Introduction to Transport Phenomena: <u>Final Exam</u>

<u>Question 1</u>

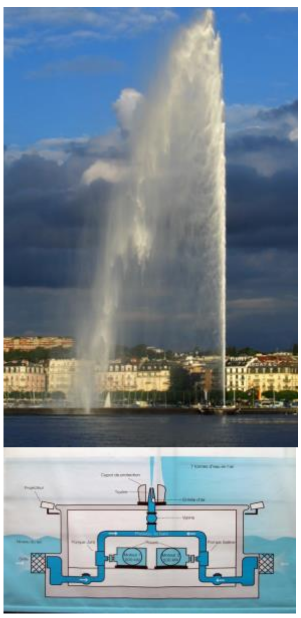
The jet d'eau is one of the famous landmarks of Geneva. The water leaves the nozzle with diameter d_{nz} with the velocity of v_{nz} and a volumetric flow of $Q = 500\,\frac{L}{s}$. The ambient pressure is $p_{atm} = 1bar$ at the day of the calculation. The density of the lake-water is assumed to be $\rho_w = 1000\,\frac{kg}{m^3}$. Acceleration due to gravity is given by $g = 9.81\,\frac{m}{s^2}$. The viscosity is given by $\mu = 8.9 * 10^{-4}\ Pa * s$.

a) **Calculate the required nozzle diameter d_{nz} to reach a fountain height of $h_2 = 140m$ neglecting all frictional losses**. The nozzle is at height $h_1 = 0m$.
In reality the diameter of the nozzle is approximately 50% smaller than the value you should get. Explain why (2-3 bullet points).

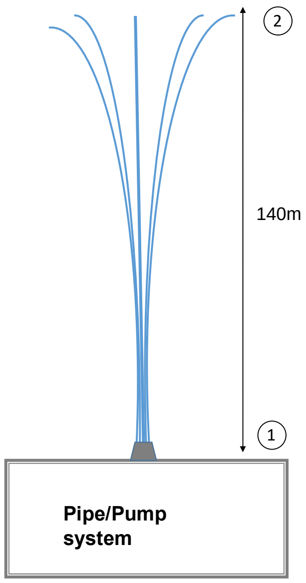
b) ***(can be solved independently of a)**.*
The water is accelerated by two pumps, each with power of 500KW, to reach the fountain height of 140m. A fraction of the pump power is required to compensate for the friction losses in the pipe system. The loss coefficients of the pipe system are:

$$\sum_i L_i = 5m$$
$$\sum_j K_j = 2$$

The internal surface roughness of the pipe is $\varepsilon = 0.1 * 10^{-3}m$. The average velocity in the pipe system until they reach the valve and nozzle is $v_{avg} = 25\,\frac{m}{s}$, the pipe diameter is $d_p = 10cm$. **Calculate the Reynolds number and use the Moody diagram (w. relative pipe roughness) to estimate the friction factor**. Then **estimate which fraction of the pumping power (out of 100%) is required to overcome the frictional losses** of the tubing in the housing (use given loss coefficients for calculations).



for illustration purpose only



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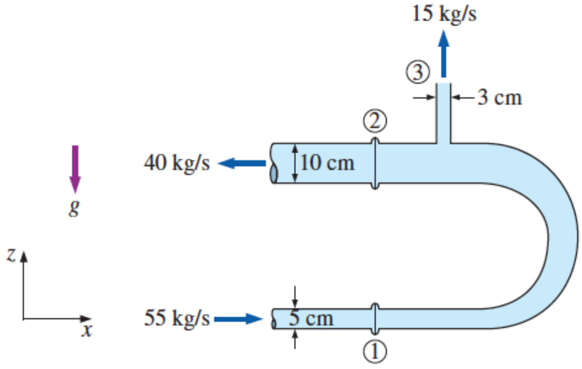
Question 2

Water is flowing into and discharging from a pipe U-section as shown in the figure.



At flange 1, the total absolute pressure is 200 kPa, and 55 kg/s flows into the pipe. At flange 2, the total absolute pressure is 150 kPa. At location (3), 15 kg/s of water discharges to the atmosphere, which is at 100 kPa. **Determine the total x-force and z-force at the two flanges connection the pipe**. **Discuss the significance of gravity force in this problem**. Consider frictionless flow.

