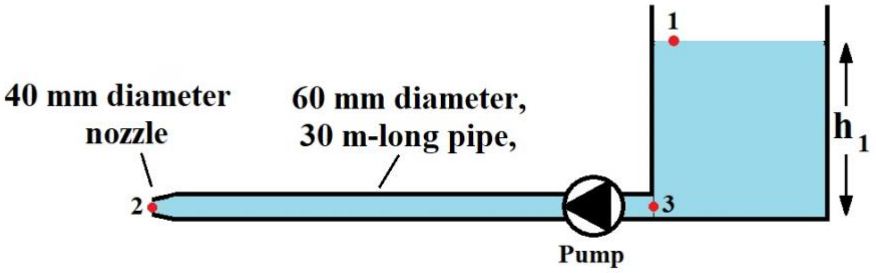
Introduction to Transport Phenomena: Final Exam

Question 1 (11 points)

In the system below, the pump provides a power of 25 kW and generates a flow rate of 0.04 m^3/s in the pipe. The fluid is water so ρ=1000kg/m^3 and μ=1.69 x 10^{-3} Pa*s. Also take $g = 9.81 m/s^2$. To calculate the friction pressure drop in the pipe, assume the roughness of the pipe ε = 6 x 10^{-6} m and neglect the minor losses. The pressure at points 1 and 2 are equal to atmospheric pressure.

a) Determine the height of water in the reservoir (h_1)

b) Determine the flow rate if the pump is removed from the system.(Use the same friction factor f_f as that calculated in part (a))



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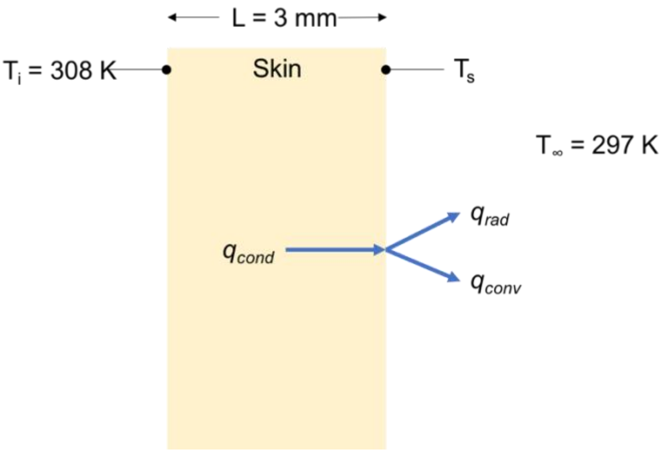
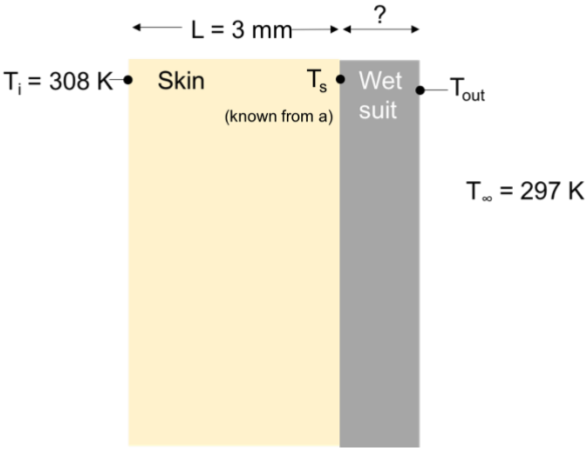
Question 2 (10 points)

Humans are able to control their heat production rate and heat loss rate to maintain a nearly constant body temperature. This process is called thermoregulation. From the perspective of calculating the heat transfer between the human body and its surroundings, we focus on a layer of skin with thickness L = 3mm and thermal conductivity k = 0.3 W/m·K. The inside temperature is T_i = 35ºC = 308 K. Consider that the total surface area of the person is A = 1.8 m^2.

a) Assume the person in a room with T_∞ = 297 K and convection heat transfer to the **air** h_{conv} = 2 W/m^2 ·K What is the skin surface temperature T_s and the **rate of heat loss**? Consider q_{rad}=$h_{rad}(T_S - T_\infty)$ with h_{rad} = 5.9 W/m^2 ·K.



b) Assume now that the person is in **water**, to make the skin surface temperature in water the same as the one in air, the person wears a **wet suit** covering the whole body. The thermal conductivity of the wet suit is k=0.03 W/m^2·K. The convection heat transfer coefficient from the wet suit to water is h = 200 W/m^2·K and radiative heat transfer is neglected. Calculate the surface temperature of the wet suit $\mathbf{T_{out}}$ and the **thickness** that the wet suit should have to get the same surface skin temperature obtained in a).



