Introduction to Transport Phenomena: <u>Simulation of the final exam</u>

<u>Question 1</u>

The jet d'eau is one of the famous landmarks of Geneva. The water leaves the nozzle with diameter d_{nz} with the velocity of v_{nz} and a volumetric flow of $Q = 500\,\frac{L}{s}$. The ambient pressure is $p_{atm} = 1bar$. The density of the lake-water is assumed to be $\rho_w = 1000\,\frac{kg}{m^3}$. Acceleration due to gravity is given by $g = 9.81\,\frac{m}{s^2}$. The viscosity is given by $\mu = 8.9 * 10^{-4}\ Pa * s$.

a) Calculate the required nozzle diameter d_{nz} to reach a fountain height of $h_2 = 140m$ neglecting all frictional losses. The nozzle is at height $h_1 = 0m$.

b) (to be solved independently from a).

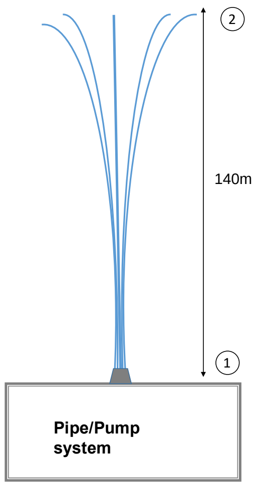
The water is accelerated by two pumps, each with power of 500KW, to reach the fountain height of 140m. A fraction of the pump power is required to compensate for the friction losses in the pipe system. The loss coefficients of the pipe system are:

$$\sum_i L_i = 5m$$

$$\sum_j K_j = 2$$

The internal surface roughness of the pipe is $\varepsilon = 0.1 * 10^{-3}m$. The average velocity in the pipe system until they reach the valve and nozzle is $v_{avg} = 25\,\frac{m}{s}$, the pipe diameter is $d_p = 10cm$. **Calculate the Reynolds number and use the Moody diagram (w. relative pipe roughness) to estimate the friction factor**. Then **estimate which fraction of the pumping power (out of 100%) is required to overcome the frictional losses** of the tubing in the housing (use given loss coefficients for calculations).

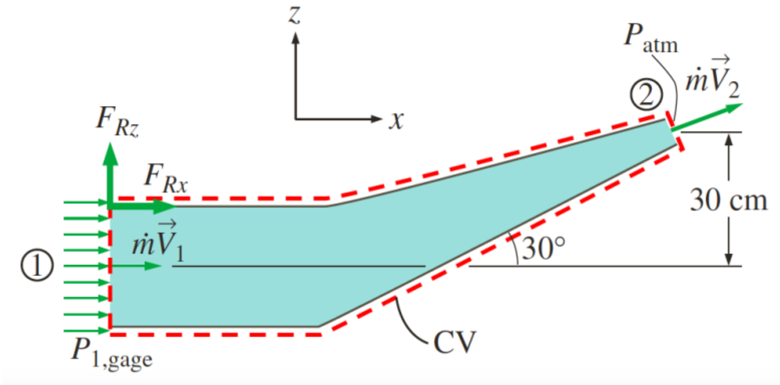
for illustration purpose only



Question 2

A reducing elbow is used to deflect water (density = 1000 kg/m^3) that flows at a rate of 14 Kg/s in a horizontal pipe upward 30degree (see figure below). The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the center of the outlet and the inlet is 30 cm. The weight of the elbow and of the water in it is considered to be negligible.

Determine (a) the gage pressure at the inlet of the elbow and b) the x- and z- components of the anchoring force needed to keep it in place.