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Question 3

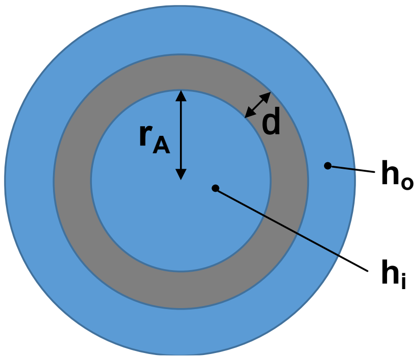
Consider a water-to-water double-pipe counter-flow heat exchanger. The inner diameter of the inner tube (**r_A**) and the thickness of the inner tube **(δ)** are **1 cm** and **0.2 mm**, respectively. The **cold water**, which flows in the inner tube, enters at **20°C** and leaves at **50°C**, while the **hot water,** which flows in the outer tube, enters at **80°C** and leaves at **45°C**.

(a) Consider the **h_i=300 W/m².°C** and **h_o=500 W/m².°C**. The inner tube is made of **aluminum**. If the **velocity of the cold water** flowing **in the inner tube** is **0.5 m/s**, **calculate the mass flow of the hot fluid and the required surface area of heat exchanger**.

(For both hot and cold water $C_p = 4180 \frac{J}{kg.°C}$ $and\ \rho = 1000\ {kg}/{m^3}$).

(b) After some time, a microbiological fouling layer forms inside the inner tube of heat exchanger. **Calculate the overall heat transfer coefficient if the thickness of the fouling layer is 0.0267cm**.

Thermal conductivity of selected materials	
Material	**K (W/m.K)**
Common solids	
Aluminum	237
Concrete	1
Copper	386
Glass	0.9
Stainless Steel	16.5
Water	0.6
Fouling materials	
Calcium Carbonate	2.93
Microbological film	0.63
Calcium Sulfate	2.31
Calcium phosphate	2.6
Magnesium phosphate	2.16
Magnetic Iron oxide	2.88
Analytic	1.27



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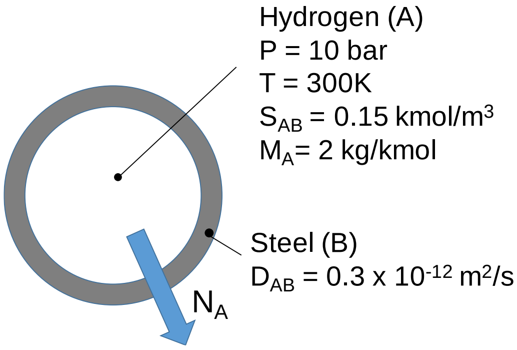
Question 4

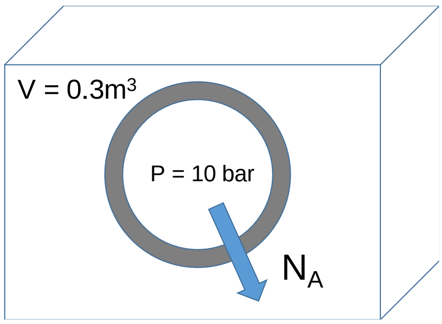
Gaseous hydrogen at 10 bars and 27ºC is stored in a 100 mm diameter spherical tank having a steel wall 2 mm thick. The solubility of hydrogen in steel is 0.15 kmol/m^3 bar and the diffusion coefficient of hydrogen in steel is approximately 0.3 x 10^{-12} m^2/s.

a) If the concentration of hydrogen at the outer surface is negligible, **calculate the initial rate of mass loss of hydrogen diffusion through the tank wall and and the initial rate of pressure drop within the tank**.



b) Now consider that the spherical tank is enclosed in a bigger container with overall volume of 0.3m^3. **Calculate how much time it will take to reach a concentration of 0.5 kmol/m^3 in this container** if you make the following assumptions:

- The only variable changing with time is the concentration of hydrogen in the bigger container.
- The concentration in the bulk of the bigger container is equal to the concentration at the steel/container interphase.



