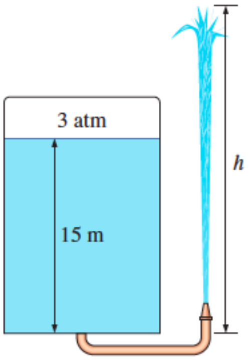
ChE 204 – Introduction to Transport Phenomena – Final Exam

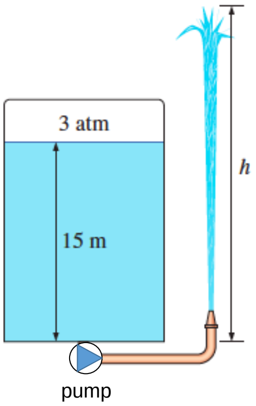
Question 1:



The water level in the tank is 15m above the ground. A hose is connected to the bottom of the tank, and the nozzle at the end of the hose is pointed straight up. The tank is closed and the pressure above the water is 3 atm.

a) Determine the maximum height to which the water stream could rise

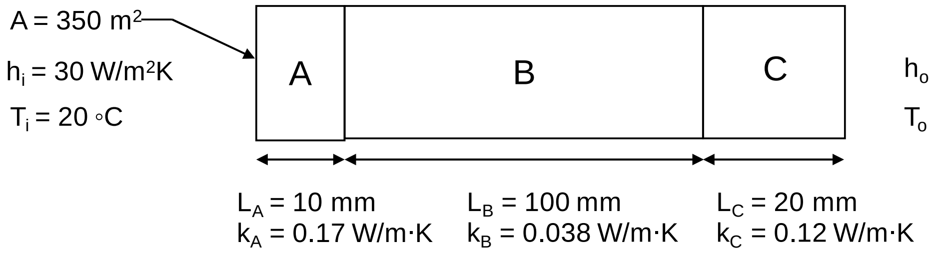
b) BONUS QUESTION: Why it the height calculated in a) “the maximum height”?



c) Calculate the power of the pump needed to have a height of 100 m in the presence of frictional losses (minor losses due to closed elements are negletted). The internal surface roughness of the pipe is $\epsilon = 0.1 * 10^{-3} m$, the pipe diameter is 20 cm, the pipe length is 5 m, the average velocity in the pipe is 50 m/s. (ρ = 1000 kg/m^3; g = 9.81 m/s^2; μ= 8.9 x 10^{-4} Pa·s)

Question 2:

Consider the composite wall below:



a) Sketch the thermal resistance circuit

b) Find the expression for the total resistance

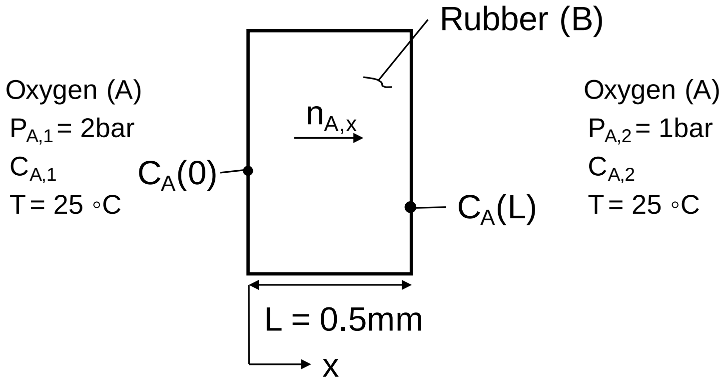
c) Calculate the total heat loss

d) What is the controlling resistance for heat loss?

e) BONUS QUESTION: What is the heat flux at AB and BC interface? Can you explain why this is the case?

Question 3:

Consider the sketch and the data on the left:



$D_{AB} = 0.21 * 10^{-9}\frac{m^2}{s}$; $S_{AB} = 3.12 * 10^{-3}\frac{kmol}{m^3 bar}$; $M_A = 32\ g/mol$; R = 0.082 $\frac{L\cdot atm}{K\cdot mol}$

Find:

a) The molar diffusion flux of oxygen

b) The mass diffusion flux of oxygen

c) The molar concentration of oxygen outside the rubber assuming perfect gas behavior

d) BONUS QUESTION: Can you briefly explain why the molar concentrations outside the membrane differ from those within the membrane?