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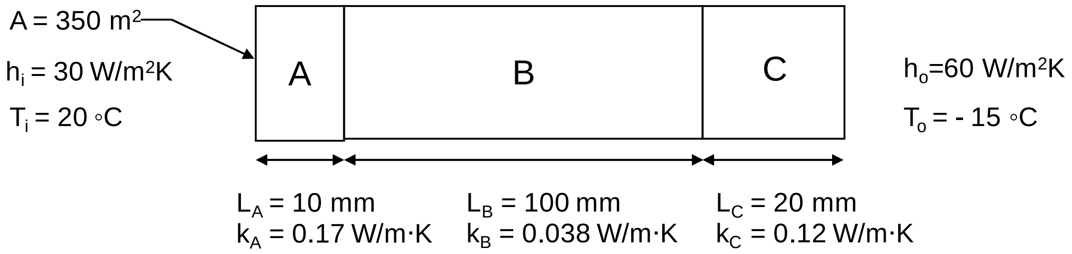
Question 3

Consider the composite wall below:



a) Sketch the thermal resistance circuit

b) Find the expression for the total resistance

c) Calculate the total heat loss

d) What is the controlling resistance for heat loss?

e) What is the heat flux at AB and BC interface? Can you explain why this is the case?

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Question 4

Consider a water-to-water double-pipe counter-flow heat exchanger. The inner diameter of the inner tube (**r_A**) and the thickness of the inner tube **(delta)** are **1 cm** and **0.2 mm**, respectively. The **cold water**, which flows in the inner tube, enters at **20°C** and leaves at **50°C**, while the **hot water,** which flows in the outer tube, enters at **80°C** and leaves at **45°C**.

Consider the **h_i=300 W/m².°C** and **h_o=500 W/m².°C**. The inner tube is made of **aluminum**. If the **velocity of the cold water** flowing **in the inner tube** is **0.5 m/s**,

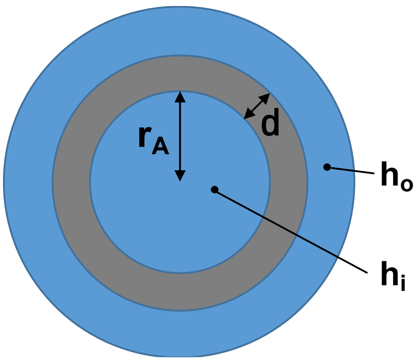
a) Calculate the mass flow of the hot fluid and the required surface area of heat exchanger.

(For both hot and cold water $C_p = 4180 \frac{J}{kg.°C}$ and $\rho = 1000 \, {}^{kg}/_{m^3}$).

After some time, a microbiological fouling layer forms inside the inner tube of heat exchanger.

b) Calculate the overall heat transfer coefficient if the thickness of the fouling layer is 0.0267cm.

Thermal conductivity of selected materials	
Material	**K (W/m.K)**
Common solids	
Aluminum	237
Concrete	1
Copper	386
Glass	0.9
Stainless Steel	16.5
Water	0.6
Fouling materials	
Calcium Carbonate	2.93
Microbological film	0.63
Calcium Sulfate	2.31
Calcium phosphate	2.6
Magnesium phosphate	2.16
Magnetic Iron oxide	2.88
Analytic	1.27



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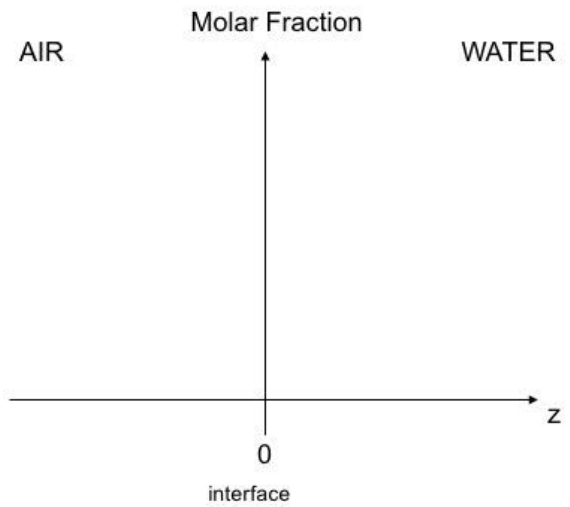
Question 5