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Question 3 (12 points)

We are looking at the transport of SO_2 (molar mass of SO_2 is 64 g/mol) from gas (air) to liquid phase (ethylene glycol) at a temperature of 40 °C. The initial partial pressure of SO_2 in air is 0.1 bar. The local mass transfer coefficient of SO_2 in air is $4.0 \cdot 10^{-3}\,\frac{cm}{s}$ and the one in ethylene glycol is $2.5 \cdot 10^{-4}\,\frac{cm}{s}$ Consider the Henry's Law $P_{SO2,G,0}=H*C_{SO2,L,0}$ where H=1.05atm/M, P_{SO2} is the pressure of SO_2 in gas phase in atm and C_{SO2} is the concentration of SO_2 in liquid phase in mol/l.

a) Calculate the **interfacial concentrations** of SO_2 in the gas and in the liquid

b) Calculate the **mass flux**

c) Determine if one side is limiting the mass transport

d) Draw a quantitative gas/liquid **concentration diagram**, which means identify the concentration values, in bulk and at the interface, along with the direction of the molar flux (in the exam sheets, not here).

e) Qualitatively identify bulk, equilibrium and interface pressure and concentration in the **equilibrium diagram** (in the exam sheets, not here).

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Question 4 (5 points)

For the system shown in Fig.1, what is the **evaporation rate in g/hr** of $CC1_3NO_2$ (chloropicrin) into air at 25°C?

Total pressure in the column	770 mm Hg
Diffusivity (CCl_3NO_2-air)	0.088 cm^2/s
Vapor pressure of CCl_3NO_2	23.81 mm Hg
Distance from liquid level to top of tube	11.14 cm
Surface area of liquid exposed for evaporation	2.29 cm^2
Molar mass of CCl_3NO_2	164.5 g/mol

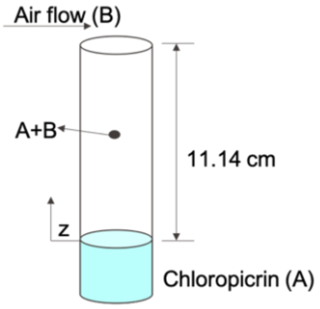

Figure 1

1 atm = 760 mm Hg

The following *assumptions* are made:

- Steady-state, one-dimensional diffusion in the column
- Constant properties
- Uniform temperature throughout the column
- CCl_3NO_2 vapor exhibits ideal gas behavior
- The change in diffusion path is negligible with respect to time.

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Question 5 (10 points)

a) A sphere of iodine (A), 0.5 cm in radius (r_1), is placed in still air (B) at 40°C and 747 mm Hg pressure as shown in Figure 2. At this temperature the vapor pressure of iodine is about 1.03 mm Hg. Estimate the **rate of sublimation in mol/s** basing calculations on the net diffusive rate (in mol/s) given by:

$$N_A = \frac{4\pi c D_{AB}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \ln \left(\frac{1 - x_{A,2}}{1 - x_{A,1}}\right)$$

Molar diffusivity (I_2-air) 0.088 cm^2/s

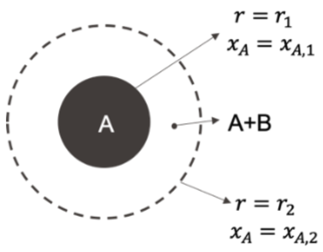

Figure 2

Hint : Assume r_2 to be very large and I_2 vapor exhibits ideal gas behavior.

b) For system shown in Figure 3, attempt to **derive the expression for the net diffusion rate (mol/s) of A through a spherical shell of stagnant gas B at steady-state conditions** as:

$$N_A = \frac{4\pi c D_{AB}}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \ln\left(\frac{1 - x_{A,2}}{1 - x_{A,1}}\right)$$

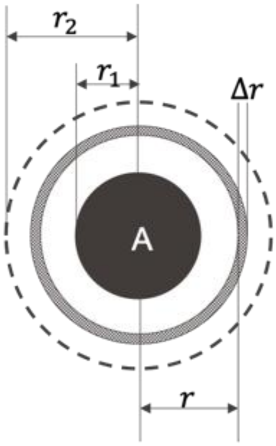

Figure 3

Where c is the mixture molar concentration, D_{AB} is the molar diffusivity of A in B, x_{A_1} and x_{A_2} are mole fractions of A at r_1 and r_2.