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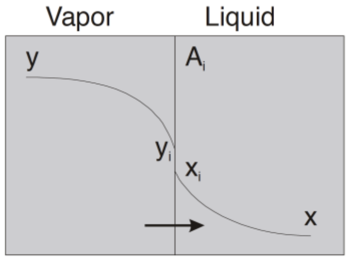
Question 5

An air-H_2S mixture is in contact with a pool of water. The H_2S is being transferred from the air to the water at a total pressure of 1.5 atm and 30 °C. The value of the gas-phase mass transfer coefficient $k_{H_2S,G,loc}$ is 9.567 x 10^{-3} m/s. At a given point the mole fraction of H_2S in the liquid at the liquid-gas interface is 2.0 x 10^{-5} and pressure of H_2S in the gas is 0.05 atm. The Henry's law equilibrium relation is P_A [atm]= 609 X_A (mole fraction in the liquid). **Calculate the transfer rate of H_2S from the gas to the liquid phase**.



Mass transfer from the gas to the liquid phase

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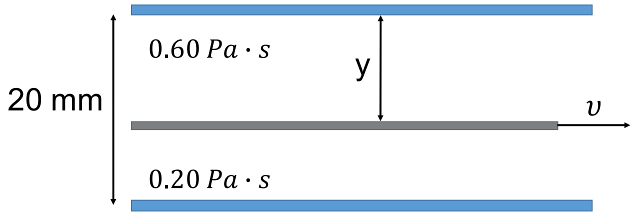
Question 6

A thin plate is placed between two flat surfaces *20* mm apart such that the viscosity of liquids on the top and bottom of the plate are 0.60 Pa · s and 0.20 Pa · s, respectively. **Determine the position of the thin plate such that the viscous resistance to uniform motion of the thin plate is minimum**.



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<u>Question 1</u>

The jet d'eau is one of the famous landmarks of Geneva. The water is accelerated by two pumps with a power of $500kW$ and leaves the nozzle with diameter d_{nz} with the velocity of v_{nz} and a volumetric flow of $Q = 500\frac{L}{s}$. The ambient pressure is $p_{atm} = 1bar$ at the day of the calculation. The density of the lake-water is assumed to be $\rho_w = 1000\frac{kg}{m^3}$. Acceleration due to gravity is given by $g = 9.81\frac{m}{s^2}$. The viscosity is given by $\mu = 8.9 * 10^{-4}\ Pa * s$.

Bernoulli equation:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_{nz}^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

Neglecting frictional losses and setting $v_2 = 0$. $P_1 = P_2 = 1atm$.

$$\cancel{P_1} + \cancel{\rho g h_1} + \frac{1}{2}\rho v_{nz}^2 = \cancel{P_2} + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \cancel{\Delta P_f}$$

The velocity at the nozzle is therefore given by:

$$v_{nz} = \sqrt{2gh_2} = \sqrt{2 * 9.81\frac{m}{s^2} * 140m} = 52.4\frac{m}{s}$$

From the given volumetric flow and the velocity, the diameter can be calculated.

$$Q = A_{nz} * v_{nz}$$

$$A_{nz} = r_{nz}^2 * \Pi$$

$$d_{nz} = 2 * r_{nz} = 2 * \sqrt{\frac{1}{\Pi} * \frac{Q}{v_{nz}}} = 2 * \sqrt{\frac{1}{3.1416} * \frac{0.5\frac{m^3}{s}}{52.4\frac{m}{s}}} = 0.11m = 11cm$$

- Same volumetric flow with a smaller diameter will increase the veloity of the water
- Therefore, the smaller nozzle/higher velocity compensates for the frictional losses in reality
- Frictional losses can be due to friction of the water jet in air, wind…

Calculate the relative pipe roughness

$$\frac{\varepsilon}{d_p} = \frac{0.0001}{0.1} = 0.001$$

$$Re = \frac{\rho * v_{avg} * d_p}{\mu} = \frac{1000\frac{kg}{m^3} * 25\frac{m}{s} * 0.1m}{8.9 * 10^{-4}\ Pa * s} = 2.8 * 10^6$$

$f_f = 0.005$ (New Moody diagram)

Calculate the frictional induced pressure loss in the pipes

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2\left(\frac{4f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

With friction factor of new diagram (at exam)

$$\Delta P_f = \frac{1}{2}1000\frac{kg}{m^3} * \left(25\frac{m}{s}\right)^2\left(\frac{4 * 0.005}{0.1m} * 5m + 2\right) = 9.375 * 10^5 Pa = 9.375bar$$

