Introduction to Transport Phenomena: <u>Final Exam</u>

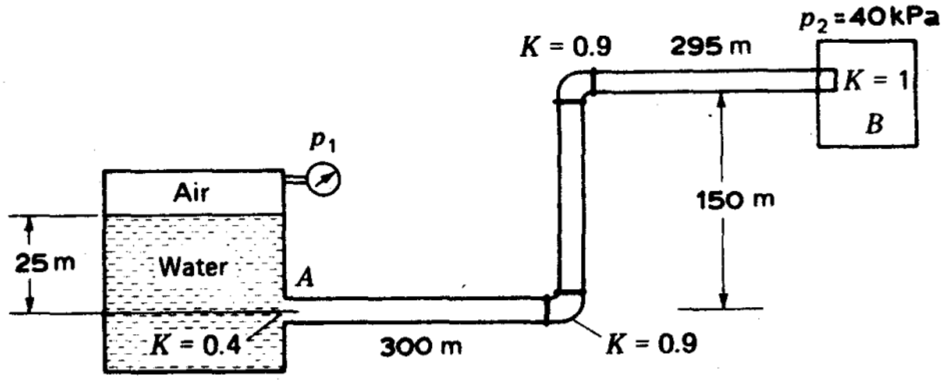
Question 1 (9 points)

The system below carries water (ρ = 1000 kg/m^3; g = 9.8 m/s^2; μ = 8.9 x 10^{-4} Pa·s) at a volumetric rate of 0.1 m^3/s. If the pipe is stainless steel (roughness factor from lecture slides) and has a diameter of 150 mm, calculate the pressure P_1 in the following cases:

a) neglecting losses

b) including losses

c) if water was discharged directly to the atmosphere in B (exclude losses)



Note: Keep in mind that a streamline can be maintained all the way from point 1 (water surface) to point 2 (B)

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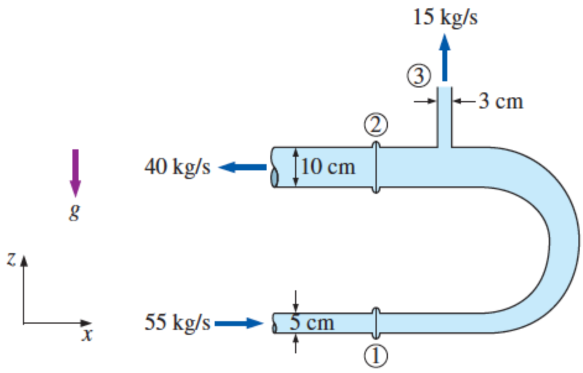
Question 2 (9 points)

Water is flowing into and discharging from a pipe U-section as shown in the figure.



At flange 1, the total absolute pressure is 200 kPa, and 55 kg/s flows into the pipe. At flange 2, the total absolute pressure is 150 kPa. At location (3), 15 kg/s of water discharges to the atmosphere, which is at 100 kPa. **Determine the total x-force and z-force at the two flanges connecting the pipe**. Consider frictionless flow.

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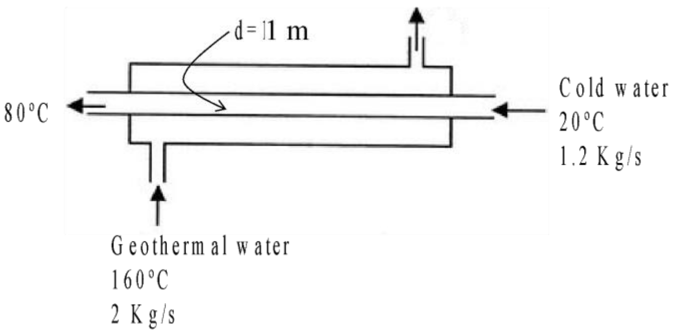
Question 3 (8 points)

A counter-flow double pipe heat exchanger has to heat water from 20°C to 80°C at a rate of 1.2 Kg/s. The heating has to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thinned walled (it means you can neglect the conduction resistance across the wall) and has a diameter of 1 m.