The following assumptions are made:

- Gas B is not soluble in A
- The net diffusive flux in radial co-ordinate system is given as:
  $$n_{A,r} = x_A\left(n_{A,r} + n_{B,r}\right) + j_{A,r}$$
- The diffusive flux in radial co-ordinate system is given as $j_{A,r} = -cD_{AB}\frac{dx_A}{dr}$
- Assume that change in $r_1$is negligible with respect to time.

Hint : Solve the equation for rate of net diffusion at steady state $\frac{d\,(n_{A,r}S)}{dr} = 0$ where S is the differential area. Refer to the derivation of the net diffusive flux in a linear system (evaporation through a column).

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Question 6 (12 points)

We want to dye a disk-shaped window. Its diameter is D=1 m, its thickness is t=1 cm and its weight is 10 kg. The window is pulled out from the dye solution with a force F of 25 N. The density of the dye solution is ρ_{dye}=1200 kg/m^3 and its dynamic viscosity is 0.5 Pa·s.

a) Look at Figure 3 below. Can you estimate the **velocity** at which the window is pulled out when it is still submerged? (Hint : Consider only the two flat basal facets and ignore the curved surface)

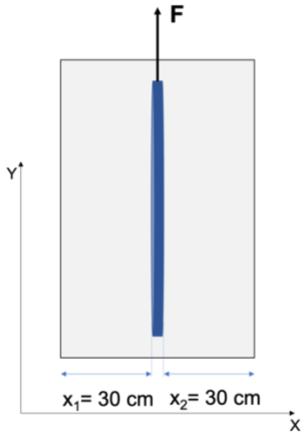

Figure 3

b) Look at Figure 4. Can you estimate the **velocity** at which the window is pulled out when it is still submerged? (Hint : Keep in mind that because the window is not in the center, the velocity gradients on the two flat sides are different. Ignore the curved surface)

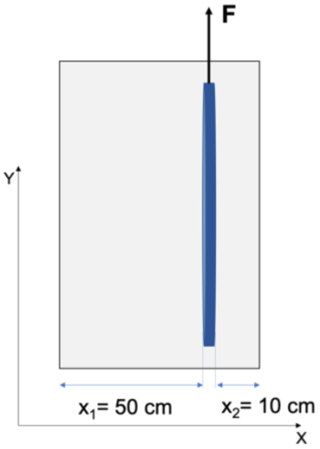

Figure 4

c) Taking the velocity of window as calculated in part b), find the **velocities of the solution layers** which are at a distance of 5 cm from the moving window on both sides. Assume a linear velocity profile on both sides (Hint : $V_y = a - bx$ and think about the right boundary conditions)

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ChE 204 – Introduction to Transport Phenomena

Solutions for Final Exam

Question 1:

a) We apply the Bernoulli's equation between point 1 and point 3. Keep in mind that for the same assumption made in the Torricelli's theorem $v_1 \ll v_3$. So, we write:

$$P_1 + \rho g h_1 + \cancelto{0}{\frac{1}{2}\rho v_1^2} = P_3 + \cancelto{0}{\rho g h_3} + \frac{1}{2}\rho v_3^2$$

Then we apply the Bernoulli equation between point 2 and point 3, $(h_2 = h_3 = 0)$.

$$P_3 + \frac{1}{2}\rho v_3^2 \; + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

Comparing the two equations, we can write:

$$P_1 + \rho g h_1 + P_{pump} = P_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

$$P_{1,gauge} = P_{2,gauge} = 0$$

$$v_2 = \frac{Q}{A_2} = \frac{0.04\frac{m^3}{s}}{\frac{\pi}{4}(0.04\,m)^2} = 31.84\,\frac{m}{s}$$

In order to find the ΔP_f, we need to find the friction factor:

$$Re = \frac{\rho * v_{pipe} * D}{\mu}$$

$$v_{pipe} = \frac{Q}{A_{pipe}} = \frac{0.04\,\frac{m^3}{s}}{\frac{\pi}{4}(0.06\,m)^2} = 14.15\,\frac{m}{s}$$

