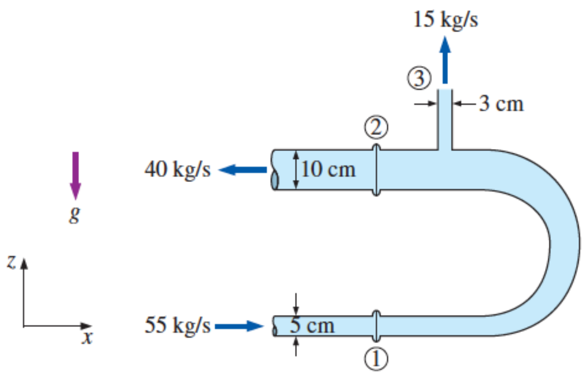
Final exam simulation

Question 1

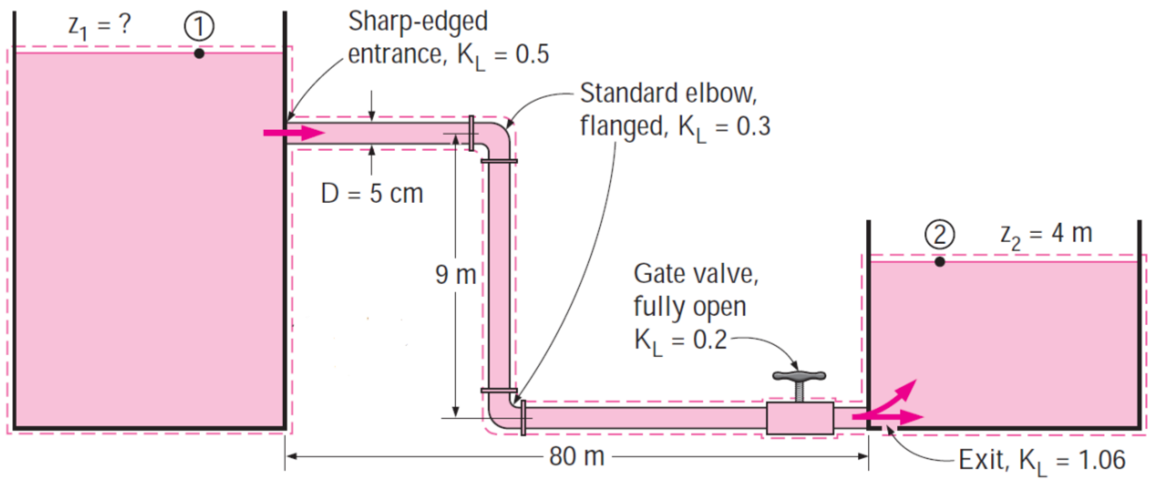
Water is flowing into and discharging from a pipe U-section as shown in the figure.



At flange (1), the total absolute pressure is 200 kPa, and 55 kg/s of water flows into the pipe. At flange (2), the total absolute pressure is 150 kPa, and water flows at 40 kg/s. At location (3), 15 kg/s of water discharges to the atmosphere, which is at 100 kPa. Determine the total x-force and z-force at the two flanges connecting the pipe. Consider frictionless flow and neglect volume forces.

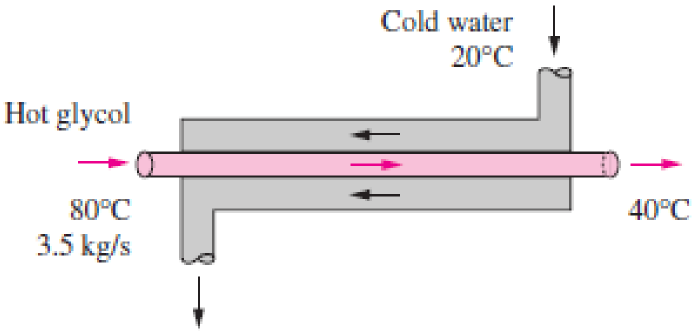
Question 2

Water flows from a large reservoir to a smaller one through a 5-cm-diameter cast iron piping system, as shown in the figure. Determine the elevation z_1 necessary for a flow rate of 6 L/s. The density and dynamic viscosity of water are $\rho = 1000\ kg/m^3$ and $\mu = 1.307\ \times 10^{-3}\ kg/m \cdot s$. The roughness of cast iron pipe is $\varepsilon = 0.00026\ m$.