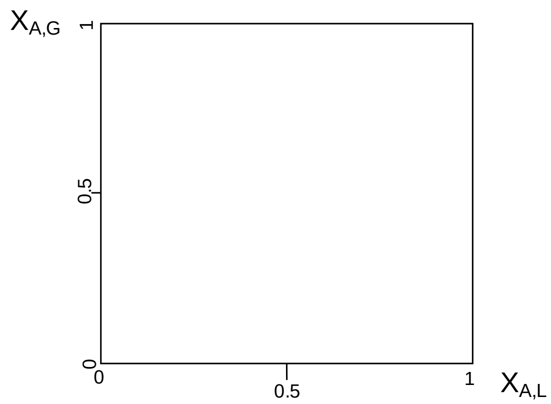
An air-H_2S mixture is in contact with a pool of water and H_2S (A) is being transferred from the air to the water at 30 °C. The value of the gas-phase local mass transfer coefficient $k_{A,G,loc}$ is 9.567 x 10^{-3} m/s. At a certain moment, the actual mole fraction of H_2S in the liquid at equilibrium with the gas phase is 2.0 x 10^{-5} and the partial pressure of H_2S in the air (bulk) is 0.05 atm. The Henry's law equilibrium relation is P_A [atm]= 609 X_A (mole fraction in the liquid).

a) Calculate the molar flux of H_2S from the gas to the liquid phase.

b) Draw the qualitative gas/liquid concentration diagram, which means identify the molar fraction values, in bulk and at the interface, along with the direction of the molar flux



c) Qualitatively identify bulk, equilibrium and interface molar fractions in the equilibrium diagram



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Question 6

We want to dye a disk-shaped window. Its diameter is D=1 m, its thickness is t=1 cm and its weight is 10 kg. The window is pulled out from the dye solution with a force F of 25 N. The density of the dye solution is 1200 kg/m^3 and its dynamic viscosity is 0.5 Pa·s.

a) Look at Figure 3 below. Can you estimate the **velocity** at which the window is pulled out when it is still submerged?

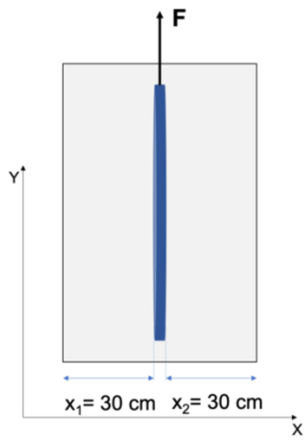

Figure 3

b) Look at Figure 4. Can you estimate the **velocity** at which the window is pulled out when it is still submerged?

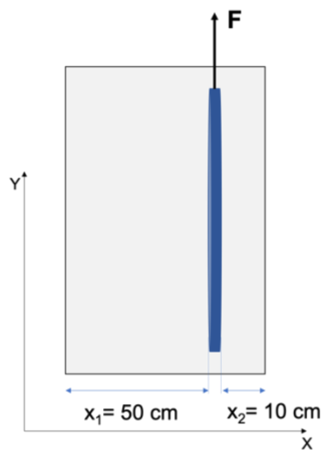

Figure 4

c) Find the **velocities of the solution layer**s which are at a distance of 5 cm from the moving window on both sides. Assume a linear velocity profile on both sides (Hint : $V_y = a - bx$ and think about the right boundary conditions)

Introduction to Transport Phenomena: Simulation of the final exam
Solution

Question 1

The jet d'eau is one of the famous landmarks of Geneva. The water is accelerated by two pumps witl power of $500kW$ and leaves the nozzle with diameter d_{nz} with the velocity of v_{nz} and a volumetric fl of $Q = 500\frac{L}{s}$. The ambient pressure is $p_{atm} = 1bar$ at the day of the calculation. The density of t lake-water is assumed to be $\rho_w = 1000\frac{kg}{m^3}$. Acceleration due to gravity is given by $g = 9.81\frac{m}{s^2}$. T viscosity is given by $\mu = 8.9 * 10^{-4}\ Pa * s$.

a) Bernoulli equation:

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_{nz}^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

Neglecting frictional losses and setting $v_2 = 0$. $P_1 = P_2 = 1atm$.

$$\cancel{P_1} + \cancel{\rho g h_1} + \frac{1}{2}\rho v_{nz}^2 = \cancel{P_2} + \rho g h_2 + \cancel{\frac{1}{2}\rho v_2^2} + \cancel{\Delta P_f}$$

The velocity at the nozzle is therefore given by:

$$v_{nz} = \sqrt{2gh_2} = \sqrt{2 * 9.81\frac{m}{s^2} * 140m} = 52.4\frac{m}{s}$$