Consider $v_{pipe} is\ v_3$

$$Re = \frac{1000\,\frac{kg}{m^3} * 14.15\,\frac{m}{s} * 0.06m}{1.69 * 10^{-3} Pas} = 5\ * 10^5$$

$$\frac{\varepsilon}{D_{pipe}} = \frac{6 * 10^{-6}\ m}{0.06\ m} = 0.0001$$

Friction factor from Moody diagram:

$$f_f\ = 0.004$$

Calculation of friction induced pressure loss term ΔP_f

$$\Delta P_f\ = \frac{1}{2}\rho v_{pipe}^2\left((\frac{4f_f}{D_{pipe}}\sum_{i}^{\square} \square\, L_i) + \sum_{j}^{\square} \square\, K_j \right)$$

$$\Delta P_f\ = \frac{1}{2} * 1000\,\frac{kg}{m^3} * \left(14.15\,\frac{m}{s}\right)^2 * \left(4 * \frac{0.004}{0.06m} * 30 + 0\right) =\ 8 * 10^5\ Pa$$

$$P_{pump} = \frac{power}{Q} = \frac{25 \times 10^3 W}{0.04\,\frac{m^3}{s}} = 625000\ Pa$$

$$h_1\rho g = \frac{\rho v_2^2}{2} + \Delta P_f - P_{pump} = \frac{1000\,\frac{kg}{m^3} * \left(31.8\,\frac{m}{s}\right)^2}{2} + 8 * 10^5 - 625000 = 680620\ Pa$$

$$h_1 = \frac{680620 Pa}{1000\,\frac{kg}{m^3} * 9.81\,\frac{m}{s^2}} = 69.4\ m$$

b)

$Q = A_{pipe} * v_{pipe} = A_2 * v_2$

$$v_2 = \frac{A_{pipe} * v_{pipe}}{A_2} = (\frac{D_{pipe}}{D_2})^2 * v_{pipe} = (\frac{0.06}{0.04})^2 * v_{pipe} = 2.25 *\ v_{pipe}$$

Without the pump: $P_{pump} = 0$ and we already calculated $h_1 = 69.4\ m$. So, we can write:

$$h_1 = 69.4\ m = \frac{v_2^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}}\sum_{i}^{\square}\square\, L_i\right) = \frac{(2.25\ \ v_{pipe})^2}{2g} + \frac{v_{pipe}^2}{2g}\left(\frac{4f_f}{D_{pipe}}\sum_{i}^{\square}\square\, L_i\right)$$

$$69.4\ m = \frac{{v_{pipe}}^2}{2 * 9.81\,\frac{m}{s^2}}\left(5.06 + \frac{4 * 0.004\ \ * 30\ m}{0.06\ m}\right)$$

$$v_{pipe} = \sqrt{\frac{69.4 * 2 * 9.81\ m^2}{13.06\ s^2}} = 10.21\,\frac{m}{s}$$

$Q = v_{pipe} * A_{pipe} = 10.21\,\frac{m}{s}\ *\ \frac{\pi}{4}(0.06\ m)^2 = 0.0288\ \frac{m^3}{s}$

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Question 2:

a) Because we are always assuming no heat losses or gain

$$q_{cond} = q_{conv} + q_{rad}$$

$$q_{cond} = k\frac{(T_i - T_s)}{L}$$

$$q_{conv} = h_{conv}(T_s - T_\infty)$$

$$q_{rad} = h_{rad}(T_s - T_\infty)$$

Hence

$$k\frac{(T_i - T_s)}{L} = h_{conv}(T_s - T_\infty) + h_{rad}(T_s - T_\infty)$$

Rearranging the terms, we get

$$T_s = \frac{k\frac{T_i}{L} + (h_{conv} + h_{rad})T_\infty}{\frac{k}{L} + (h_{conv} + h_{rad})}$$

$$T_s = \frac{0.3\frac{W}{m\cdot K}\frac{308\ K}{3\times 10^{-3}\ m} + \left(2\frac{W}{m^2\cdot K} + 5.9\frac{W}{m^2\cdot K}\right).297\ K}{\frac{308\ K}{3\times 10^{-3}\ m} + \left(2\frac{W}{m^2\cdot K} + 5.9\frac{W}{m^2\cdot K}\right)}$$