Question 3

A double-pipe counter-flow heat exchanger is to cool ethylene glycol ($C_p = 2560$ J/kg · °C) flowing at a rate of 3.5 kg/s from 80°C to 40°C by water ($C_p = 4180$ J/kg · °C) that enters at 20°C and leaves at 55°C. The overall heat transfer coefficient based on the inner surface area of the tube is 250 W/m^2 · °C. Determine (*a*) the rate of heat transfer, (*b*) the mass flow rate of water, and (*c*) the heat transfer surface area on the inner side of the tube.

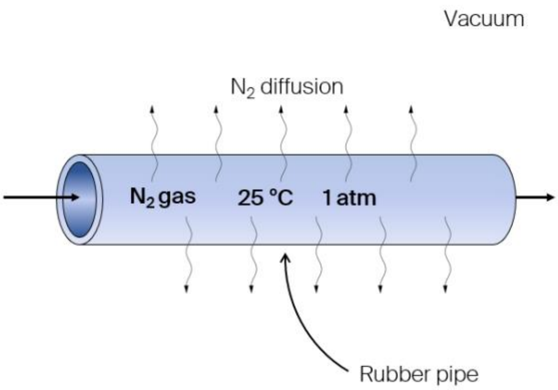


Question 4

Consider a carbonated drink in a bottle at 27°C and 130 kPa. Assuming the gas space above the liquid consists of a saturated mixture of CO_2 and water vapor and treating the drink as water, determine (*a*) the mole fraction of the water vapor in the CO_2 gas and (*b*) the mass of dissolved CO_2 in a 200-ml drink. The saturation pressure of water at 27°C is 3.60 kPa. Henry's constant for CO_2 dissolved in water at 27ºC (300 K) is H = 1710 bar. Molar masses of CO_2 and water are 44 and 18 kg/kmol, respectively.

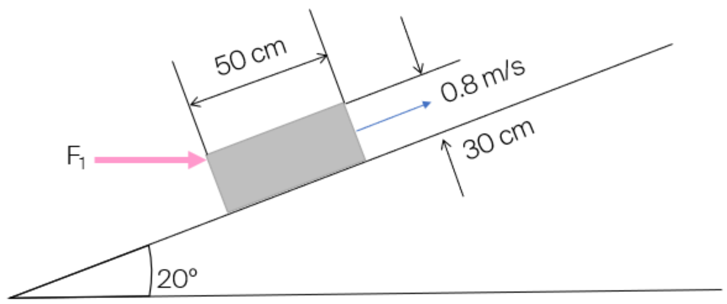
Question 5

Pure N_2 gas at 1 atm and 25°C is flowing through a 10-m-long, 3-cm-inner diameter pipe made of 1-mm-thick rubber. Determine the rate at which N_2 leaks out of the pipe if the medium surrounding the pipe is (*a*) a vacuum and (*b*) atmospheric air at 1 atm and 25°C with 21 percent O_2 and 79 percent N_2. The diffusivity and solubility of nitrogen in rubber at 25°C are 1.5×10^{-10} m^2/s and 0.00156 9 kmol/m^3.bar, respectively.



Question 6

A 50-cm x 30-cm x 20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (a) Determine the force F that needs to be applied in the horizontal direction to achieve the desired velocity. Start by drawing the force vectors corresponding to the normal force (F_N), the friction force (F_f) and the weight (W). (b) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa·s is applied between the block and inclined surface, determine the percent reduction in the required force.



Reminder: mathematically, the friction coefficient f corresponds to F_f/F_N.

Considerations: **1** The inclined surface is plane (perfectly flat, although tilted). **2** The oil film thickness is uniform.

Final exam simulation - Solutions

Question 1.