From the given volumetric flow and the velocity, the diameter can be calculated.

$$Q = A_{nz} * v_{nz}$$
$$A_{nz} = r_{nz}^2 * \Pi$$

$$d_{nz} = 2 * r_{nz} = 2 * \sqrt{\frac{1}{\Pi} * \frac{Q}{v_{nz}}} = 2 * \sqrt{\frac{1}{3.1416} * \frac{0.5\frac{m^3}{s}}{52.4\frac{m}{s}}} = 0.11m = 11cm$$

b) Calculate the relative pipe roughness

$$\frac{\varepsilon}{d_p} = \frac{0.0001}{0.1} = 0.001$$

$$Re = \frac{\rho * v_{avg} * d_p}{\mu} = \frac{1000\frac{kg}{m^3} * 25\frac{m}{s} * 0.1m}{8.9 * 10^{-4}\ Pa * s} = 2.8 * 10^6$$

$f_f = 0.005$

Calculate the frictional induced pressure loss in the pipes

$$\Delta P_f = \frac{1}{2}\rho v_{avg}^2\left(\frac{4f_f}{D}\sum_i L_i + \sum_j K_j\right)$$

With friction factor of new diagram (at exam)

$$\Delta P_f = \frac{1}{2}1000\frac{kg}{m^3} * \left(25\frac{m}{s}\right)^2\left(\frac{4 * 0.005}{0.1m} * 5m + 2\right) = 9.375 * 10^5 Pa = 9.375bar$$

With calculate value given in this sub-question

$$Q_2 = A * v_{avg} = \frac{(0.1)^2}{4} * \Pi * 25\,\frac{\mathrm{m}}{\mathrm{s}} = 0.196\,\frac{\mathrm{m}^3}{s}$$

$$P_{pump,f_2} = \Delta P_f * Q_2 = 9.375 * 10^5 Pa * 0.196\,\frac{m^3}{s} = 184kW$$

$$F_2 = \frac{P_{pump,f}}{P_{pump,tot}} = 18.4\%$$

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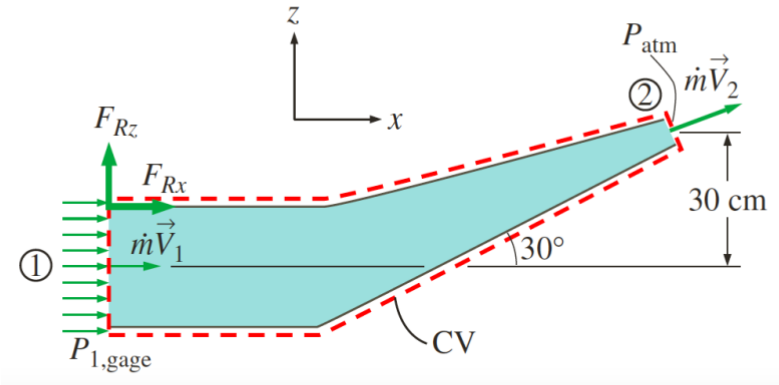
Question 2



a) Applying continuity equation at points 1 and 2 to calculate the inlet and outlet velocities:

$$v_1 = \frac{\dot{m}}{\rho A_1} = \frac{14\left(\frac{kg}{s}\right)}{1000\left(\frac{kg}{m^3}\right) \times 0.0113\left(\frac{m^2}{s}\right)} = 1.24\frac{m}{s}$$

$$v_2 = \frac{\dot{m}}{\rho A_2} = \frac{14\left(\frac{kg}{s}\right)}{1000\left(\frac{kg}{m^3}\right) \times 7 \times 10^{-4}\left(\frac{m^2}{s}\right)} = 20\frac{m}{s}$$