Question 4:

Jasmone ($C_{11}H_{16}O$) is a valuable material in the perfume industry, used in many soap and cosmetics. Suppose we are recovering this material from a **water** suspension (concentration of jasmone in water is 5mM) by an extraction with **benzene**. The local mass transfer coefficient in water is $3.0 \cdot 10^{-4}\,\frac{cm}{s}$ and the local mass transfer coefficient in benzene is $2.4 \cdot 10^{-3}\,\frac{cm}{s}$. The interfacial concentrations are in equilibrium according to the following equation $C_{j,benzene} = 170\ C_{j,water}$.

a) Calculate the interfacial concentration of jasmone in water and in benzene

b) Calculate the molar flux

c) Determine if one side is limiting the mass transport

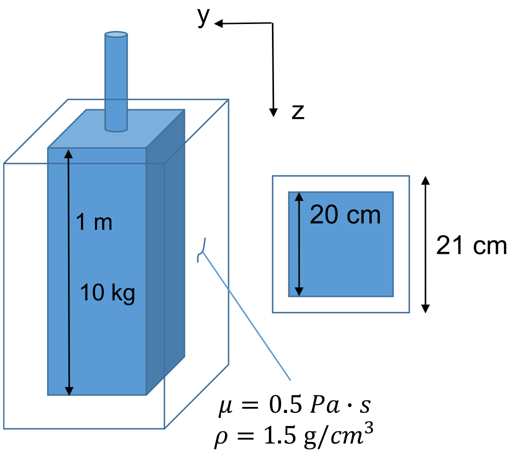
d) Draw the qualitative liquid/liquid **concentration diagram**, which means identify the concentration values, in bulk and at the interface, along with the direction of the molar flux (in the exam sheets, not here).

e) Qualitatively identify bulk, equilibrium and interface concentrations in the **equilibrium diagram** (in the exam sheets, not here).

f) BONUS QUESTION: Can you briefly explain in which cases we introduce the overall mass transfer coefficient? Do the equilibrium concentration have any physical meaning?

Question 5:

Look at the sketch and put attention to the coordinate system:



a) Calculate the maximum rate when the piston slides in the outside container.

b) Can you draw the direction of the shear force? (in the exam sheet)

c) Can you draw the direction of the momentum flux? (in the exam sheet)

d) BONUS QUESTION: Assume there is an additional force pressing the piston downwards. Do you expect the module of the shear force to increase or to decrease? Explain why.

Introduction to Transport Phenomena: Solutions to Final exam

Question 1

a) Applying Bernoulli between point 1 and point 2

$$P_1 + \rho . g . h_1 + \frac{1}{2} . \rho . v_1^2 = P_2 + \rho . g . h_2 + \frac{1}{2} . \rho . v_2^2$$

$$3 \times 10^5 + 1000 \times 9.81 \times 15 = 1 \times 10^5 + 1000 \times 9.81 \times h$$

$$h = 35.38\ m$$

b) Because Bernoulli's equation doesn't take into account the friction losses.

c) Now we must extend the Bernoulli's Equation to include the pump and losses.

$$P_1 + \rho g\ h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g\ h_2 + \frac{1}{2}\rho v_2^2 + f_f \frac{4L}{D}\frac{\rho v_{avg}^2}{2}$$

We have to calculate the relative pipe roughness

$$\frac{\varepsilon}{d_p} = \frac{0.0001}{0.2} = 0.0005$$

$$Re = \frac{\rho * v_{avg} * d_p}{\mu} = \frac{1000\frac{kg}{m^3} * 50\frac{m}{s} * 0.2m}{8.9 * 10^{-4}\ Pa * s} = 1.12 * 10^7$$