a) If the overall heat transfer coefficient of the heat exchanger is 640W/m^2 C determine the length of the heat exchanger required to achieve the desired heating. ($C_{p,water}$ = 4.187 kJ/ kg K).
b) Assuming h_c = 1280 W/m^2K in the inside tube, calculate h_h in the outside tube.
c) Calculate a new overall heat transfer coefficient in the presence of a fouling layer with thickness 5 cm and with k_{foul}= 10 W/m^2K (tip: you need to use the expression for the overall heat transfer coefficient in a cylindrical geometry)

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Question 4 (8 points)

Hydrogen gas at 10 bars and 27ºC is stored in a 100 mm diameter spherical tank having a steel wall 2 mm thick. The solubility of hydrogen in steel is 0.15 kmol/m^3 bar and the diffusion coefficient of hydrogen in steel is approximately 0.3 x 10^{-12} m^2/s.

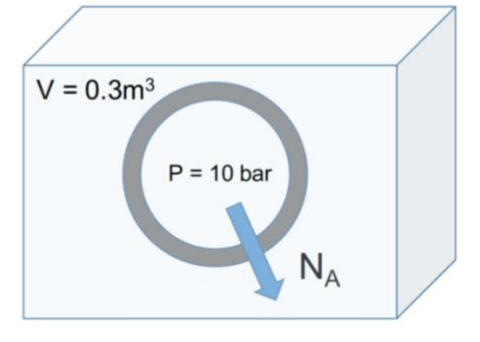
a) If the concentration of hydrogen at the outer surface is negligible, **calculate the initial rate of mass loss of hydrogen diffusion through the tank wall and the initial rate of pressure drop within the tank**.

bar

b) Now consider that the spherical tank is enclosed in a bigger container with overall volume of 0.3m^3. **Calculate how much time it will take to reach a concentration of 0.5 kmol/m^3 in this container** if you make the following assumptions:

1) The only variable changing with time is the concentration of hydrogen in the bigger container.

2) The concentration in the bulk of the bigger container is equal to the concentration at the steel/container interphase.



c) After having calculating the time that will take to reach 0.5 kmol/m^3 in the container, can you comment on why the assumptions given in b) hold?

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Question 5 (12 points)

We are looking at the transport of SO_2 from gas (air) to liquid phase (ethylene glycol) at a temperature of 40 °C. The initial partial pressure of SO_2 in air is 0.1 bar. The local mass transfer coefficient of SO_2 in air is $4.0 \cdot 10^{-3} \frac{cm}{s}$ and the one in ethylene glycol is $2.5 \cdot 10^{-4} \frac{cm}{s}$ Consider the Henry's Law $P_{SO2}=H*C_{SO2}$ where H=1.05atm/M

a) Calculate the **interfacial concentrations** of SO_2 in the gas and in the liquid

b) Calculate the **mass flux**

c) Determine if one side is limiting the mass transport

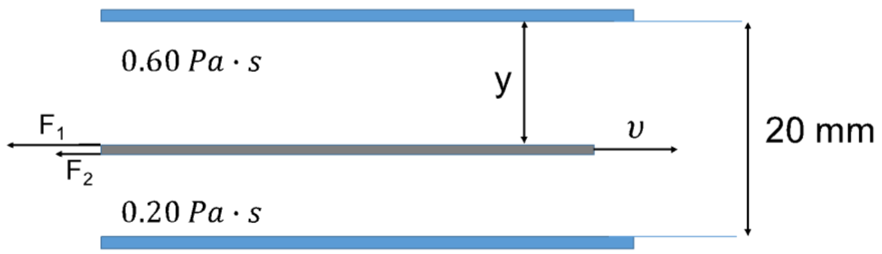
d) Draw a quantitative gas/liquid **concentration diagram**, which means identify the concentration values, in bulk and at the interface, along with the direction of the molar flux (in the exam sheets, not here).

e) Qualitatively identify bulk, equilibrium and interface pressure and concentration in the **equilibrium**

Question 6 (4 points)

A thin plate is placed between two fixed surfaces *20* mm apart and the viscosity of the liquids on the top and bottom of the plate are 0.60 Pa · s and 0.20 Pa · s, respectively.