Substituting the numbers, we get

$$T_s = 307.2\ K$$

For the rate of heat loss,

$$\dot{Q} = q_{cond}\ A = kA\frac{(T_i - T_s)}{L} = 146\ W$$

b) Now we have an additional conductive resistance, so the energy conservation equation becomes:

$$q_{cond(skin)} = q_{cond(wet\ suit)} = q_{conv}$$

$$q_{cond(skin)} = k_{skin}\frac{(T_i - T_s)}{L_{skin}}$$

$$q_{cond(wet\ suit)} = k_{wet\ suit}\frac{(T_{suit(in)} - T_{suit(out)})}{L_{wet\_suit}}$$

$$q_{conv} = h_{conv}(T_{suit(out)} - T_\infty)$$

We know that

$$T_{suit(in)} = T_{skin} = 307.2K$$

$$0.3\frac{W}{m.K} \times\ 1.8\ m^2\ \times \frac{(308 - 307.2)K}{3 \times 10^{-3}m} = 146\ W = 200\ K\ \times\ (T_{suit(out)} - 297)K$$

$$T_{suit(out)} = 297.3\ K$$

$$146\,W = 0.03\,\frac{W}{m.K}.\times\ 1.8\,m^2\ \times \frac{(307.2-297.3)K}{L_{wetsuit}}$$

$$L_{wetsuit} = 3.7\,mm$$

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Question 3:

We refer to SO_2 as A, air as G and ethylene glycol as L

a) If we equate the fluxes of SO_2 in the two phases

$$n_A = k_G^{loc}\left(C_{A,G,bulk} - C_{A,G,o}\right) = k_L^{loc}\left(C_{A,L,o} - C_{A,L,bulk}\right)\ (Eq.1)$$

Also, it is given that the interface concentration are in equilibrium as per the following relation:

$$P_{A,G,o} = H * C_{A,L,o}$$

We consider SO_2 as an ideal gas, which means that:

$C_{A,G,o} = \frac{P_{A,G,o}}{RT}$ and $C_{A,G,bulk} = \frac{P_{A,G,bulk}}{RT}$

Also, we can assume that $C_{A,L,bulk} = 0$

In order to solve for $P_{A,G,o}$:

$$k_G^{loc}\frac{1}{RT}\left(P_{A,G,bulk} - P_{A,G,o}\right) = k_L^{loc}\frac{P_{A,G,0}}{H}$$

Thus, we get

$$P_{A,G,o} = \frac{\frac{k_G^{loc} P_{A,G,bulk}}{RT}}{\left(\frac{k_L^{loc}}{H} + \frac{k_G^{loc}}{RT}\right)}$$

If we put the values from the question,

$$P_{A,G,o} = \frac{\frac{4 \times 10^{-5} \times 0.1}{0.082 \times 313}}{\left(\frac{2.5 \times 10^{-6}}{1.05} + \frac{4 \times 10^{-5}}{0.082 \times 313}\right)}$$

$$P_{A,G,o} = 4 \times 10^{-2} atm$$

Thus:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} = 1.6 \times 10^{-3} M$$

$$C_{A,L,o} = \frac{P_{A,G,o}}{H} = 3.8 \times 10^{-2} M$$

b) In order to solve for the mass flux, we need first the molar flux. We can use the $(Eq. 1)$

$$n_A = k_L^{loc} \times C_{A,L,o} =$$

$$= 2.5 \times 10^{-6} m/s \ \times 0.038 \times 10^3 mol/m^3$$

$$n_A = 0.095 \times 10^{-3} mol/m^2 s$$

thus

$$\dot{m}_A = M_A \times n_A = \frac{64g}{mol} \times 0.095 \times \frac{10^{-3} mol}{m^2 s} = 6 \times 10^{-3} g/m^2 s$$

c) To determine if mass transfer is limited on one side, we have to calculate m_G, m_L and m_{avg} and also the C^{eq}values

$$m_G = \frac{c_{A,G,0} - c_{A,G}^{eq}}{c_{A,L,0} - c_{A,L,bulk}} = \frac{1.6 \times 10^{-3} - 0}{3.8 \times 10^{-2} - 0} = 0.042$$

$$m_L = \frac{c_{A,G,bulk} - c_{A,G,0}}{c_{A,L}^{eq} - c_{A,L,0}} = \frac{4 \times 10^{-3} - 1.6 \times 10^{-3}}{0.095 - 3.8 \times 10^{-2}} = 0.042$$

