Final Exam Simulation

Question 1

The water from a river is flowing into a cylindric tank through a smooth short pipe. **Calculate the time t that takes to fill the tank up to $h = H$.**

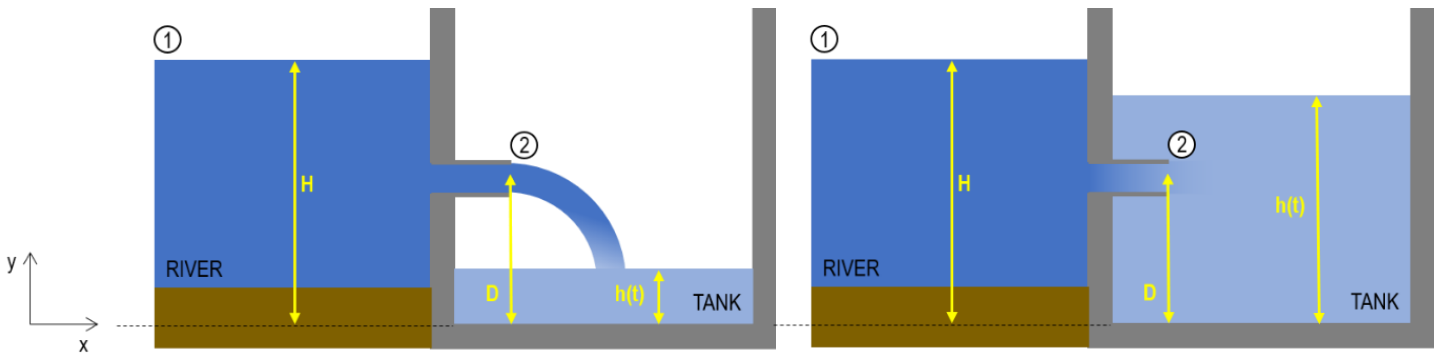
Tip: When $h > D$, the pressure at the exit of the short pipe is the hydrostatic pressure, which is equal to $\rho g h(t)$.

$A_{tank} = 1000\ m^2$ (surface area of the tank)

$$D = 20\ m$$

$$H = 30\ m$$

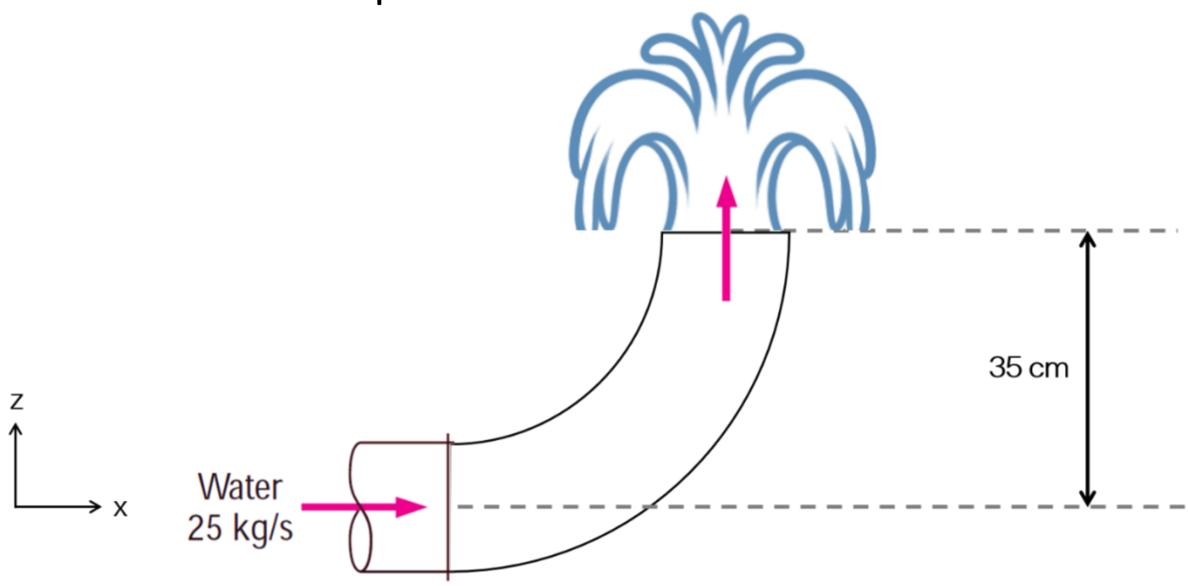
$A_{pipe} = 0.05\ m^2$ (cross-sectional area of the short pipe)



Tip: you need to split the problem in two steps, which are illustrated by the figures above.