Determine the position of the thin plate such that the total viscous resistance force to uniform motion of the thin plate is minimum $(Impose\ \frac{dF}{dy} = 0\ to\ find\ the\ minimum)$



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ChE 204 – Introduction to Transport Phenomena

Solutions for Final Exam

Question 1:

$$pipe\ area = \pi \times r^2$$

$$A = \pi \times 0.\frac{15^2}{4} = 0.0176\ m^2$$

$$velocity = Q \times \frac{1}{A}$$

$$velocity = 0.1 \times \frac{1}{0.0176} = 5.68\frac{m}{s}$$

a) Applying Bernoulli between point 1 and point 2

$$P_1 + \rho.g.h_1 + \frac{1}{2}.\rho.v_1^2 = P_2 + \rho.g.h_2 + \frac{1}{2}.\rho.v_2^2$$

$$P_1 + \rho.g.25 = 40 \times 10^3 + \rho.g.150 + \frac{1}{2}.\rho.5.68^2$$

$$P_1 = 1.28\ MPa$$

b) Now we must consider losses, so calculating the Reynolds number, relative roughness and the fanning friction factor

$$R_e = \rho \times v \times \frac{D}{\mu}$$

$R_e = 1000 \times 5.68 \times 0.\frac{15}{8.9\times10^{-4}} = 10^6$

$$\frac{\varepsilon}{D} = 0.\frac{015}{150} = 10^{-4}$$

Checking the corresponding value on the moody chart for the fanning friction factor,

$$f_f = 0.0035$$

$$P_1 + \rho.g.h_1 + \frac{1}{2}.\rho.v_1^2 = P_2 + \rho.g.h_2 + \frac{1}{2}.\rho.v_2^2 + \frac{1}{2}.\rho.v_2^2\,(\frac{4f_f}{D}.\left(\sum L_i\right) + \sum K_j)$$

$$P_1 + \rho.g.25 = \rho.g.150 + \frac{1}{2}.\rho.5.68^2 + 40\times10^3 + \frac{1}{2}.\rho.5.68^2(\left(4\times\frac{f_f}{0.15}(300+150+295)\right) + (0.4+0.9+0.9+1))$$

$$P_1 = 2.35\ MPa$$

c) Essentially same as part 1, just P_2 changes to atmospheric pressure

$$P_1 + \rho.g.h_1 + \frac{1}{2}.\rho.v_1^2 = P_2 + \rho.g.h_2 + \frac{1}{2}.\rho.v_2^2$$

$$P_1 + \rho.g.25 = 100\times10^3 + \rho.g.150 + \frac{1}{2}.\rho.5.68^2$$

$$P_1 = 1.34\ MPa$$

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Question 2:

In order to determine the total x-force and y-force acting on the control volume, which is represented by the U-section of the pipe and the water, we have to apply the equation for conservation of momentum considering the velocities and pressures. at the various inlets and outlets (gauge pressure might be use as atmospheric pressure acts on all the surfaces).

We can easily calculate the velocities first:

$$v_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{55\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.05^2}{4}\ m^2} = 28.01\ m^2$$

$$v_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{40\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.1^2}{4}\ m^2} = 5.093\ m^2$$

$$v_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{15\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.03^2}{4}\ m^2} = 21.22\ m^2$$

Equation for momentum conservation:

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . \hat{n_i}$$

Resolving for X:

$$\sum F_{volume} = 0$$

$$\sum F_{surface-x} = F_{reaction-x} - P_1.A_1.\widehat{n_1} - P_2.A_2.\widehat{n_2}$$

$$\sum F_{surface-x} = F_{reaction-x} + P_1.A_1 + P_2.A_2$$

(this is not a scalar product, the signs come from the coordinate reference system)

$$\sum F_{surface-x} = F_{reaction-x} + [(200-100)\times 10^3 \times \pi \times \frac{0.05^2}{4}] + [(150-100)\times 10^3 \times \pi \times \frac{0.1^2}{4}]$$

$$\sum F_{surface-x} = F_{reaction-x} + 196.34N + 392.69N$$

$$\sum_{i=1}^{N}\int_{A_i}\rho v(v\cdot\hat{n})dA_i = \rho v_1(v_1\cdot\widehat{n_1})A_1 + \rho v_2(v_2\cdot\widehat{n_2})A_2$$