where $c_{A,L}^{eq} = \frac{p_{A,G,bulk}}{H}$

$$m_{avg} = \frac{1}{2}(m_L + m_G) = 0.042$$

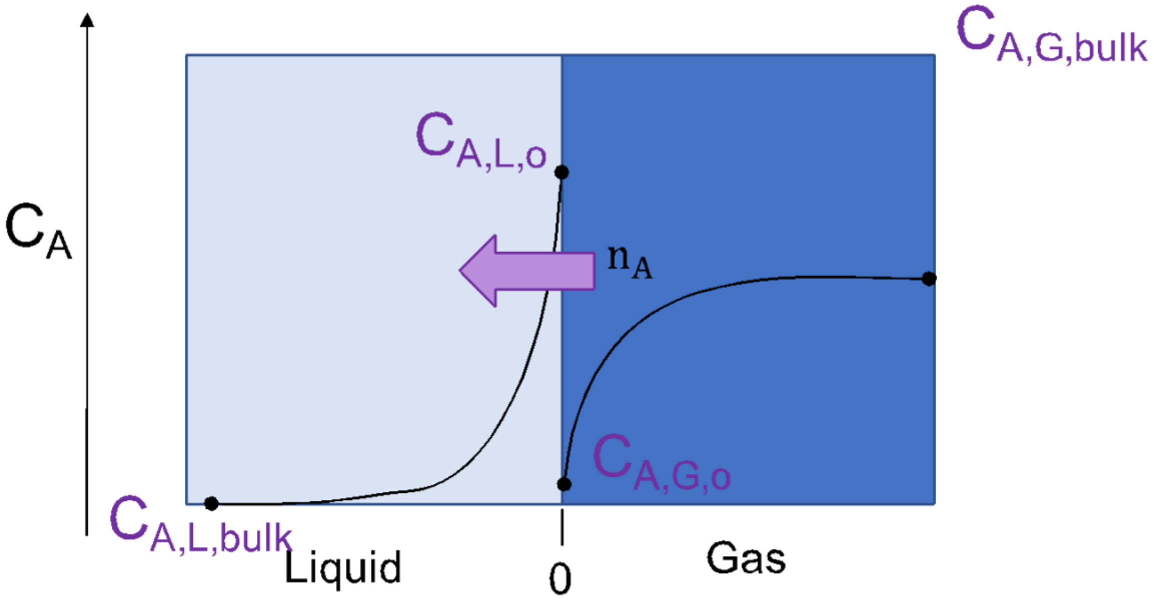
$$\frac{k_L^{loc}}{m_{avg}k_G^{loc}} = \frac{2.5 \times 10^{-6}\,\frac{m}{s}}{0.042 \times 4 \times 10^{-5}\,\frac{m}{s}} = 1.5$$

1 pt

Since $\frac{k_W^{loc}}{m_{avg}k_B^{loc}} > 1$, the mass transfer is mildly gas phase controlled.

2 pt

d)