We choose points 1 and 2 at the free surface of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and that the fluid velocities at both points are zero ($V_1 = V_2 = 0$), the energy equation between these two points corresponds to:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

$$\Downarrow$$

$$\rho g h_1 = \rho g h_2 + \Delta P_f$$

Where:

$$\Delta P_f = \frac{1}{2}\rho v^2 \left(\left(\frac{4 f_f}{D} \sum_{i=1}^{3} L_i\right) + \sum_{j=1}^{2} K_j \right)$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are:

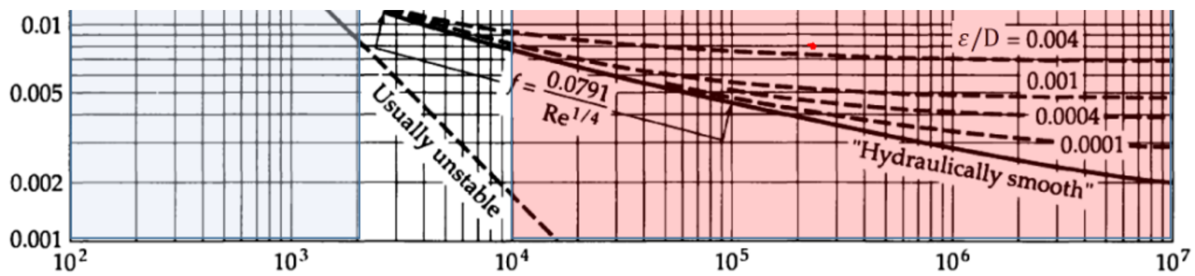
$$v = \frac{Q}{A} = \frac{0.006\ m^3/s}{\pi (0.025\ m)^2} = 3.06\ m/s$$

$$Re = \frac{\rho \cdot v \cdot D}{\mu} = \frac{(1000 \frac{kg}{m^3}) \cdot (3.06\ m/s) \cdot (0.05\ m)}{1.307\ \cdot\ 10^{-3}\ kg/m \cdot s} = 117000 \approx 1.2 \times 10^5$$

The flow is turbulent since Re > 4000.

$$\frac{\varepsilon}{D} = \frac{0.00026\ m}{0.05\ m} = 0.0052$$

From the Moody chart:



$$f_f = 0.008$$

$$\sum_{i=!}^{2} L_i = 80m + 9m = 89m$$

$$\sum_{j=1}^{2} K_j = 0.5 + 0.3 + 0.3 + 0.2 + 1.06 = 2.36$$

The pressure drop is then:

$$\Delta P_f = \frac{1}{2}\rho v^2 \left((\frac{4f_f}{D}\sum_{i=1}^{3} L_i) + \sum_{j=1}^{2} K_j \right) = \frac{1}{2} \cdot (1000\frac{kg}{m^3} \cdot \left(3.06\frac{m}{s}\right)^2) \times \left[\frac{4 \cdot (0.008)}{0.05\, m} \cdot 89\, m + 2.36\right]$$
$$= 277.7\, kPa$$

$$\rho g h_1 = \rho g h_2 + \Delta P_f$$

$$h_1 = \frac{\rho g h_2 + \Delta P_f}{\rho g} = \frac{(1000\frac{kg}{m^3} \cdot 9.8\frac{m}{s^2 \cdot 4\, m) + \ 277.7\, kPa}}{(1000\frac{kg}{m^3} \cdot 9.8\frac{m}{s^2})} = 32.3\, m$$

Therefore, the free surface of the first reservoir must be 32.3 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Question 2.

In order to determine the total x-force and y-force acting on the control volume, which is represented by the U-section of the pipe and the water, we have to apply the equation for conservation of momentum considering the velocities and pressures. at the various inlets and outlets (gauge pressure might be use as atmospheric pressure acts on all the surfaces).