Question 2

A 90 degrees elbow is used to direct water flow at a rate of 25 kg/s in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be negligible. **Determine (a) the gauge pressure at the inlet of the elbow and (b) the total anchoring force needed to hold the elbow in place.**



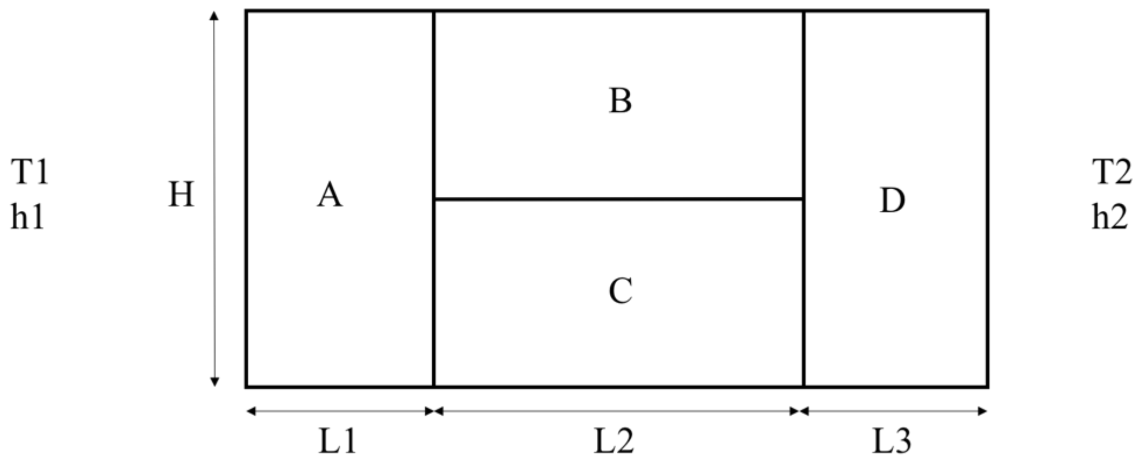
Question 3

Consider the composite wall illustrated below for high H = 3m, thickness 1m, $H_B = H_C = 1.5$ m, L1 = L3 = 0.05 m, L2 = 0.1 m, $k_A = k_D = 50$ W/mK, $k_B = 10$ W/mK and $k_C = 1$ W/mK.

a) Sketch the **thermal resistance circuit**.

b) Under conditions for which T1 = 200 °C, h1 = 50W/m^{2}K, T2=25 °C and h2=10W/m^{2}K, what is the **rate of heat transfer through the wall**?

c) What are the **interface temperatures**?



Question 4

You've probably noticed that balloons inflated with helium rise in the air the first day but begin to fall down the next day and act like ordinary balloons filled with air. This is because helium slowly leaks out through the wall of the balloon while air diffuses in.

Consider a balloon that is made of 0.1 mm thick, soft rubber and has a diameter of 15 cm when inflated (assume that the balloon is a sphere and that the volume does not change). The pressure and temperature inside the balloon are initially 110 kPa and 25°C. The diffusion coefficients of helium, oxygen, and nitrogen in rubber at 25 °C are $2.33 * 10^{-11}$, $1.75 * 10^{-11}$, and $6.44 * 10^{-12}\ m^2/s$, respectively. The molar mass of helium is 4 g/mol and the gas constant $R = 8.314\ \left(\frac{m^3 \cdot Pa}{K \cdot mol}\right)$.

Determine the initial rates of diffusion of helium, oxygen, and nitrogen through the balloon wall in molar flow rates and the mass fraction of helium that escapes the balloon during the first 5 hours assuming the helium pressure inside the balloon remains constant. Assume air to be 21 mole percent oxygen and 79 mole percent nitrogen and assume room conditions of 100 kPa and 25 °C.

Was it a good assumption that the pressure of helium inside the balloon remains constant over 5 hours? Explain with words *and* calculation.