We use Bernoulli equation to calculate the pressures. Taking the center of the inlet cross section as the reference level (z_1=0) and writing the pressure value in gauge, $P_{2,gauge}$ = 0, the Bernoulli equation for the streamline going through the center of the elbow is expressed as:

$$P_{1,gauge} + \frac{\rho v_1^2}{2} + \rho g z_1 = P_{2,gauge} + \frac{\rho v_2^2}{2} + \rho g z_2$$

$$P_{1,gauge} - P_{2,gauge} = \frac{\rho(V_2^2 - V_1^2)}{2} + \rho g(z_2 - z_1)$$

$$P_{1,gauge} - 0 = \frac{1000\left(\frac{kg}{m^3}\right) \times (20^2 - 1.238^2)\left(\frac{m^2}{s^2}\right)}{2} + 1000\left(\frac{kg}{m^3}\right) \times 9.81\left(\frac{m}{s^2}\right) \times 0.3(m)$$

$$P_{1,gauge} = 202.2\ kPa$$

b)

$$F_{rxn,x} + \left(-P_{1,gauge}A_1\widehat{n_1}\right) + \left(-P_{2,gauge}A_2\widehat{n_2}\right) = \int \rho V_1\,(V_1.\widehat{n_1})dA_1 + \int \rho V_2\,(V_2.\widehat{n_2})dA_2$$

$$F_{rxn,x} + \left(-P_{1,gauge}A_1(-1)\right) + \left(-P_{2,gauge}A_2(1)\cos 30°\right) = \rho V_1^2 A_1(-1) + \rho V_2^2 A_2(1)\cos 30°$$

$$F_{rxn,x} = -\left(202.2 \times 10^3 (Pa) \times 113 \times 10^{-4} (m^2)\right) + 0 - \left(10^3 \left(\frac{kg}{m^3}\right) \times 1.238^2 \left(\frac{m^2}{s^2}\right) \times 113 \times 10^{-4} (m^2)\right) + \left(10^3 \left(\frac{kg}{m^3}\right) \times 20^2 \left(\frac{m^2}{s^2}\right) \times 7 \times 10^{-4} (m^2)\right)$$

$F_{rxn,x} = -2053N$

Similarly, the momentum equation for steady flow in the z-direction can be written as:

$$F_{rxn,z} + \left(-P_{1,gauge} A_1 \widehat{n_1}\right) + \left(-P_{2,gauge} A_2 \widehat{n_2}\right) = \int \rho V_1 \,(V_1.\widehat{n_1}) dA_1 + \int \rho V_2 \,(V_2.\widehat{n_2}) dA_2$$

However, there is no momentum terms in the z-component at the inlet.

$$F_{rxn,z} + 0 + (-P_2 A_2 (1) \sin 30°) = 0 + \rho V_2^2 A_2 (1) \sin 30°$$

$$F_{rxn,z} = 0 + (10^3 \left(\frac{kg}{m^3}\right) \times 20^2 \left(\frac{m^2}{s^2}\right) \times\ 7 \times 10^{-4} (m^2) \times \sin 30°)$$

$F_{rxn,z} = 144\ N$

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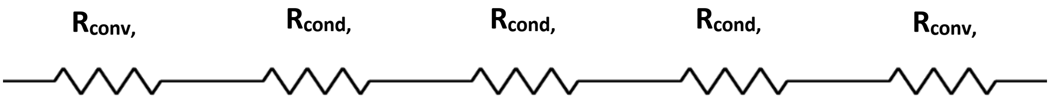
Question 3

a) The thermal resistance circuit is as follows



b) All the elements are in series. Hence the net resistance is evaluated as follows:

$$R_{tot} = \frac{1}{A} \cdot [\left(\frac{1}{h_i}\right) + \left(\frac{L_a}{k_A}\right) + \left(\frac{L_b}{k_b}\right) + \left(\frac{L_c}{k_c}\right) + \left(\frac{1}{h_o}\right)]$$

$$= \frac{1}{350} \cdot \left[\left(\frac{1}{30}\right) + \left(\frac{10 \cdot 10^{-3}}{0.17}\right) + \left(\frac{100 \cdot 10^{-3}}{0.038}\right) + \left(\frac{20 \cdot 10^{-3}}{0.12}\right) + \left(\frac{1}{60}\right)\right] =$$

$$= \frac{1}{350} \cdot [0.03 + 0.06 + 2.63 + 0.16 + 0.016\,] =$$

$$R_{tot} = 0.0083 \frac{W}{K}$$

c) $\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{20-(-15)}{0.0083} = \frac{35}{0.0083} = 4.21\ kW$

d) The controlling resistance is $R_{cond,B}$

e) The heat flux through the two interfaces mentioned is the same as they are in series and we consider that there are no heat losses.