With value given above a)

$$P_{pump,f} = \Delta P_f * Q = 9.375 * 10^5 Pa * 0.5 \frac{m^3}{s} = 469kW$$

With calculate value given in this sub-question

$$Q_2 = A * v_{avg} = \frac{(0.1)^2}{4} * \Pi * 25 \frac{\mathrm{m}}{\mathrm{s}} = 0.196 \frac{\mathrm{m}^3}{s}$$

$$P_{pump,f_2} = \Delta P_f * Q_2 = 9.375 * 10^5 Pa * 0.196 \frac{m^3}{s} = 184kW$$

Both solutions are accepted!

$$F = \frac{P_{pump,f}}{P_{pump,tot}} = 46.9\%$$

$$F_2 = \frac{P_{pump,f}}{P_{pump,tot}} = 18.4\%$$

Both solutions are accepted!

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Question 2

In order to determine the total x-force and y-force acting on the control volume, which is represented by the U-section of the pipe and the water, we have to apply the equation for conservation of momentum considering the velocities and pressures. at the various inlets and outlets (gauge pressure might be use as atmospheric pressure acts on all the surfaces).

We can easily calculate the velocities first:

$$v_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{55\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.05^2}{4}\ m^2} = 28.01\ m^2$$

$$v_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{40\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.1^2}{4}\ m^2} = 5.093\ m^2$$

$$v_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{15\ kg.s^{-1}}{1000\ kg.m^{-3}\ \times\ \pi\ \times \frac{0.03^2}{4}\ m^2} = 21.22\ m^2$$

Equation for momentum conservation:

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v(v \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i.A_i.\widehat{n_i}$$

Resolving for X:

$$\sum F_{volume} = 0$$

$$\sum F_{surface-x} = F_{reaction-x} - P_1.A_1.\widehat{n_1} - P_2.A_2.\widehat{n_2}$$

$$\sum F_{surface-x} = F_{reaction-x} + P_1.A_1 + P_2.A_2$$

(this is not a scalar product, the signs come from the coordinate reference system)

$$\sum F_{surface-x} = F_{reaction-x} + [(200-100)\times 10^3 \times \pi \times \frac{0.05^2}{4}] + [(150-100)\times 10^3 \times \pi \times \frac{0.1^2}{4}]$$

$$\sum F_{surface-x} = F_{reaction-x} + 196.34N + 392.69N$$

$$\sum_{i=1}^{N} \int_{A_i} \rho v(v\cdot\hat{n})dA_i = \rho v_1(v_1\cdot\widehat{n_1})A_1 + \rho v_2(v_2\cdot\widehat{n_2})A_2$$

$$= -\rho v_1^2 A_1 - \rho v_2^2 A_2$$

$$= -1000[\left\{28.01^2 \times \pi \times \frac{0.05^2}{4}\right\} + \{5.09^2 \times \pi \times \frac{0.1^2}{4}\}]$$

$$= -1540.48\ N - 203.48N$$

Therefore,

$$F_{reaction-x} = -196.34N - 392.69N - 1540.48N - 203.48N = -2333N$$

Now resolving for Z,

$$\sum F_{volume} = 0$$

$$\sum F_{surface-z} = F_{reaction-z} - P_3.A_3.n_3$$

$$\sum F_{surface-z} = F_{reaction-z} + 0\ (Working\ with\ gauge\ pressure)$$

$$\sum_{i=1}^{N}\int_{A_i}\rho v(v.n)dA_i = +\rho v_3^2 A_3$$

$$= 1000 \times 21.22^2 \times \pi \times \frac{0.03^2}{4} = 318N$$

Therefore,

$$F_{reaction-z} = 318N$$

Comments on the weight: Negligible contribution.

Question 3

(a) Firstly we should calculate the heat transferred from the hot media to the cold media.

$$\dot{m}_{cold} = \rho_{water} * V_{cold} * A_{cold} \rightarrow 1000 * 0.5 * \pi * \frac{0.01^2}{4} = 0.03927\ kg/s$$

After that for the calculation of heat transfer we have:

$$\dot{Q}_{cold} = \dot{m}_{cold} * C_{p_{cold}} * \Delta T_{cold} = 0.03927 \frac{kg}{s} * 4180 \frac{J}{kg.°C} * (50-20)\,°C = 4924.458\,W$$

For the calculation of the flow for the hot media, we have:

$$\dot{Q}_{Hot} = \dot{Q}_{cold} = 4924.458\,W$$

$$\dot{m}_{Hot} = \frac{\dot{Q}_{Hot}}{C_{p_{Hot}} * \Delta T_{Hot}} = \frac{4924.458}{4180 * (80-45)} = 0.03366\,kg/s$$

For the calculation of the surface area, we need to calculate two things: The log mean temperature difference and the overall heat transfer coefficient.