Question 5

We are considering the transport of SO_2 from a gas (air) to a liquid phase (ethylene glycol) at a temperature of 40°C. The initial partial pressure of SO_2 in the air is 0.1 bar. The local mass transfer coefficient of SO_2 in air is $4.0 \cdot 10^{-3} \frac{cm}{s}$ and the one in ethylene glycol is $2.5 \cdot 10^{-4} \frac{cm}{s}$ Consider the Henry's Law relation P_{SO2}=H*C_{SO2} where H=1.05atm/M.

a) Calculate the **interfacial concentrations** of SO_2 in the gas and in the liquid.

b) Calculate the **mass flux.**

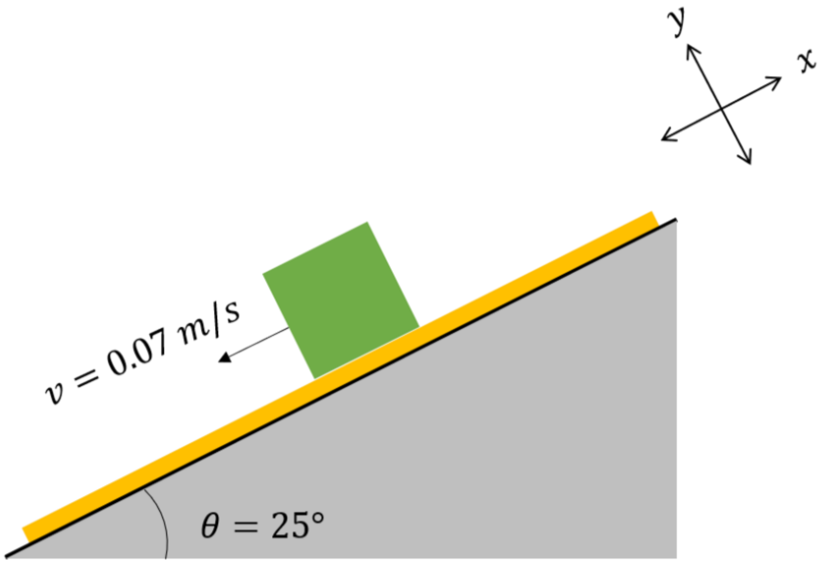
c) Determine if one side is limiting the mass transport.

d) Draw a *quantitative* gas/liquid **concentration diagram**, i.e., identify the concentration values in the bulk and at the interface, along with the direction of the molar flux.

e) Qualitatively identify bulk, equilibrium and interface pressure and concentration in the **equilibrium diagram**.

Question 6

A cube (50 cm x 50 cm x 50 cm) weighing 50 N is placed on an inclined ramp with a thin oil film between them (film thickness h=0.8 mm). The plate slides down the ramp at a constant speed of 0.07 m/s. Calculate the viscosity of the oil assuming a linear velocity profile. Draw a free body diagram of the cube indicating the forces acting on it.



QC1. How does the dynamic viscosity of liquids and gases vary with temperature? (Elaborate on the reason)

QC2. How does advective mass transport differ from diffusive mass transport?

Final exam simulation solution

Solution 1

We approach this problem by dividing it into two parts. First, we calculate the time needed for to fill the tank up to D and then from D to H.

First, let's calculate the time it takes to fill the tank up to the pipe:

$$time = \frac{volume}{Q} = \frac{A_{tank} \times D}{A_{pipe} \times v_2}$$

We need to calculate v_2, so we apply Bernoulli's theorem between points 1 and 2:

$$\rho g h_1 + P_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + P_2 + \frac{1}{2}\rho v_2^2 \qquad (1)$$

At this point, we have that:

- h_1 is H.
- h_2 is D.
- $P_{1,gauge} = P_{2,gauge} = 0\ atm$
- The level of water in the river is nominally not going to be affected by the filling of the tank, so we have that $v_1 \rightarrow 0$. Another way to see it is through the continuity equation: as the surface area of the river (A_1) is going to be extremely bigger than the area of the pipe (A_{pipe}), v_1 will tend to be so small that it can be neglected.