We can easily calculate the velocities first:

$$v_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{55\, kg.s^{-1}}{1000\, kg.m^{-3}\ \times\ \pi\ \times \frac{0.05^2}{4}\ m^2} = 28\, m/s$$

$$v_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{40\, kg.s^{-1}}{1000\, kg.m^{-3}\ \times\ \pi\ \times \frac{0.1^2}{4}\ m^2} = 5.1\, m/s$$

$$v_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{15\, kg.s^{-1}}{1000\, kg.m^{-3}\ \times\ \pi\ \times \frac{0.03^2}{4}\ m^2} = 21.2\, m/s$$

Equation for momentum conservation:

$$\sum F_{surface} + \sum F_{volume} = \sum_{i=1}^{N} \int_{A_i} \rho v (v \cdot \hat{n}) dA_i$$

$$\sum F_{surface} = \sum F_{friction} + \sum F_{pressure} + \sum F_{reaction}$$

$$\sum F_{friction} = 0$$

$$\sum F_{pressure} = -P_i . A_i . \widehat{n_i}$$

$$\sum F_{volume} = 0$$

<u>Solving for X:</u>

$$F_{surface-x} = \sum_{i=1}^{N} \int_{A_i} \rho v (v \cdot \hat{n}) dA_i = \rho v_1 (v_1 \cdot \widehat{n_1}) A_1 + \rho v_2 (v_2 \cdot \widehat{n_2}) A_2$$

$$\rho v_1 (v_1 \cdot \widehat{n_1}) A_1 + \rho v_2 (v_2 \cdot \widehat{n_2}) A_2 = F_{reaction-x} - P_1 . A_1 . \widehat{n_1} - P_2 . A_2 . \widehat{n_2}$$

$$-\rho v_1^2 A_1 - \rho v_2^2 A_2 = F_{reaction-x} + P_1 . A_1 + P_2 . A_2$$

$$F_{reaction-x} = -P_1 . A_1 - P_2 . A_2 - \rho v_1^2 A_1 - \rho v_2^2 A_2$$

$$F_{reaction-x} = -\left[(200-100) \times 10^3 \times \pi \times \frac{0.05^2}{4}\right] - [(150-100) \times 10^3 \times \pi \times \frac{0.1^2}{4}] - 1000[\left\{28^2 \times \pi \times \frac{0.05^2}{4}\right\} + \{5.1^2 \times \pi \times \frac{0.1^2}{4}\}]$$

$$F_{reaction-x} = -196.34N - 392.69N - 1540.48N - 203.48N = -2333N$$

Solving for Z:

$$\sum F_{surface-z} = F_{reaction-z} - P_3 . A_3 . \widehat{n_3}$$

$$+\rho v_3^2 A_3 = F_{reaction-z} - P_3 . A_3 . n_3$$

$$+\rho v_3^2 A_3 = F_{reaction-z} + 0 \ (Working\ with\ gauge\ pressure)$$

$$F_{reaction-z} = 1000 \times 21.2^2 \times \pi \times \frac{0.03^2}{4} = 318N$$

Question 3.

(a) The rate of heat transfer is:

$$Q = m\, C_p (T_{in} - T_{out})_{glycol} = 3.5 \frac{kg}{s} \left(\frac{2.56kJ}{kg°C} \right) (80 - 40)°C = 358.4\ kW$$

(b) The rate of heat transfer from water must be equal to the rate of heat transfer to the glycol. Then:

$$Q = mC_p (T_{out} - T_{in})_{water} \rightarrow m_{water} = \frac{358.4 \frac{kJ}{s}}{\left(4.18 \frac{kJ}{kg°C}\right) (55 - 20)°C} = 2.45\ kg/s$$

(c) The temperature differences at the two ends of the heat exchanger are:

$$\Delta T_1 = 80°C - 55°C = 25°C$$

$$\Delta T_2 = 40°C - 20°C = 20°C$$

And,

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{ln\ ln\ \left(\frac{\Delta T_1}{\Delta T_2}\right)} = 22.4°C$$

Then the heat transfer surface area becomes:

$$Q = UA\Delta T_{lm} \rightarrow A = \frac{Q}{U\Delta T_{lm}} = \frac{358.4kW}{\left(\frac{0.25kW}{m^2°C}\right)(22.4°C)} = 64.0\ m^2$$

Question 4.