Checking the corresponding value on the moody chart for the fanning friction factor,

$$f_f = 0.004$$

Replacing the value of f_f in Bernoulli's equation:

$$3\times10^5 + 1000\times9.81\times15 + P_{pump}$$

$$= 1\times10^5 + 1000\times9.81\times100 + \frac{1}{2}\times1000\times50^2\left(4\times\frac{0.004}{0.2}(5)\right)$$

$$P_{pump} = 1133850\ Pa = 1134\ kPa$$

$Power = Q\rho\ gH_p$

To calculate the power, we need to find the pump head and volumetric flow rate:

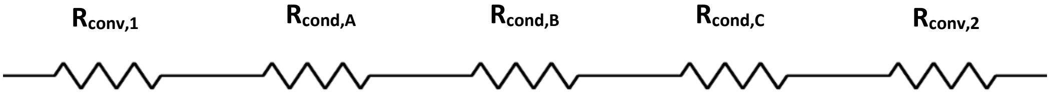
$H_P = \frac{P_{pump}}{\rho g} = \frac{1133850}{1000\times9.81} = 115.58\ m$

$Q\ = A * v_{avg} = \frac{(0.2)^2}{4} * \Pi * 50\,\frac{\mathrm{m}}{\mathrm{s}} = 1.57\,\frac{\mathrm{m}^3}{s}$

$Power = 1.57\times1000\times9.81\times115.58 = \mathbf{1.8\ MW}$

Question 2

a) The thermal resistance circuit is as follows



b) All the elements are in series. Hence the net resistance is evaluated as follows:

$$R_{tot} = \frac{1}{A} \cdot [\left(\frac{1}{h_i}\right) + \left(\frac{L_a}{k_A}\right) + \left(\frac{L_b}{k_b}\right) + \left(\frac{L_c}{k_c}\right) + \left(\frac{1}{h_o}\right)]$$

$$= \frac{1}{350} \cdot \left[\left(\frac{1}{30}\right) + \left(\frac{10 \cdot 10^{-3}}{0.17}\right) + \left(\frac{100 \cdot 10^{-3}}{0.038}\right) + \left(\frac{20 \cdot 10^{-3}}{0.12}\right) + \left(\frac{1}{60}\right)\right] =$$

$$= \frac{1}{350} \cdot [0.03 + 0.06 + 2.63 + 0.16 + 0.016\ ] =$$

$$R_{tot} = 0.0083 \frac{W}{K}$$

c) $\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{20-(-15)}{0.0083} = \frac{35}{0.0083} = 4.21\ kW$

d) The controlling resistance is medium $R_{cond,B}$

e) BONUS QUESTION: Heat flux through the two interfaces mentioned is the same as they are in series and we consider that there are no heat losses.

Question 3

a) $C_{A,0} = S_{AB} \cdot P_{A,1} = 3.12 \cdot 2 = 6.24\ mol/m^3$

$$C_{A,L} = S_{AB} \cdot P_{A,2} = 3.12 \cdot 1 = 3.12\ mol/m^3$$

$$n_A = -\frac{D_{AB}\left(C_{A,L} - C_{A,0}\right)}{L} = \frac{0.21 \cdot 10^{-9} \cdot 3.12}{5 \cdot 10^{-4}} = 1.31 \cdot 10^{-6}\ mol/(m^2 \cdot s)$$

b) $\dot{m}_A = n_A \cdot M_A = 1.31 \cdot 10^{-6} \cdot 32 = 41.93 \cdot 10^{-6}\ \frac{g}{m^2 \cdot s}$

c) Concentrations outside the rubber on either side can be found using the ideal gas law

$$C_{A,1} = \frac{P_{A,1}}{R \cdot T}$$

$$C_1 = \frac{2}{0.082 \cdot 298} = 0.08 \frac{mol}{L}$$

Similarly,

$$C_{A,2} = \frac{P_{A,2}}{R \cdot T}$$

$$C_2 = \frac{1}{0.082 \cdot 298} = 0.04 \frac{mol}{L}$$

d) BONUS QUESTION: The concentration of the oxygen outside the membrane is determined directly by the partial pressure of oxygen, whereas in the membrane one needs to consider the solubility of the oxygen in the membrane material.