Therefore, if substituted in (1), we obtain:

$$gH = gD + \frac{1}{2}v_2^2$$

$$v_2 = \sqrt{2g(H-D)} = \sqrt{2 \times 9.8 \times (30-20)} = 14\ m/s$$

$$time = \frac{A_{tank} \times D}{A_{pipe} \times v_2} = \frac{1000 \times 20}{0.05 \times 14} = 28571\ seconds \approx 8\ hours$$

Once the pipe is submerged in the water, we have to account that P_2 is no longer atmospheric pressure, but that it is under hydrostatic pressure. The hydrostatic pressure is going to increase as the water level raises in the tank, so it will be time dependent:

$$P_2 = \rho g(h(t) - D)$$

We can again describe the Bernoulli equation between points 1 and 2:

$$\rho g h_1 + P_1 + \frac{1}{2}\rho v_1^2 = \rho g h_2 + P_2 + \frac{1}{2}\rho v_2^2$$

Now we have that:

- h_1 is H.
- h_2 is D.
- $P_{gauge} = P_1 = 0\ atm$
- $v_1 \to 0$
- $P_2 = \rho g(h(t) - D)$

$$gH = gD + g(h(t) - D) + \frac{1}{2}v_2^2$$

$$v_2^2 = 2 \times (gH - gD - gh(t) + gD)$$

$$v_2 = \sqrt{2g(H - h(t))}$$

The *volumetric flow rate* must be equal to the *rate of water volume increase in the tank*:

$$v_2 \times A_{pipe} = A_{tank} \times \frac{dh}{dt}$$

$$dt = \frac{A_{tank}}{A_{pipe}} \frac{dh}{v_2} = \frac{A_{tank}}{A_{pipe}} \frac{dh}{\sqrt{2g(H - h(t))}}$$

Boundaries:

time: 0 and t

height: D and H

$$\int_0^t dt = \frac{A_{tank}}{A_2} \int_D^H \frac{dh}{\sqrt{2g(H-h)}}$$

$$time = \frac{A_{tank}}{A_{pipe}} \frac{\sqrt{2g(H-D)}}{g} = \frac{1000}{0.05} \frac{\sqrt{2 \times 9.8 \times (30-20)}}{9.8} = 28571\ seconds \approx 8\ hours$$

The total time required is therefore $\approx 16\ hours$

Solution 2

(a) The Bernoulli equation applied between the inlet and the outlet allows us to determine the gauge pressure at the inlet of the elbow

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$v_1 = v_2 = \frac{Q}{\rho A} = \frac{25\ kg/s}{(1000\ {}^{kg}/_{m^3}) \cdot [\pi(0.05m)^2]} = 3.18\ m/s$$

$$P_{2,gauge} = 0\ , v_1 = v_2$$

$$P_1 = \rho g(h_2 - h_1) = \left(1000\ {}^{kg}/_{m^3}\right) \cdot \left(9.8\ {}^{m}/_{s^2}\right) \cdot (0.35\ m - 0\ m) = 3.43\ kPa$$

(b) Using the momentum balance equation to obtain the reaction force:

$$\sum F_{surface} + \sum F_{volume} = \sum_i^N \int_{A_i} \rho \bar{v}(\bar{v} \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

Volume and friction are neglected:

$$\sum F_{volume} = 0, \sum F_{friction} = 0$$

$$\sum_i^N F_{pressure} = -P_i . A_i . \widehat{n_i}$$

Solving for x:

$$F_{reaction-x} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \bar{v}_1(\bar{v}_1 \cdot \widehat{n_1}) A_1 + \rho \bar{v}_2(\bar{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-x} + P_1 A_1 = -\rho v_1^2 A_1$$

$$F_{reaction-x} + 3430 \cdot \pi \cdot (0.05)^2 = -1000 \cdot (3.18^2) \cdot \pi \cdot (0.05)^2$$

$$F_{reaction-x} = -109\ N$$

Solving for z:

$$F_{reaction-Z} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2} = \rho \bar{v}_1(\bar{v}_1 \cdot \widehat{n_1}) A_1 + \rho \bar{v}_2(\bar{v}_2 \cdot \widehat{n_2}) A_2$$