$$= -\rho v_1^2 A_1 - \rho v_2^2 A_2$$

$$= -1000[\left\{28.01^2 \times \pi \times \frac{0.05^2}{4}\right\} + \{5.09^2 \times \pi \times \frac{0.1^2}{4}\}]$$

$$= -1540.48\,N - 203.48N$$

Therefore,

$$F_{reaction-x} = -196.34N - 392.69N - 1540.48N - 203.48N = -2333N$$

Now resolving for Z,

$$\sum F_{volume} = 0$$

$$\sum F_{surface-z} = F_{reaction-z} - P_3.A_3.n_3$$

$$\sum F_{surface-z} = F_{reaction-z} + 0\ (Working\ with\ gauge\ pressure)$$

$$\sum_{i=1}^{N} \int_{A_i} \rho v(v.n) dA_i = +\rho v_3^2 A_3$$

$$= 1000 \times 21.22^2 \times \pi \times \frac{0.03^2}{4} = 318N$$

Therefore,

$$F_{reaction-z} = 318N$$

Comments on the weight: Negligible contribution.

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Question 3:

a) Overall heat transfer coefficient, U = 640 $\frac{W}{m^2}$

$$m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right) = m_{hot}c_{p_{hot}}\left(T_{hot_{in}} - T_{hot_{out}}\right)$$

$$T_{hot_{out}} = T_{hot_{in}} - \frac{m_{cold}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{m_{hot}}$$

$$T_{hot_{out}} = 160 - \left(1.2 * \frac{60}{2}\right) = 124°\text{C}$$

$$\Delta T_{lm} = \frac{(160-80)-(124-20)}{\ln\left(\frac{160-80}{124-20}\right)} = 91.5°\text{C}$$

$\dot{Q}$= UAΔT_{lm} = $m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)$

Area, $A = 2\pi rL$

- **0.5 point for getting the formula right.**

$$L = \frac{m_{cold}c_{p_{cold}}\left(T_{cold_{out}} - T_{cold_{in}}\right)}{U\Delta T_{lm}2\pi rL}$$

$$\boldsymbol{L = \frac{1.2 * 4187 * 60}{640 * 91.5 * 2 * \pi * 0.5} = 1.639\ m}$$

b) The tube is thin walled and hence the resistance to heat transfer from the walls of the tube is neglected.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$h_o = \frac{1}{\left(\frac{1}{U} - \frac{1}{h_i}\right)} = 1280\ \text{W m}^{-2}\text{K}$$

c)

$$\frac{1}{r_o U_{new}} = \frac{1}{(r_o - \delta_{foul})h_i} + \frac{1}{h_o r_o} + \frac{\ln\left(\frac{r_o}{(r_o - \delta_{foul})}\right)}{k_{foul}}$$

$$\frac{1}{U_{new}} = \left(\frac{1}{(0.5 - 0.05) * 1280} + \frac{1}{1280 * 0.5} + \frac{\ln\left(\frac{0.5}{(0.5 - 0.05)}\right)}{10} \right) * 0.5$$

$$\boldsymbol{U_{new} = 145.62\ Wm^{-2}K}$$

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Question 4:

Assumptions:

- Since diameter >> thickness we approximate the spherical shell as a plane wall
- Concentration of hydrogen outside the chamber is negligible
- Hydrogen behaves as an ideal gas

a)

We apply the Fick's Law and write:

$$N_A = -AD_{AB}\frac{C_{A,o} - C_{A,i}}{L}$$

In this formula $C_{A,o}$ indicates the concentration at the steel/air interface, which is initially zero. $C_{A,i}$ is the concentration at internal steel/tank interface, which can be calculated using the solubility:

$$C_{A,i} = P_A \cdot S = 10\ bar\ * 0.15\ \frac{kmol}{m^3 bar} = 1.5\frac{kmol}{m^3}$$

So we write:

$$N_A = AD_{AB}\frac{C_{A,i}}{L} = 3.14X\ 4X(0.05^2)m^2\ * 0.3\ * 10^{-12}\frac{m^2}{s} * \ 1.5\frac{kmol}{m^3} * \frac{1}{0.002\ m}$$
$$= 7 * 10^{-12}\frac{kmol}{s}$$