Question 4

a) If we equate the local fluxes of jasmine

$$n_j = k_W^{loc}\left(C_{j,W,b} - C_{j,W,o}\right) = k_B^{loc}\left(C_{j,B,o} - C_{j,B,b}\right)(\boldsymbol{Eq.1})$$

Also, it is given that the interface concentration are in equilibrium as per the following relation:

$$C_{j,B,o} = 170 * C_{j,W,o}\ (\boldsymbol{Eq.2})$$

In order to solve for $C_{j,W,o}$:

$$k_W^{loc}\left(C_{j,W,b} - C_{j,W,o}\right) = k_B^{loc}(170 * C_{j,W,o} - C_{j,B,b})$$

Thus, we get

$$C_{j,W,o} = \frac{\left(k_W^{loc} C_{j,w,b} + k_B^{loc} C_{j,B,b}\right)}{\left(170 k_B^{loc} + k_W^{loc}\right)}(\boldsymbol{Eq.3})$$

If we put the values from the question,

$$C_{j,W,o} = \frac{(3 \times 10^{-6} \times 0.005 + 2.4 \times 10^{-5} \times 0)}{(170 \times 2.4 \times 10^{-5} + 3 \times 10^{-6})}$$

$$C_{j,W,o} = 3.6 \times 10^{-6} M$$

Since $C_{j,W,o}$is in equilibrium with $C_{j,B,o}$ as per $\boldsymbol{Eq.2},$

$$C_{j,B,o} = 170 * C_{j,W,o} = 0.612 \times 10^{-3} M$$

b) In order to solve for the molar flux, we can use the $(\boldsymbol{Eq.1})$

$$n_j = k_W^{loc}\left(C_{j,W,b} - C_{j,W,o}\right)$$

$$n_j = 3 \times 10^{-6} m/s \times (0.005 - 0.0000036) \times 10^3 mol/m^3$$

$$n_j = 0.015 \times 10^{-3} mol/m^2 s$$

c) To determine if mass transfer is limited on one side, we have to calculate m_{B}, m_{W} and m_{avg}

$$m_{\mathrm{B}} = \frac{\mathrm{c}_{\mathrm{j,B,0}} - \mathrm{c}_{\mathrm{j,B,b}}^{eq}}{\mathrm{c}_{\mathrm{j,W,0}} - \mathrm{c}_{\mathrm{j,W,b}}} = \frac{0 - 170 \times 3.6 \times 10^{-6}}{3.6 \times 10^{-6} - 0.005} = 0.1226$$