where

$$C_{A,L,bulk} = 4 \times 10^{-3} M$$

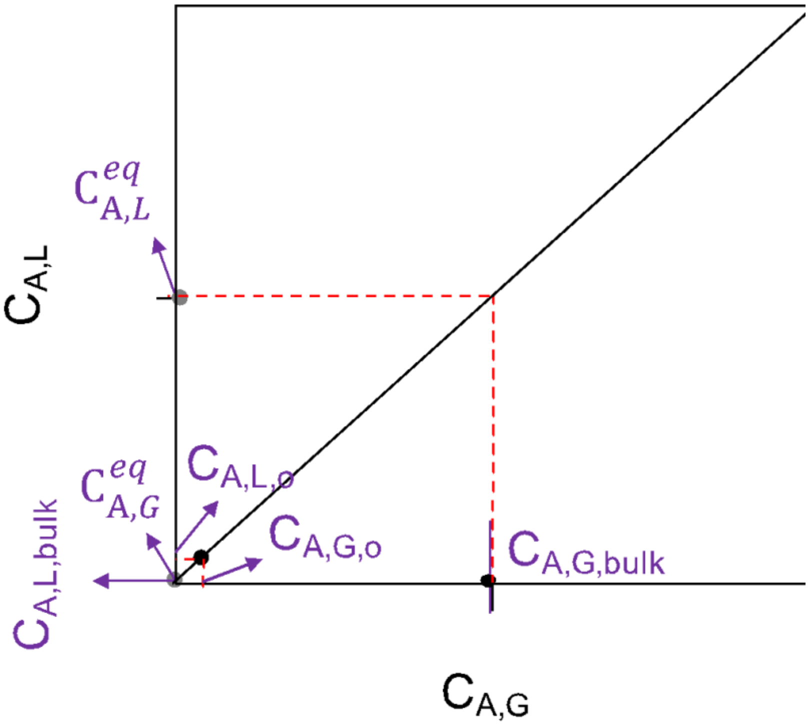
$$C_{A,G,o} = 1.6 \times 10^{-3} M$$

$$C_{A,L,bulk} = 0$$

$$C_{A,L,o} = 3.8 \times 10^{-2} M$$

e) The graph below is quantitative, a qualitative version is also accepted considering the challenge in positioning $C_{A,L,bulk}$ and $C_{A,G,eq}$ at the origin.

2 pt



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Question 4:

We are looking at the evaporation of A through flowing gas (B). So you can directly use from the course slides:

$$n_{A,z} = \frac{cD_{AB}}{L} ln\left(\frac{1 - x_{A,L}}{1 - x_{A,0}}\right)$$

where c is the mixture concentration determined from the ideal gas law as:

$$c = \frac{P}{RT} = \frac{\frac{770}{760} \times 101.3 \times 10^3\ (Pa)}{8.314\ (J\ mol^{-1}K^{-1}) \times 298(K)} = 41.42\ mol/m^3$$

The mole fractions at x=0 and x=L are

1 pt

$$x_{A,L} = 0$$

$$x_{A,0} = \frac{p_A}{P} = \frac{23.81}{770} = 0.03$$

where P_A is the saturation pressure of A at T=298K.

$$n_{A,z} = \frac{41.42 \frac{mol}{m^3} \times 0.088 \times 10^{-4} m^2/s}{0.1114\ m} ln\ (\frac{(1-0)}{(1-0.03)})$$

$$n_{A,z} = 0.99 \times \frac{10^{-4} mol}{m^2 s}$$

The evaporation rate $N_A = n_{A,z} \times A$

Substituting the values, we get $N_A = 0.99 \times 10^{-4} \times 2.29 \times 10^{-4} = 2.27 \times 10^{-8} mol/s$

In order to convert N_A in terms of g/hr :

$$\dot{m} = N_A \times M_A$$

$$\dot{m} = 0.99 \times 10^{-4} \left(\frac{mol}{m^2 s}\right) \times 164.5 \left(\frac{g}{mol}\right) \times 3600 \left(\frac{s}{hr}\right) = 0.013 \frac{g}{hr}$$

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Question 5:

a) Rate of sublimation is given by

$$N_A = \frac{4\pi c D_{AB} \; ln \, ln \left(\frac{(1-x_{A,2})}{(1-x_{A,1})}\right)}{\left(\frac{1}{r_1} - 0\right)}$$

$$c = \frac{P}{RT} = \frac{\frac{747}{760} \times 101.3 \times 10^3 \; (Pa)}{8.314 \; (J \, mol^{-1} K^{-1}) \times 313(K)} = 38.26 \; mol/m^3$$

$$x_{A,2} = 0$$

$$x_{A,1} = \frac{p_A}{P} = \frac{1.03}{747} = 0.0013$$

$$N_A = \frac{4\pi \times 38.26 \; (\frac{mol}{m^3}) \times 0.088 \times 10^{-4} (\frac{m^2}{s}) \; ln \, ln \left(\frac{(1-0)}{(1-0.0013)}\right)}{\left(\frac{1}{0.005} - 0\right)}$$

$$N_A = 2.7 \; \times 10^{-8} \; (\frac{mol}{s})$$

b) The differential area, $S = 4\pi r^2$

$$\frac{d(n_{A,r} S)}{dr} = 0$$

$$\frac{d(n_{A,r} 4\pi r^2)}{dr} = 0$$

$$\frac{d(n_{A,r} r^2)}{dr} = 0$$

$$n_{A,r} \, r^2 = C_1$$

Since B is not diffusing into A, we can assume $n_{B,r} = 0$

$$n_{A,r} = x_A n_{A,r} + j_{A,r}$$

$$n_{A,r} = -\frac{c D_{AB}}{(1-x_A)} \frac{dx_A}{dr}$$

So, we get

$$n_{A,r} = \frac{C_1}{r^2}$$

$$-\frac{cD_{AB}}{(1-x_A)}\frac{dx_A}{dr} = \frac{C_1}{r^2}$$

$$\int_{\square}^{\square} \square \frac{-cD_{AB}}{(1-x_A)} dx_A = \int_{\square}^{\square} \square \frac{C_1}{r^2} dr$$