(a) Noting that the CO_2 gas in the bottle is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 27°C, $P_{vapor} = P_{sat@27°C} = 3.60\ kPa$.

Assuming both CO_2 and vapor to be ideal gases, the mole fraction of water vapor in the CO_2 gas becomes:

$$y_{vapor} = \frac{P_{vapor}}{P} = \frac{3.5kPa}{130kPa} = 0.0277$$

(b) Noting that the total pressure is 130 kPa, the partial pressure of CO_2 is:

$$P_{CO_2gas} = P - P_{vapor} = 130 - 3.60 = 126.4\ kPa = 1.264\ bar$$

From Henri's law, the mole fraction of CO_2 in the drink is determined to be:

$$y_{CO_2,liquid} = \frac{P_{CO_2,gas}}{H} = \frac{1.264bar}{1710bar} = 7.39 \cdot 10^{-4}$$

Then the mole fraction of water in the drink becomes:

$$y_{water,liquid} = 1 - y_{CO_2,liquid} = 1 - 7.39 \cdot 10^{-4} = 0.9993$$

The mass and mole fractions of a mixture are related to each other by:

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \left(\frac{M_i}{M_m}\right)$$

Where the apparent molar mass of the drink (liquid water – CO_2 mixture) is:

$$M_m = \Sigma y_i M_i = y_{liquid\ water} M_{water} + y_{CO_2} M_{CO_2} = 0.9993 \cdot 18 + 7.39 \cdot 10^{-4} \cdot 44 = 18.02 \frac{kg}{kmol}$$

Then the mass fraction of dissolve CO_2 gas in liquid water becomes:

$$w_{CO_2,liquid\ side} = \frac{y_{CO_2,liquid\ side} M_{CO_2}}{M_m} = 7.39 \cdot 10^{-4} \cdot \frac{44}{18.02} = 0.0018$$

Therefore the mass of dissolved CO_2 in a 200 mL = 200g drink is:

$$m_{CO_2} = w_{CO_2} m_m = 0.00180 \cdot 200g = 0.360g$$

Question 5.

$$P_{N_2} = y_{N_2} P = (0.79)(100\ kPa) = 79\ kPa = 0.79\ bar$$

$$P_{O_2} = y_{O_2} P = (0.21)(100\ kPa) = 21\ kPa = 0.21\ bar$$

When solubility data is available, the molar flow rate of a gas through a solid can be determined by replacing the molar concentration by $C_{A,\ solid}\ (0) = S_{AB} \cdot P_{A,\ gas}\ (0)$ where S_{AB} is the solubility and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of gas A on the two sides of the wall. For a cylindrical pipe the molar rate of diffusion can be expressed in terms of solubility as:

$$J_{A,cylinder} = \frac{2\pi L D_{AB} S_{AB} \left(P_{A,1} - P_{A,2}\right)}{ln\ ln\ \left(\frac{r_2}{r_1}\right)}$$

(a) The pipe is in vacuum and thus $P_{A,2} = 0$:

$$J_{N_2,cyl,vacuum} = \frac{2\pi(10\ m)\left(1.5 \cdot 10^{-10} 10\frac{m^2}{s}\right)\left(0.00156\frac{kmol}{m^3} \cdot s \cdot bar\right)(1-0)bar}{ln\ ln\ \left(\frac{0.031}{0.03}\right)} = 4.483 \cdot 10^{-10}\ kmol/s$$

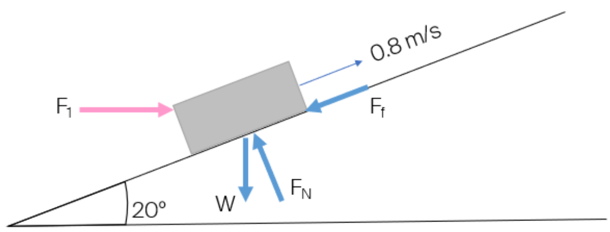
(b) The pipe is in atmospheric air and thus $P_{A,2} = 0.79$ bar:

$$J_{N_2,cyl,atm} = \frac{2\pi(10\ m)\left(1.5 \cdot 10^{-10} 10\frac{m^2}{s}\right)\left(0.00156\frac{kmol}{m^3} \cdot s \cdot bar\right)(1-0.79)bar}{ln\ ln\ \left(\frac{0.031}{0.03}\right)} = 9.416 \cdot 10^{-11}\ kmol/s$$

In the case of a vacuum environment, the diffusion rate of nitrogen from the pipe is about 5 times the rate in atmospheric air. This is expected since mass diffusion is proportional to the concentration difference.