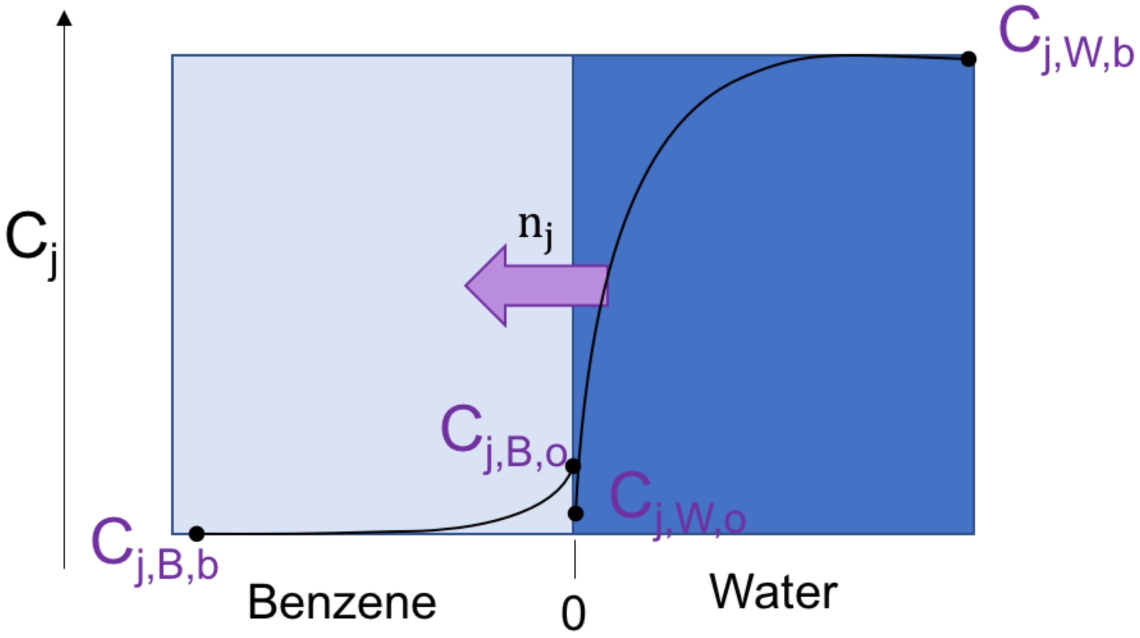
$$m_{\mathrm{W}} = \frac{\mathrm{c}_{\mathrm{j,B,b}} - \mathrm{c}_{\mathrm{j,B,0}}}{\mathrm{c}_{\mathrm{j,W}}^{eq} - \mathrm{c}_{\mathrm{j,W,0}}} = \frac{0 - 0.612 \times 10^{-3}}{0 - 3.6 \times 10^{-6}} = 170$$

$$m_{avg} = \tfrac{1}{2}(m_{\mathrm{B}} + m_{\mathrm{W}}) = \tfrac{1}{2}(0.1226 + 170) = 85.06$$

$$\frac{\boldsymbol{k_W^{loc}}}{\boldsymbol{m_{avg}k_B^{loc}}} = \frac{\boldsymbol{3 \times 10^{-6}\,\frac{m}{s}}}{\boldsymbol{85.06 \times 2.4 \times 10^{-5}\,\frac{m}{s}}} = \boldsymbol{0.0014}$$

Since $\frac{\boldsymbol{k_W^{loc}}}{\boldsymbol{m_{avg}k_B^{loc}}} \ll \boldsymbol{1},$ the mass transfer is liquid phase controlled.

d)