$$F_{reaction-Z} = \rho v_2^2 A_2$$

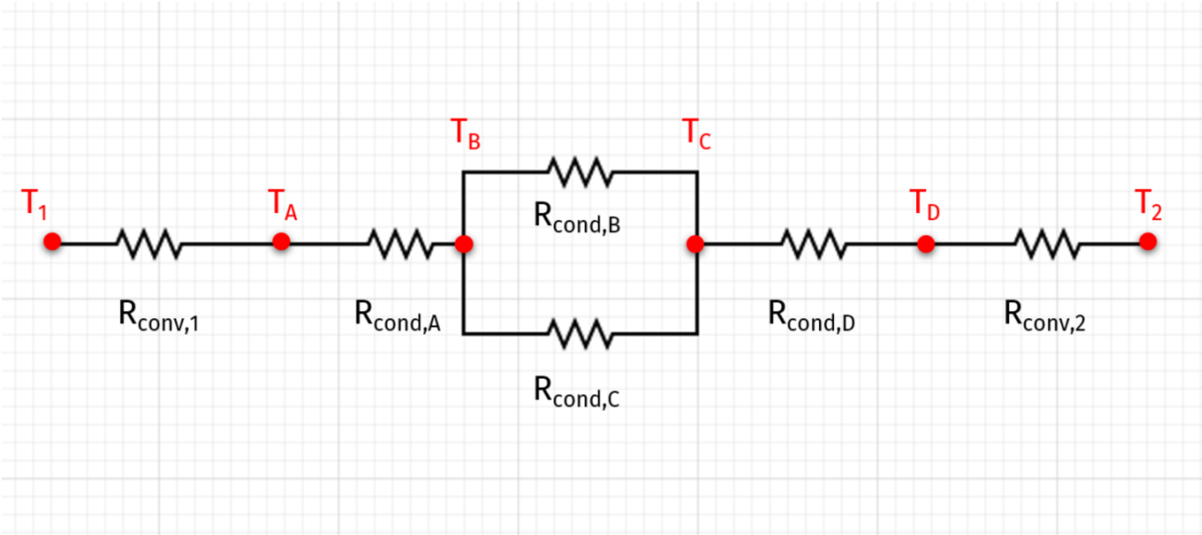
$$F_{reaction-Z} = -1000 \cdot (3.18^2) \cdot \pi \cdot (0.05)^2$$

$$F_{reaction-Z} = 81.9\ \mathrm{N}$$

$$F_r = \sqrt{F_{r-x}^2 + F_{r-Z}^2} = 136\ N$$

Solution 3

a) The first thing we do for these heat transfer problems is to draw the thermal circuit:



b)

$$R_{tot} = \frac{1}{h_1 A_1} + \frac{L_A}{k_A A_A} + \frac{1}{\frac{k_B A_B}{L_B} + \frac{k_C A_C}{L_C}} + \frac{L_D}{k_D A_D} + \frac{1}{h_2 A_2}$$

$$R_{tot} = \frac{1}{50 * 3} + \frac{0.05}{50 * 3} + \frac{1}{\frac{10 * 1.5}{0.1} + \frac{1 * 1.5}{0.1}} + \frac{0.05}{50 * 3} + \frac{1}{10 * 3}$$

$$R_{tot} = 0.0067 + 0.0003 + 0.0060 + 0.0003 + 0.0333 = 0.0466℃W^{-1}$$

$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{200 - 25}{0.0466} = \mathbf{3755.4\ W}$$

c) For the interface temperatures,

$$\dot{Q} = h_1 A_1 (T_1 - T_A)$$

So, $T_A = T_1 - \frac{\dot{Q}}{h_1 A_1} = 200 - \frac{3755.4}{50*3} = \mathbf{174.96}℃$

Similarly, $\dot{Q} = h_2 A_2 (T_D - T_2)$

So, $T_D = T_2 + \frac{\dot{Q}}{h_2 A_2} = 25 + \frac{3755.4}{10*3} = \mathbf{150.18}℃$

$$T_B = T_A - \frac{\dot{Q}}{\frac{k_A A_A}{L_A}}$$

$$T_B = 174.96 - R_{COND,A}\dot{Q} = 174.96 - 0.0003 * 3755.4 = \mathbf{173.8}℃$$

Similarly,

$$T_C = T_D + R_{COND,D}\dot{Q} = 150.18 + 0.0003 * 3755.4 = \mathbf{151.3}℃$$

Solution 4

In air, the partial pressures of oxygen and nitrogen are

$$P_{N_2} = y_{N_2}P = (0.79)(100\ kPa) = 79\ kPa = 0.79\ bar = 79{,}000\ Pa$$

$$P_{O_2} = y_{O_2}P = (0.21)(100\ kPa) = 21\ kPa = 0.21\ bar = 21{,}000\ Pa$$

Partial pressure of helium in the air is negligible.