Question 6.

As the velocity of the block is constant, the acceleration and the net force acting on it must be zero. A free body diagram of the block looks like:



With the friction force parallel to the surface, the normal force perpendicular to the surface and the weight in the direction of gravity acceleration.

The force balance decomposed in x and y gives:

$$\sum F_x = 0: \quad F_1 - F_f cos\,(20°) - F_{N1} sin\,(20°) = 0$$

$$\sum F_y = 0: \quad F_{N1} cos\,(20°) - F_f sin\,(20°) - W = 0$$

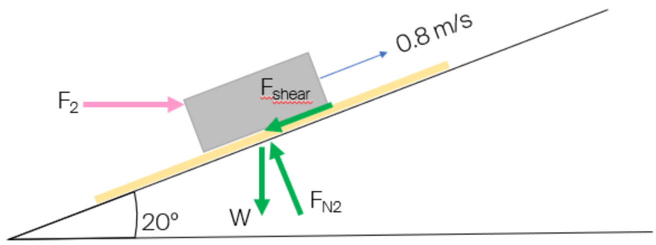
Considering: $F_f = f \cdot F_{N1}$, this is a system with three equations and three components. Solving for F_{N1} gives:

$$F_{N1} = \frac{W}{cos\,cos\,(20°)\ - f \cdot sin\,(20°)} = \frac{150\ N}{cos\,cos\,(20°)\ - 0.27 \cdot sin\,(20°)} = 177\ N$$

Then:

$$F_1 = F_f\ cos\,cos\,(20°)\ + F_{N1}\ sin\,sin\,(20°)\ = (0.27 \cdot 177N)\ cos\,cos\,(20°)\ + (177N)\ sin\,sin\,(20°) = 105.5\ N$$

When a layer of oil is placed on the inclined surface, the friction force is replaced by the shear force applied on the bottom surface of the block due to the oil's presence. We can now draw again the free body diagram of the block:



As seen, the shear force is expressed as:

$$F_{shear} = \tau \cdot A = \left(\mu \frac{V}{y}\right) \cdot A$$

$$F_{shear} = \left(0.012\ N \cdot \frac{s}{m^2}\right)(0.5 \cdot 0.2\ m^2)\frac{0.8\frac{m}{s}}{4 \cdot 10^{-4}\ m} = 2.4\ N$$

Decomposing the forces in x and y as we did in part (a) gives:

$$\sum F_x = 0: \quad F_2 - F_{shear} cos\,(20°) - F_{N2} sin\,(20°) = 0$$

$$\sum F_y = 0 : \quad F_{N2} cos\,(20°) - F_{shear} sin\,(20°) - W = 0$$

Solving for F_{N2} gives:

$$F_{N2} = \frac{F_{shear}\ sin\ sin\ (20°)\ + W}{cos\ cos\ (20°)} = \frac{(2.4\ N)\ sin\ sin\ (20°) +\ 150\ N}{cos\ cos\ (20°)} = 160.5\ N$$

And solving for F_2 gives:

$$F_2 = F_{shear}\ cos\ cos\ (20°)\ + F_{N2}\ sin\ sin\ (20°)\ = (2.4\ N)\ cos\ cos\ (20°)\ + (160.5\ N)\ sin\ sin\ (20°) = 57.2\ N$$

Based on this result, the percentage reduction of the required force is:

$$\frac{F_1 - F_2}{F_1} = 0.458 = 45.8\ \%$$

The force required to push the block on the inclined surface reduces significantly by oiling the surface.