To calculate the mass loss of hydrogen by diffusion:

$$\dot{m}_A = M_A \cdot N_A = 2\ \frac{kg}{kmol} * \ 7 * 10^{-12}\frac{kmol}{s} = 14 * 10^{-12}\frac{kg}{s}$$

For the pressure loss, we consider hydrogen as an ideal gas. So, because $P = RT\frac{n_A}{V}$ where n_A is the number of moles

$$\frac{dP}{dt} = -\frac{RT}{\pi\frac{4}{3}r^3}N_A = -\frac{\left(0.08314\ \frac{m^3 bar}{kmol\ K}\right)300K}{3.14\ \frac{4}{3}(0.05^3)m^3}\ 7 * 10^{-12}\frac{kmol}{s} = -3.2 * 10^{-7}\frac{bar}{s}$$

b) In the previous question, we have assumed that the concentration $C_{A,o}$ was zero. Now we are considering it as the only variable in time.

Therefore we can write:

$$N_A(t) = -AD_{AB}\frac{C_{A,o}(t) - C_{A,i}}{L}$$

We can also write the equation which correlated the flow rate to the concentration change of hydrogen in the big container

$$N_A(t) = -\frac{d(C_{A,o} \cdot V)}{dt} = -V\frac{dC_{A,o}}{dt}$$

Please note that here the volume is the volume of the container

Therefore $-V\frac{dC_{A,o}}{dt} = -AD_{AB}\frac{C_{A,o}(t)-C_{A,i}}{L}$

$$\frac{dC_{A,o}}{dt} = \frac{AD_{AB}}{V} \cdot \frac{C_{A,o}(t) - C_{A,i}}{L}$$

$$\frac{dC_{A,o}}{\left(C_{A,o}(t) - C_{A,i}\right)} = \frac{AD_{AB}}{VL}dt$$

$$\int_{C_{A,0}(0)}^{C_{A,0}(t_f)} \frac{dC_{A,o}}{\left(C_{A,o}(t) - C_{A,i}\right)} = \int_{t=0}^{t=t_f} \frac{AD_{AB}}{VL}dt$$

$$\ln\frac{\left(C_{A,0}(t_f) - C_{A,i}\right)}{\left(C_{A,0}(0) - C_{A,i}\right)} = \frac{AD_{AB}}{VL}t_f$$

$$C_{A,0}(t_f) = 0.5\frac{kmol}{m^3}$$

$$C_{A,i} = 1.5 \frac{kmol}{m^3}$$

$$C_{A,0}(0) = 0$$

As we found for exercize 2 in the Recap of Module 4 and 5

$$t_f = -\frac{VL}{AD_{AB}} \ln\left(1 - \frac{C_{A,0}(t_f)}{C_{A,i}}\right) = \frac{6 * 10^{-4}}{9.4245 * 10^{-15}} (0.4) = 2.5 * 10^{10}\ s$$

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Question 5:

We refer to SO_2 as A, air as G and ethylene glycol as L

a) If we equate the fluxes of SO_2 in the two phases

$$n_A = k_G^{loc}\left(C_{A,G,bulk} - C_{A,G,o}\right) = k_L^{loc}\left(C_{A,L,o} - C_{A,L,bulk}\right) \ (\boldsymbol{Eq.1})$$ 1 pt

Also, it is given that the interface concentration are in equilibrium as per the following relation:

$$P_{A,G,o} = H * C_{A,L,o}$$

We consider SO_2 as an ideal gas, which means that:

$C_{A,G,o} = \frac{P_{A,G,o}}{RT}$ and $C_{A,G,bulk} = \frac{P_{A,G,bulk}}{RT}$

Also, we can assume that $C_{A,L,bulk} = 0$

In order to solve for $P_{A,G,o}$:

$$k_G^{loc}\frac{1}{RT}\left(P_{A,G,bulk} - P_{A,G,o}\right) = k_L^{loc}\frac{P_{A,G,0}}{H}$$

Thus, we get

$$P_{A,G,o} = \frac{\frac{k_G^{loc} P_{A,G,bulk}}{RT}}{\left(\frac{k_L^{loc}}{H} + \frac{k_G^{loc}}{RT}\right)}$$