Inside the balloon, the initial partial pressure of helium in the balloon is 110 kPa while the partial pressure of nitrogen and oxygen are zero.

The molar flow rate is given by

$$J_A = A * D_{AB} * \left(\frac{dc_A}{dy}\right) \rightarrow A * D_{AB} * \frac{c_A}{Y}$$

For a system where the concentration on one side of the wall is negligible.

When substituting pressure for concentration, as seen in class, we obtain:

$$J_A = \frac{AD_{AB}}{RT} * \frac{p_{A_1} - p_{A_2}}{Y}$$

The balloon can be treated as an infinitesimally thin sphere.

To calculate the surface area of the balloon

$$A = \pi D^2 = \pi * (0.15m)^2 = 0.07069m^2$$

To calculate the initial rates of diffusion for the three gases we have

$$J_{He} = \frac{\left(0.07069m^2 * 2.33 * 10^{-11}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{110{,}000Pa}{0.1 * 10^{-3}m}\right) = 7.31 * 10^{-7}mol/s$$

$$J_{O_2} = \frac{\left(0.07069m^2 * 1.75 * 10^{-11}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{-21{,}000Pa}{0.1 * 10^{-3}m}\right) = -1.05 * 10^{-7}mol/s$$

$$J_{N_2} = \frac{\left(0.07069m^2 * 6.44 * 10^{-12}\left(\frac{m^2}{s}\right)\right)}{8.314\left(\frac{m^3 \cdot Pa}{K \cdot mol}\right) * 298\ K} * \left(\frac{-79{,}000Pa}{0.1 * 10^{-3}m}\right) = -1.15 * 10^{-7}mol/s$$

The initial mass of helium that escapes during the first 5 hours is

$$m_{diff,He} = M * J_{He} * \Delta t = \frac{4kg}{kmol} * 7.31 * \frac{10^{-7}mol}{s} * 5hr * \frac{3600s}{hr} = 5.26 * 10^{-5}kg$$
$$= 52.6\ mg$$

Initial mass of helium in balloon is

$$m_{initial} = \frac{PV}{RT} * M = \frac{110{,}000Pa * 4 * \pi * (0.075m)^3 * 4}{8.314 * 298 * 3} = 0.315\ g = 315\ mg$$

Fraction of helium that has escaped is therefore 16.7% so our assumption of constant helium pressure in the balloon is not great.

Solution 5

We refer to SO_2 as A, air as G and ethylene glycol as L.

a) If we equate the fluxes of SO_2 in the two phases

$$n_A = k_G^{loc}\left(C_{A,G,bulk} - C_{A,G,o}\right) = k_L^{loc}\left(C_{A,L,o} - C_{A,L,bulk}\right)\ (Eq.1)$$

Also, it is given that the interface concentrations are in equilibrium according to the following relation:

$$P_{A,G,o} = H * C_{A,L,o}$$

We consider SO_2 as an ideal gas, which means that:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} \text{ and } C_{A,G,bulk} = \frac{P_{A,G,bulk}}{RT}$$

Also, we can assume that $C_{A,L,bulk} = 0$

To solve for $P_{A,G,o}$:

$$k_G^{loc}\frac{1}{RT}\left(P_{A,G,bulk} - P_{A,G,o}\right) = k_L^{loc}\frac{P_{A,G,0}}{H}$$

Thus, we get

$$P_{A,G,o} = \frac{\dfrac{k_G^{loc}P_{A,G,bulk}}{RT}}{\left(\dfrac{k_L^{loc}}{H} + \dfrac{k_G^{loc}}{RT}\right)}$$