where

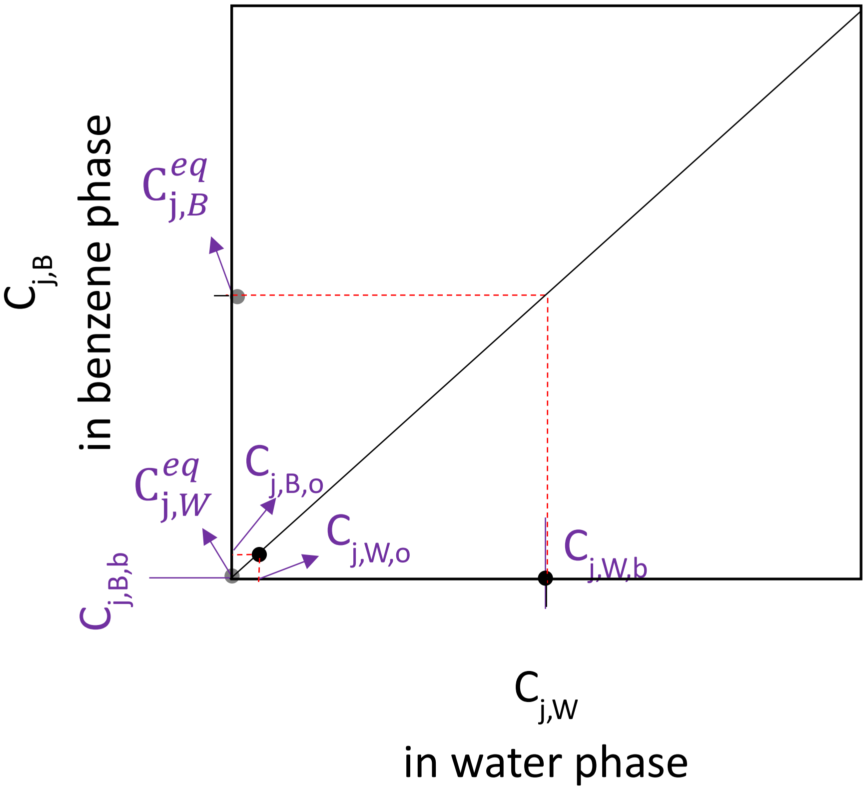
$$C_{j,W,b} = 0.005M$$

$$C_{j,W,o} = 0.0000036M$$

$$C_{j,B,b} = 0$$

$$C_{j,B,o} = 0.0006M$$

e) The graph below is quantitative, a qualitative version is also accepted considering the challenge in positioning $C_{j,B,b}$ and $C_{j,w,eq}$ at the origin.



$$C_{j,W}^{eq} = \frac{C_{j,B,b}}{170} = 0$$

$$C_{j,B}^{eq} = 170 \times C_{j,W,b} = 0.85M$$

f) BONUS QUESTION: The interface concentrations are not normally known and only the bulk concentrations and local mass transfer co-efficients are known in real life problems. Hence, it is easier to have overall mass transfer coefficients relating the bulk concentrations. In order to convert the local mass transfer to overall mass transfer coeffiecients, we use theoretical values as equilibrium concentration. These equilibrium concentrations do not physically appear anywhere in the system and is just a simplification to solve the problems.

Indeed, for the sake of knowledge, if we rigorously solve the equations of local mass transfer coefficient to relate the flux to known bulk concentrations, interface equilibrium relation (solubility curve) and local mass transfer co-efficients, we can derive the theoretical equilibrium concentrations as shown below:

$$n_j = k_B^{loc}\left(C_{j,B,o} - C_{J,B,b}\right) = K_B\left(C_{j,B}^{eq} - C_{j,B,b}\right)\boldsymbol{Eq.5}$$

If we express $C_{j,B,o}$in terms of known quantities like k_B^{loc}, $k_W^{loc}$and $C_{J,W,b}$, from $\boldsymbol{Eq.2}$ and $\boldsymbol{Eq.3}$, we get

$$n_j = k_B^{loc}\left(\frac{\left(170k_W^{loc}C_{j,W,b} + 170k_B^{loc}C_{j,B,b}\right)}{170k_B^{loc} + k_W^{loc}} - C_{J,B,b}\right)$$

$$n_j = k_B^{loc}\left(\frac{\left(170k_W^{loc}C_{j,W,b} + 170k_B^{loc}C_{j,B,b}\right) - \left(170k_B^{loc}C_{j,B,b} + k_W^{loc}C_{j,B,b}\right)}{170k_B^{loc} + k_W^{loc}}\right)$$

$$n_j = k_B^{loc}\left(\frac{170k_W^{loc}C_{j,W,b} - k_W^{loc}C_{j,B,b}}{170k_B^{loc} + k_w^{loc}}\right)$$

$$n_j = \left(\frac{k_B^{loc}k_W^{loc}}{170k_B^{loc} + k_w^{loc}}\right)(170C_{j,W,b} - C_{j,B,b})$$

$$n_j = \left(\frac{1}{\frac{170}{k_W^{loc}} + \frac{1}{k_B^{loc}}}\right)(170C_{j,W,b} - C_{j,B,b})$$

This equation when equated to the overall mass transfer coefficient from $\boldsymbol{Eq.5,}$

$$\left(\frac{1}{\frac{170}{k_W^{loc}} + \frac{1}{k_B^{loc}}}\right)\left(170C_{j,W,b} - C_{j,B,b}\right) = K_B\left(C_{j,B}^{eq} - C_{j,B,b}\right)$$