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{(80-50)-(45-20)}{\ln \frac{(80-50)}{(45-20)}} = 27.4\,°C$$

As for the tube we have $\frac{\delta}{r} < 0.1$, we can use the linear approximation for the overall heat transfer coefficient and thus we will have (we can extract the thermal conductivity for aluminum from the table given in the question):

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{\delta_w}{k_w} + \frac{1}{h_o}} = \frac{1}{\frac{1}{300} + \frac{0.002}{237} + \frac{1}{500}} = \frac{1}{0.00333 + 1.6878 * 10^{-5} + 0.002} = \frac{1}{0.00535}$$
$$= 186.204\ {}^{W}/_{m^2.°C}$$

One should consider that the external convective heat transfer resistance, conductive heat transfer resistance and internal convective heat transfer resistance are $0.0033, 8.4388 * 10^{-6}, 0.002$, respectively.

Now, we can calculate the surface area:

$$Q = U_o * A_o * \Delta T_{LMTD} \rightarrow A_o = \frac{Q}{U_o * \Delta T_{LMTD}} = \frac{4924.458}{186.204 * 27.4} = 0.9652\,m^2$$

(b) To check if the use of linear approximation has been appropriate we can calculate $\frac{\delta_w + \delta_{foul}}{r} = \frac{0.0267+0.02}{0.5} = 0.0934 < 10.$ So, our linear approximation has been valid. Thus we will have (the thermal conductivity of the microbiological film can be extracted from the given table) :

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{\delta_w}{k_w} + \frac{\delta_{foul}}{k_{foul}} + \frac{1}{h_o}$$

$$= \frac{1}{300} + \frac{0.002}{237} + \frac{2.67 * 10^{-4}}{0.63} + \frac{1}{500} = \frac{1}{174.4598} \rightarrow U_o = 174.4598\ {}^{W}/_{m^2.°C}$$

Question 4

Assumptions:

- Since diameter >> thickness we approximate the spherical shell as a plane wall
- Concentration of hydrogen outside the chamber is negligible
- Hydrogen behaves as an ideal gas

4a.

We apply the Fick's Law and write:

$$N_A = -AD_{AB}\frac{C_{A,o} - C_{A,i}}{L}$$

In this formula $C_{A,o}$indicates the concentration at the steel/air interface, which is initially zero. $C_{A,i}$ is the concentration at internal steel/tank interface, which can be calculated using the solubility:

$$C_{A,i} = P_A \cdot S = 10\ bar\ * 0.15\ \frac{kmol}{m^3 bar} = 1.5\frac{kmol}{m^3}$$

So we write:

$$N_A = AD_{AB}\frac{C_{A,i}}{L} = 3.14\ (0.1^2)m^2\ * 0.3\ * 10^{-12}\frac{m^2}{s} *\ 1.5\frac{kmol}{m^3} * \frac{1}{0.002\ m} = 7 * 10^{-12}\frac{kmol}{s}$$

To calculate the mass loss of hydrogen by diffusion:

$$n_A = M_A \cdot N_A = 2\ \frac{kg}{kmol} *\ 7 * 10^{-12}\frac{kmol}{s} = 7 * 10^{-12}\frac{kg}{s}$$

For the pressure loss, we consider hydrogen as an ideal gas. So, because $P = RT\frac{n_A}{V}$

$$\frac{dP}{dt} = -\frac{6RT}{\pi D^3}\ N_A = -\frac{6 * \left(0.08314\ \frac{m^3 bar}{kmol\ K}\right) 300K}{3.14\ (0.1^3)m^3}\ 7 * 10^{-12}\frac{kmol}{s} = -3.34 * 10^{-7}\frac{bar}{s}$$

4.b.