If we put the values from the question,

$$P_{A,G,o} = \frac{\dfrac{4\times10^{-5}\times0.1}{0.082\times313}}{\left(\dfrac{2.5\times10^{-6}}{1.05} + \dfrac{4\times10^{-5}}{0.082\times313}\right)}$$

$$P_{A,G,o} = 4\times10^{-2}atm$$

Thus:

$$C_{A,G,o} = \frac{P_{A,G,o}}{RT} = 1.6\times10^{-3}M$$

$$C_{A,L,o} = \frac{P_{A,G,o}}{H} = 3.8\times10^{-2}M$$

b) In order to solve for the mass flux, we need first the molar flux. We can use the $(Eq.1)$

$$n_A = k_L^{loc}\times C_{A,L,o} =$$

$$= 2.5\times10^{-6}m/s\ \times 0.038\times10^{3}mol/m^3$$

$$n_A = 0.095\times10^{-3}mol/m^2s$$

thus

$$\dot{m}_A = M_A \times n_A = \frac{64g}{mol} \times 0.095 \times \frac{10^{-3}mol}{m^2s} = 6 \times 10^{-3} g/m^2s$$

c) To determine if mass transfer is limited on one side, we have to calculate m_G, m_L and m_{avg} and also the C^{eq}values

$$m_G = \frac{c_{A,G,0} - c_{A,G}^{eq}}{c_{A,L,0} - c_{A,L,bulk}} = \frac{1.6 \times 10^{-3} - 0}{3.8 \times 10^{-2} - 0} = 0.042$$

$$m_L = \frac{c_{A,G,bulk} - c_{A,G,0}}{c_{A,L}^{eq} - c_{A,L,0}} = \frac{4 \times 10^{-3} - 1.6 \times 10^{-3}}{0.095 - 3.8 \times 10^{-2}} = 0.042$$

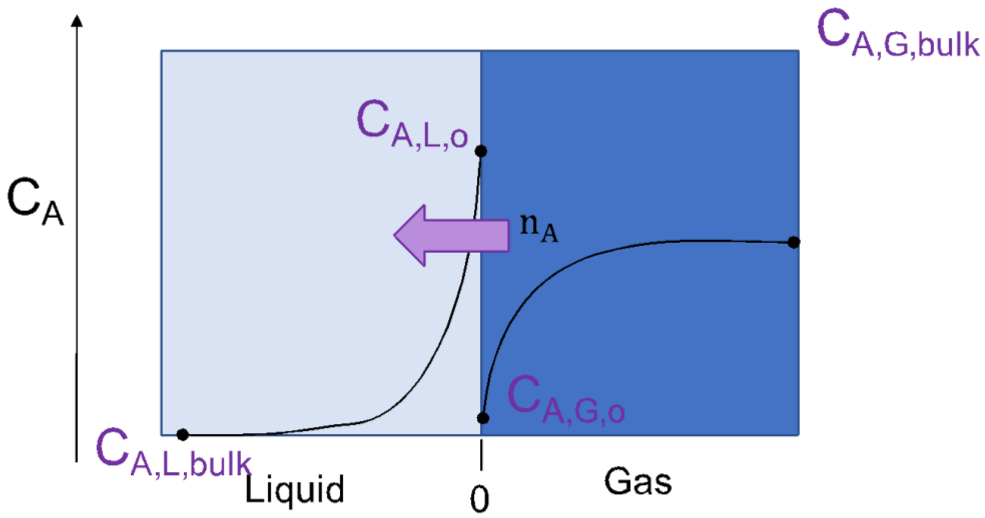
where $c_{A,L}^{eq} = \frac{P_{A,G,bulk}}{H}$

$$m_{avg} = \frac{1}{2}(m_L + m_G) = 0.042$$

$$\frac{k_L^{loc}}{m_{avg}k_G^{loc}} = \frac{2.5 \times 10^{-6}\frac{m}{s}}{0.042 \times 4 \times 10^{-5}\frac{m}{s}} = 1.5$$

Since $\frac{k_W^{loc}}{m_{avg}k_B^{loc}} > 1$, the mass transfer is moderately gas phase controlled.