If we put the values from the question,

$$P_{A,G,o} = \frac{\frac{4 \times 10^{-5} \times 0.1}{0.082 \times 313}}{\left(\frac{2.5 \times 10^{-6}}{1.05} + \frac{4 \times 10^{-5}}{0.082 \times 313}\right)}$$

$$P_{A,G,o} = 4 \times 10^{-2} atm$$

Thus:

$$\boldsymbol{C_{A,G,o}} = \frac{P_{A,G,o}}{RT} = \mathbf{1.6 \times 10^{-3} \boldsymbol{M}}$$

$$\boldsymbol{C_{A,L,o}} = \frac{P_{A,G,o}}{H} = \mathbf{3.8 \times 10^{-2} \boldsymbol{M}}$$

b) In order to solve for the mass flux, we need first the molar flux. We can use the ($\boldsymbol{Eq.\,1}$)

$$n_A = k_L^{loc} \times C_{A,L,o} =$$

$$= 2.5 \times 10^{-6} m/s \ \times 0.038 \times 10^3 mol/m^3$$

$$n_A = 0.095 \times 10^{-3} mol/m^2 s$$

thus

$$\dot{m}_A = M_A \times n_A = \frac{64g}{mol} \times 0.095 \times \frac{10^{-3} mol}{m^2 s} = 6 \times 10^{-3} g/m^2 s$$

c) To determine if mass transfer is limited on one side, we have to calculate m_{G}, m_{L} and m_{avg} and also the C^{eq}values

$$m_{\mathrm{G}} = \frac{\mathrm{c_{A,G,0}} - \mathrm{c}_{\mathrm{A,G}}^{eq}}{\mathrm{c_{A,L,0}} - \mathrm{c_{A,L,bulk}}} = \frac{1.6 \times 10^{-3} - 0}{3.8 \times 10^{-2} - 0} = 0.042$$

$$m_{\mathrm{L}} = \frac{\mathrm{c_{A,G,bulk}} - \mathrm{c_{A,G,0}}}{\mathrm{c}_{\mathrm{A,L}}^{eq} - \mathrm{c_{A,L,0}}} = \frac{4 \times 10^{-3} - 1.6 \times 10^{-3}}{0.095 - 3.8 \times 10^{-2}} = 0.042$$

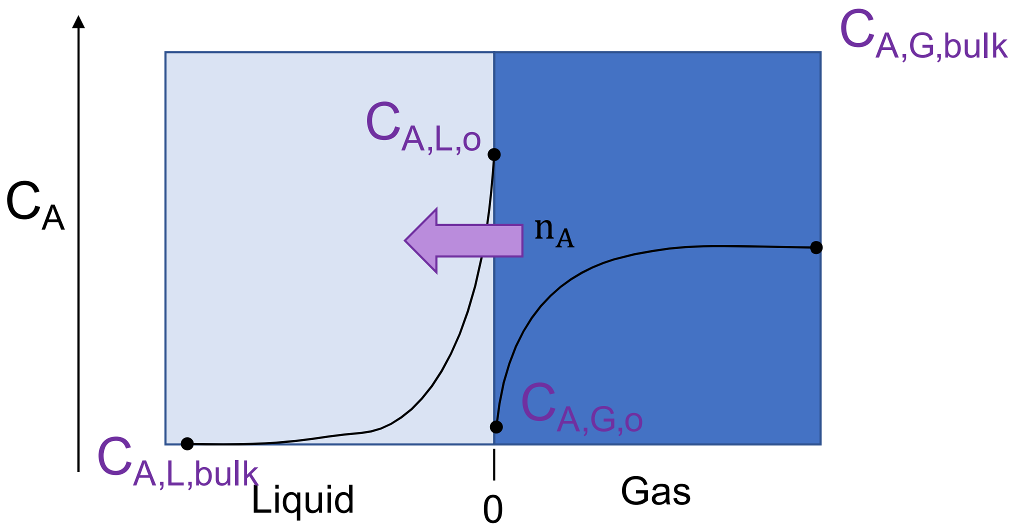
where $\mathrm{c}_{\mathrm{A,L}}^{eq} = \frac{P_{A,G,bulk}}{H}$

$$m_{avg} = \frac{1}{2}(m_{\mathrm{L}} + m_{\mathrm{G}}) = 0.042$$

$$\boldsymbol{\frac{k_L^{loc}}{m_{avg} k_{\mathrm{G}}^{loc}} = \frac{2.5 \times 10^{-6} \frac{m}{s}}{0.042 \times 4 \times 10^{-5} \frac{m}{s}} = 1.5}$$