Solving the integral gives :

$$cD_{AB}\ ln\ ln\ (1-x_A) = \frac{-C_1}{r} + C_2$$

We know the following boundary conditions :

$$r = r_1, \quad x_A = x_{A,1}$$

$$r = r_2, \quad x_A = x_{A,2}$$

$$cD_{AB}\ ln\ ln\left(1-x_{A,1}\right) = \frac{-C_1}{r_1} + C_2$$

$$cD_{AB}\ ln\ ln\left(1-x_{A,2}\right) = \frac{-C_1}{r_2} + C_2$$

Solving the above equation gives :

$$C_1 = \frac{-cD_{AB}\ ln\ ln\left(\frac{\left(1-x_{A,1}\right)}{\left(1-x_{A,2}\right)}\right)}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

$$C_2 = cD_{AB}\ ln\ ln\left(1-x_{A,1}\right) - \frac{cD_{AB}\ \ ln\ ln\left(\frac{\left(1-x_{A,1}\right)}{\left(1-x_{A,2}\right)}\right)}{r_1\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

The net diffusion rate (N_A) is given by

$$N_A \ = n_{A,r} \times 4\pi r^2$$

$$n_{A,r} = \frac{C_1}{r^2} = \frac{-cD_{AB}\ ln\ ln\ \left(\frac{(1 - x_{A,1})}{(1 - x_{A,2})}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r^2}$$

$$N_A \ = \frac{-cD_{AB}\ ln\ ln\ \left(\frac{(1 - x_{A,1})}{(1 - x_{A,2})}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right) r^2} \times 4\pi r^2$$

$$N_A = \frac{4\pi cD_{AB}\ ln\ ln\ \left(\frac{(1 - x_{A,2})}{(1 - x_{A,1})}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

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Question 6:

a) Force balance on the disk gives us :

$$F_{applied} = F_{gravity} - F_{buoyancy} + F_{shear}$$

$$F_{applied} = 25\ N$$

$$F_{gravity} = mg = 98\ N$$

$$F_{buoyancy} = \pi \left(\frac{d}{2}\right)^2 t \times \rho_{dye} \times g = \pi(0.5)^2(0.01) \times 1.2 \times 10^3 \times 9.8 = 92\ N$$

$$F_{shear} = 19N$$

Considering that shear force is acting on both faces of the disk,

$$F_{shear} = 2\mu A \left(\frac{dv}{dx}\right) = 2\mu A v (\frac{1}{x_1})$$

$$F_{shear} = 2 \times 0.5\pi(0.5)^2\ v\ \ (\frac{1}{0.3})$$

$$19\ N = \frac{\pi \times 6.67 \times v}{8}$$

$$v = 7.25\ m/s$$

b) Writing the expression for shear force in the new position of the window according to figure 4,

$$F_{shear} = \mu A \left(\frac{dv_{left}}{dx_1} + \frac{dv_{right}}{dx_2}\right) = \mu A (\frac{v_{left}}{x_1} + \frac{v_{right}}{x_2})$$

Clearly $v_{right} = v_{left} = v$

$$F_{shear} = 0.5 \times \pi(0.5)^2 \times v \times (\frac{1}{0.5} + \frac{1}{0.1})$$

$$19\ N = \frac{\pi \times 12 \times v}{8}$$

$$v = 4\ m/s$$

c) We now know the speed of the window. Hence we can evaluate the gradients on both sides. Given the linear velocity profile and the fact that the velocity of the dye solution will decrease as we go from window to the static wall of the container :

$$V_y = a - bx$$

Considering that for x=0 (at the window) $V_y = \frac{4m}{s}$

$$V_{5cm}^{left} = V - \frac{dv}{dx_1} 0.05 = 4 - \frac{4}{0.5} 0.05 = 3.6\ m/s$$

$$V_{5cm}^{right} = V - \frac{dv}{dx_2} 0.05 = 4 - \frac{4}{0.1} 0.05 = 2\ m/s$$