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Question 4

a) Firstly we should calculate the heat transferred from the hot media to the cold media.

$$\dot{m}_{cold} = \rho_{water} * V_{cold} * A_{cold} \rightarrow 1000 * 0.5 * \pi * \frac{0.01^2}{4} = 0.03927\ kg/s$$

After that for the calculation of heat transfer we have:

$$\dot{Q}_{cold} = \dot{m}_{cold} * C_{p_{cold}} * \Delta T_{cold} = 0.03927 \frac{kg}{s} * 4180 \frac{J}{kg.°C} * (50 - 20)\ °C = 4924.458\ W$$

For the calculation of the flow for the hot media, we have:

$$\dot{Q}_{Hot} = \dot{Q}_{cold} = 4924.458\ W$$

$$\dot{m}_{Hot} = \frac{\dot{Q}_{Hot}}{C_{p_{Hot}} * \Delta T_{Hot}} = \frac{4924.458}{4180 * (80 - 45)} = 0.03366\ kg/s$$

For the calculation of the surface area, we need to calculate two things: The log mean temperature difference and the overall heat transfer coefficient.

$$\Delta T_{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{(80 - 50) - (45 - 20)}{\ln \frac{(80 - 50)}{(45 - 20)}} = 27.4\ °C$$

As for the tube we have $\frac{\delta}{r} < 0.1$, we can use the linear approximation for the overall heat transfer coefficient and thus we will have (we can extract the thermal conductivity for aluminum from the table given in the question):

$$U_o = \frac{1}{\frac{1}{h_i} + \frac{\delta_w}{k_w} + \frac{1}{h_o}} = \frac{1}{\frac{1}{300} + \frac{0.002}{237} + \frac{1}{500}} = \frac{1}{0.00333 + 1.6878 * 10^{-5} + 0.002} = \frac{1}{0.00535}$$
$$= 186.204\ {}^{W}/_{m^2.°C}$$

One should consider that the external convective heat transfer resistance, conductive heat transfer resistance and internal convective heat transfer resistance are $0.0033, 8.4388 * 10^{-6}, 0.002$, respectively.

Now, we can calculate the surface area:

$$Q = U_o * A_o * \Delta T_{LMTD} \rightarrow A_o = \frac{Q}{U_o * \Delta T_{LMTD}} = \frac{4924.458}{186.204 * 27.4} = 0.9652\ m^2$$

b) To check if the use of linear approximation has been appropriate we can calculate $\frac{\delta_w + \delta_{foul}}{r} = \frac{0.0267+0.02}{0.5} = 0.0934 < 10$. So, our linear approximation has been valid. Thus we will have (the thermal conductivity of the microbiological film can be extracted from the given table) :

$$\frac{1}{U_o} = \frac{1}{h_i} + \frac{\delta_w}{k_w} + \frac{\delta_{foul}}{k_{foul}} + \frac{1}{h_o}$$
$$= \frac{1}{300} + \frac{0.002}{237} + \frac{2.67 * 10^{-4}}{0.63} + \frac{1}{500} = \frac{1}{174.4598} \rightarrow U_o = 174.4598\ {}^{W}/_{m^2.°C}$$

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Question 5

a) Considering the units of the mass transfer coefficient (module 4, slide 42), thus we write:

$$n_A = k_{A,G,loc}\left(C_{A,G,bulk} - C_{A,G,0}\right) = k_{A,L,loc}\left(C_{A,L,0} - C_{A,L,bulk}\right)$$

If we assume that H_2S behaves as an ideal gas:

$C_{A,G,bulk} = \frac{P_A}{RT}$

$$C_{A,G,0} = \frac{P_{A,0}}{RT}$$

where P_A and $P_{A,0}$ are partial pressures

$$P_A = 0.05\ atm$$

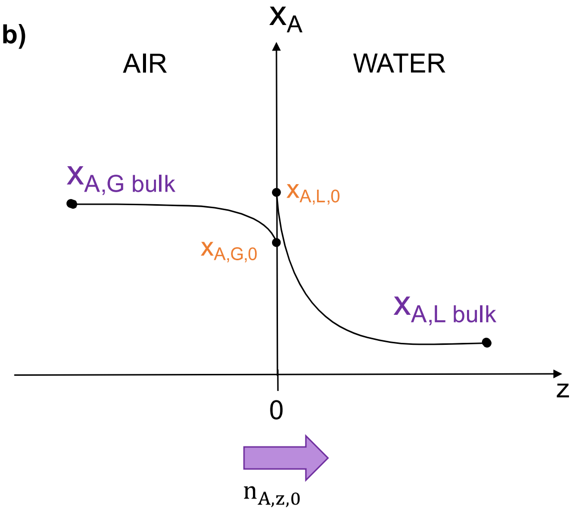
$$P_{A,G,0} = 609\chi_{A,L,0} = 609 * 2.0 * 10^{-5} = 1.218 * 10^{-2}\ atm$$

Thus we can write:

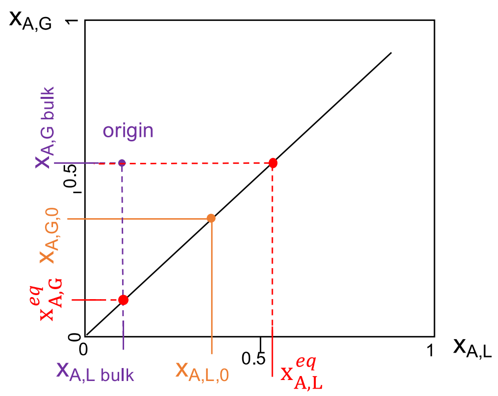
$$n_A = k_{A,G,loc}\frac{1}{RT}\left(P_A - P_{A,0}\right) = 9.567 * 10^{-3}\frac{m}{s}\frac{1}{\left(0.082\frac{m^3 atm}{mol\ K}\right)(303K)}(0.05 - 0.012)atm$$

$$= 14.77\frac{mmol}{m^2 s}$$

b)



c)



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Question 6

a) Force balance on the disk gives us :

$$F_{applied} = F_{gravity} - F_{buoyancy} + F_{shear}$$

$$F_{applied} = 25\ N$$

$$F_{gravity} = mg = 98\ N$$

$$F_{buoyancy} = \pi\left(\frac{d}{2}\right)^2 t \times \rho_{dye} \times g = \pi(0.5)^2(0.01) \times 1.2 \times 10^3 \times 9.8 = 92\ N$$

$$F_{shear} = 19N$$

Considering that shear force is acting on both faces of the disk,

$$F_{shear} = 2\mu A\left(\frac{dv}{dx}\right) = 2\mu Av(\frac{1}{x_1})$$

$$F_{shear} = 2 \times 0.5\pi(0.5)^2\ v\ (\frac{1}{0.3})$$

$$19\ N = \frac{\pi \times 6.67 \times v}{8}$$

$$\boldsymbol{v = 7.25\ m/s}$$

b) Writing the expression for shear force in the new position of the window according to figure 4,

$$F_{shear} = \mu A\left(\frac{dv_{left}}{dx_1} + \frac{dv_{right}}{dx_2}\right) = \mu A(\frac{v_{left}}{x_1} + \frac{v_{right}}{x_2})$$

Clearly $v_{right} = v_{left} = v$

$$F_{shear} = 0.5 \times \pi(0.5)^2 \times v \times (\frac{1}{0.5} + \frac{1}{0.1})$$

$$19\ N = \frac{\pi \times 12 \times v}{8}$$

$$\boldsymbol{v = 4\ m/s}$$

c) We now know the speed of the window. Hence we can evaluate the gradients on both sides. Given the linear velocity profile and the fact that the velocity of the dye solution will decrease as we go from window to the static wall of the container:

$$\mathrm{V_y = a - bx}$$

Considering that for x=0 (at the window) $V_y = \frac{4m}{s}$

$$V_{5cm}^{left} = V - \frac{dv}{dx_1} 0.05 = 4 - \frac{4}{0.5} 0.05 = 3.6\ m/s$$

$$V_{5cm}^{right} = V - \frac{dv}{dx_2} 0.05 = 4 - \frac{4}{0.1} 0.05 = 2\ m/s$$

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