Since $\boldsymbol{\frac{k_W^{loc}}{m_{avg} k_{\mathrm{B}}^{loc}}} > \mathbf{1}$**,** the mass transfer is mildly gas phase controlled.

d)



where

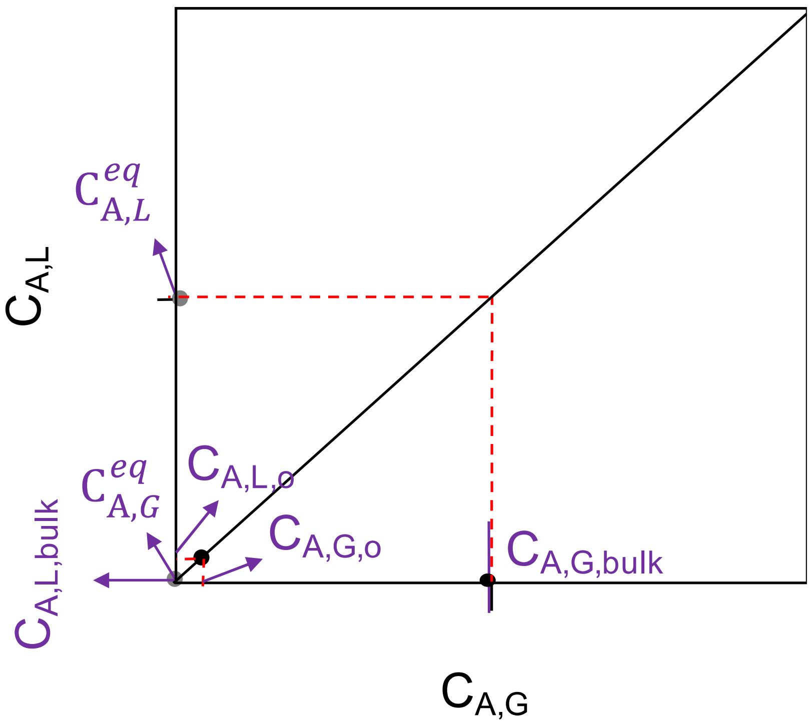
$$C_{A,L,bulk} = 4 \times 10^{-3} M$$

$$C_{A,G,o} = 1.6 \times 10^{-3} M$$

$$C_{A,L,bulk} = 0$$

$$C_{A,L,o} = 3.8 \times 10^{-2} M$$

e) The graph below is quantitative, a qualitative version is also accepted considering the challenge in positioning $C_{A,L,bulk}$ and $C_{A,G,eq}$ at the origin.


$C^{eq}_{A,L}$
$C_{A,L}$
$C^{eq}_{A,G}$
$C_{A,L,o}$
$C_{A,G,o}$
$C_{A,L,bulk}$
$C_{A,G,bulk}$
$C_{A,G}$

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Question 6:

We can write the shear stress on the top of the plate:

$$\tau_1 = -\mu_1 \frac{dv}{dy} = \mu_1 \frac{v}{y}; \qquad F_1 = A * \tau_1$$

and on the bottom of the plate:

$$\tau_2 = -\mu_2 \frac{dv}{dy} = \mu_2 \frac{v}{20-y}; \qquad F_2 = A * \tau_2$$

The viscous resistance force is $F = F_1 + F_2$, thus:

$$F = A * (\tau_1 + \tau_2) = Av \left(\frac{\mu_1}{y} + \frac{\mu_2}{20-y}\right)$$

For F to be minimum:

$$\frac{dF}{dy} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(20-y)^2} = 0$$

$$\frac{y^2}{(20-y)^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{y}{20-y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$y = 12.6\ mm$$