Thus, we get

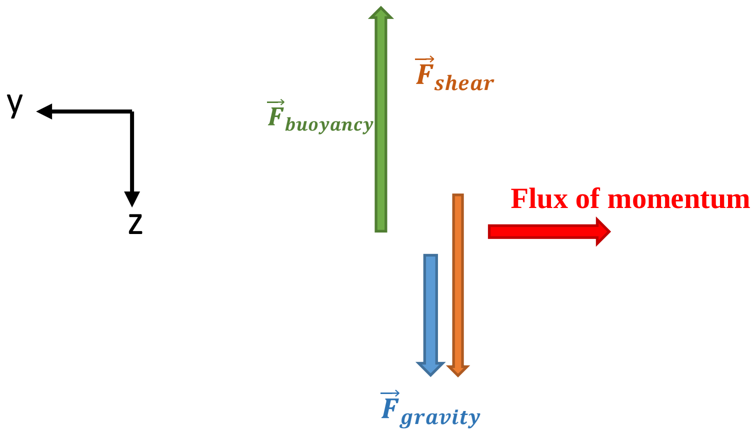
$$\frac{1}{K_B} = \frac{170}{k_W^{loc}} + \frac{1}{k_B^{loc}}$$

and

$$C_{j,B}^{eq} = 170C_{j,W,b}$$

where $C_{j,B}^{eq}$ which is the theoretical concentration of jasmone in benzene in equilibrium with the concentration of jasmone in water bulk. So, it is correct to use this equilibrium concentration for overall mass transfer coefficient equation despite the fact that the bulk concentrations are not in true equilibrium with each other.

Question 5



a) The force balance on the piston gives:

$$\vec{F}_{tot} = \vec{F}_{gravity} + \vec{F}_{shear} + \vec{F}_{buoyancy}$$

When the piston is at maximum speed (i.e. steady state), the forces are compensating each other. Therefore:

$$\vec{F}_{gravity} + \vec{F}_{shear} + \vec{F}_{buoyancy} = 0$$

The buoyancy force is given by:

$$\vec{F}_{buoyancy} = -V_{piston} \cdot \rho_{liquid} \cdot g$$

Volume of the piston = $V_{piston} = 0.2 \times 0.2 \times 1 = 4 \times 10^{-2}\ m^3$

$$\vec{F}_{buoyancy} = -4 \times 10^{-2} \times 1500 \times 9.81 = -588\ N$$

The gravitational force calculation:

$$\vec{F}_{gravity} = +mg = 10 \times 9.81 = 98.1\ N$$

Net upward force to be balanced by the shear force due to friction:

$$\vec{F}_{shear} = -\left(\vec{F}_{buoyancy} + \vec{F}_{gravity}\right) = -\left(98.1 + (-588\,)\right) = +489.9\ N$$

Therefore, the piston has to move upwards for the shear force to act upwards.

The shear stress on the piston is given by:

$$\tau_{yz} = -\mu \frac{dv_z}{dy} = -\mu \frac{v_{max}}{0.5 \times 10^{-2}}$$

$$F_{shear} = \tau_{yz} \ \times \ Area$$

$$Area = \ 4 \cdot A = 4 \times 1 \times 0.2 = 0.8 \, m^2$$

So, we have:

$$\boldsymbol{v_{max}} = \frac{\vec{F}_{shear} \times 0.5 \times 10^{-2}}{Area \ \ \mu} = -(\frac{489.9 \times 0.5 \times 10^{-2}}{0.8 \times 0.5}) = 6.1 \frac{m}{s}$$

b) and c) The directions were drawn in part (a).

d) BONUS QUESTION: In this case, the gravity and buoyancy forces remain constant. Depending on the magnitude, the applied force is acting to either reduce the magnitude of the velocity in the original direction, or increase it beyond its original value in the opposite direction. Thus, depending on how the final velocity compares with the calculated value in the question, the magnitude of shear force will either increase or decrease.

$$\vec{F}_{tot} = \vec{F}_{gravity} + \vec{F}_{shear} + \vec{F}_{buoyancy} + \vec{F}_{applied}$$