We write the formulation for the bigger tank:

$$n_{H2,tank} = V_{container}C_{H2,tank} \rightarrow \frac{dN_{H2,container}}{dt} = V_{container}\frac{dC_{H2,container}}{dt} = \dot{N}_{H2}$$

$$= -AD_{H2,steel}\frac{C_{H2\,,o}(t) - C_{H2,i}(t)}{L}$$

$$\xrightarrow{As\ it\ has\ been\ mentioned\ in\ the\ question:\ C_{H2\,,o}(t)=C_{H2,container}} \frac{dC_{H2,container}}{dt} = -\frac{AD_{H2,steel}}{VL}(C_{H2,container} - C_{H2,i}(t))$$

In the question $C_{H2\,,o}(t)$ is the concentration of the hydrogen at the outer surface of steel and $C_{H2,i}(t)$ is the concentration of hydrogen at the inner surface of steel layer.

As it has mentioned in the question that the pressure inside the smaller tank is supposed to be constant, therefore we always have $C_{H2,i}(t) = 1.5\ \frac{kmol}{m^3}$.

$$\frac{dC_{H2,container}}{\left(C_{H2,container} - C_{H2,i}(t)\right)} = -\frac{AD_{H2,steel}}{VL}dt$$

$$\to \int_{C_{H2,container}=0}^{C_{H2,container}=0.5} \frac{dC_{H2,container}}{\left(C_{H2,container} - C_{H2,i}(t)\right)} = \int_{t=0}^{t=t_f} -\frac{AD_{H2,steel}}{VL}dt$$

$$\to \ln\frac{\left(C_{H2,container}(t_f) - C_{H2,i}(t)\right)}{\left(C_{H2,container}(0) - C_{H2,i}(t)\right)} = -\frac{AD_{H2,steel}}{VL}t_f$$

$$\xrightarrow{we\ have\ C_{H2,container}(0)=0} \ln\left(\frac{\left(C_{H2,container}(t_f)\right)}{\left(-C_{H2,i}(t)\right)} + 1\right) = -\frac{AD_{H2,steel}}{VL}t_f \qquad \to t_f = -\frac{VL}{AD_{H2,steel}}\ln\left(1 - \frac{0.5}{1.5}\right) = \frac{6 * 10^{-4}}{9.4245 * 10^{-15}}\ (0.4) = 2.5 * 10^{10}\ s$$

Question 5

Considering the units of the mass transfer coefficient (module 4, slide 44), if we indicate H_2S as component A, we write:

$$n_{A,0} = k_{A,G,loc}\left(C_{A,G,bulk} - C_{A,G,0}\right) = k_{A,L,loc}\left(C_{A,L,0} - C_{A,L,bulk}\right)$$

If we assume that H_2S behaves as an ideal gas:

$$C_{A,G,bulk} = \frac{P_A}{RT} \text{ and } C_{A,G,0} = \frac{P_{A,0}}{RT}$$

where P_A and $P_{A,0}$ are partial pressures

$$P_A = 0.05\ atm$$

$$P_{A,0} = 609\chi_{A,i,0} = 609 * 2.0 * 10^{-5} = 1.218 * 10^{-2}\ atm$$

Thus we can write:

$$n_{A,0} = k_{A,G,loc}\frac{1}{RT}\left(P_A - P_{A,0}\right) = 9.567 * 10^{-3}\frac{m}{s}\frac{1}{\left(0.082\frac{m^3 atm}{mol\ K}\right)(300K)}(0.05 - 0.012)atm$$

$$= 14.77\ \frac{mmol}{m^2 s}$$

Question 6

We can write the shear stress on the top of the plate:

$$\tau_1 = -\mu_1 \frac{dv}{dy} = \mu_1 \frac{v}{y}$$

and on the bottom of the plate:

$$\tau_2 = -\mu_2 \frac{dv}{dy} = \mu_2 \frac{v}{20-y}$$

The viscous force (viscous resistance) $F = A * \tau$, thus:

$$F = A * \tau = A * (\tau_1 + \tau_2) = Av \left(\frac{\mu_1}{y} + \frac{\mu_2}{20-y}\right)$$

For F to be minimum:

$$\frac{dF}{dy} = 0$$

$$-\frac{\mu_1}{y^2} + \frac{\mu_2}{(20-y)^2} = 0$$

$$\frac{y^2}{(20-y)^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{y}{20-y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$$y = 12.6\, mm$$

The solution with shear stress was accepted too.

$$\tau = (\tau_1 + \tau_2) = v \left(\frac{\mu_1}{y} + \frac{\mu_2}{20-y}\right)$$

$$\frac{d\tau}{dy} = 0$$