0.2\, Pa \cdot s\, \frac{3\, m/s}{0.01\, m} = 60\, Pa$$

$$F_{total} = 1\, m^2 * (180 + 60) Pa = 240N$$

This force is projected in the same direction and contrary sense to the velocity vector.

b) We can write the shear stress on the top of the plate:

$$\tau_1 = -\mu_1 \frac{dv}{dy} = \mu_1 \frac{v}{y}; \qquad F_1 = A * \tau_1$$

and on the bottom of the plate:

$$\tau_2 = -\mu_2 \frac{dv}{dy} = \mu_2 \frac{v}{20 - y}; \qquad F_2 = A * \tau_2$$

The viscous resistance force is $F = F_1 + F_2$, thus:

$$F = A * (\tau_1 + \tau_2) = Av\left(\frac{\mu_1}{y} + \frac{\mu_2}{20 - y}\right)$$

For F to be minimum:

$$\frac{dF}{dy} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(20 - y)^2} = 0$$

$$\frac{y^2}{(20 - y)^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{y}{20-y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$y = 12.6\ mm$$