d)



where

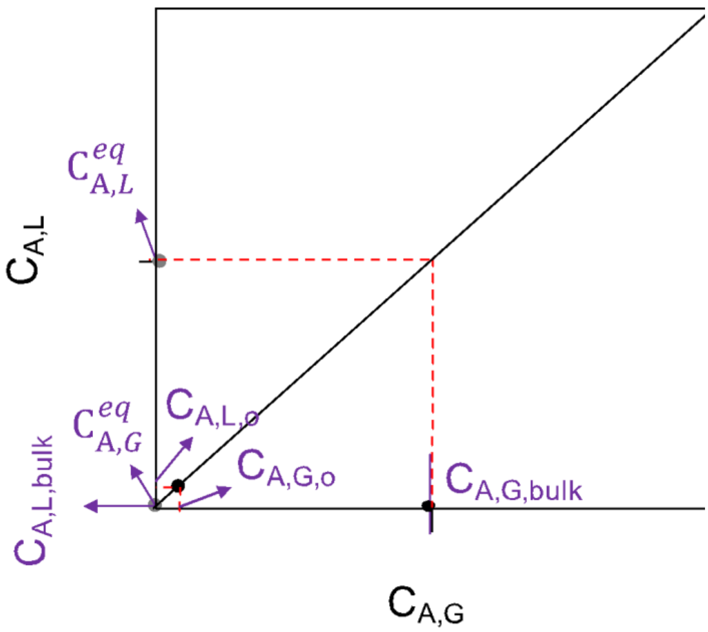
$$C_{A,L,bulk} = 4 \times 10^{-3} M$$

$$C_{A,G,o} = 1.6 \times 10^{-3} M$$

$$C_{A,L,bulk} = 0$$

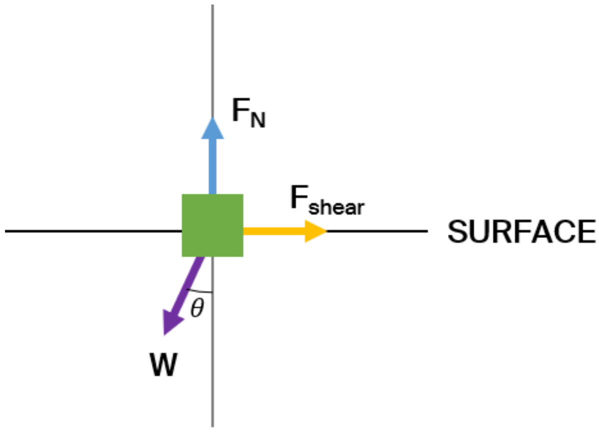
$$C_{A,L,o} = 3.8 \times 10^{-2} M$$

e) The graph below is quantitative, a qualitative version is also accepted considering the challenge in positioning $C_{A,L,bulk}$ and $C_{A,G,eq}$ at the origin.



Solution 6

As the velocity of the block is constant, the acceleration and the net force acting on it must be zero. A free body diagram of the block looks like:



The force due to viscosity is by definition:

$$F_{shear} = \tau A = \mu \frac{dv}{dy} A = \mu \frac{v}{h} A$$

The shear force can be equated to the component of weight along the ramp plane:

$$F_{shear} = W \cdot sin\theta$$

Equating the two forces expressions:

$$\mu \frac{v}{h} A = W \cdot sin\theta$$

$$\mu = \frac{W \cdot h \cdot sin\theta}{v \cdot A} = \frac{50\ N \cdot 0.0008\ m \cdot \sin(25°)}{0.07 \frac{m}{s} \cdot (0.5\ m)^2} = 0.966\ Pa \cdot s$$

QC1. How does the dynamic viscosity of liquids and gases vary with temperature?

The viscosity of gases increases with temperature. According to the kinetic theory of gases, viscosity is proportional to the square root of the absolute temperature. The higher the temperature, the faster the gas molecules move and the more efficient momentum transfer is.

The viscosity of liquids decreases with temperature. Indeed, as temperature increases the intermolecular forces which are responsible for viscosity weaken, which causes such a decrease.

QC2. How does advective mass transport differ from diffusive mass transport?

Advective mass transport refers to the *transport of a fluid on a macroscopic level* from one location to another, wherein the transport is induced by an external force, such as a fun or a pump or gravity. Instead, diffusive mass transport requires the presence of *two regions at different concentration* as it refers to the movement of molecules from a *high concentration region towards a lower concentration*.