Introduction to Transport Phenomena: Solutions Module 2

Solution 2.1

We can apply the mass balance between point 1 and point 2 :

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 v_{1,avg} A_1 = \rho_2 v_{2,avg} A_2$$

Therefore $$v_{2,avg} = \frac{\rho_1 v_{1,avg} A_1}{\rho_2 A_2} = \frac{\rho_1 Q}{\rho_2 A_2}$$

Here we have A_1, A_2, Q but we need that ρ_2

It is given that $\rho_1 = 1.17 \frac{kg}{m^3}$ at $p_1 = 1.25 bar$

QUESTION: Why we cannot apply the continuity equation from Module 1? Which assumption is not valid for gases?

Considering that we are dealing with a perfect gas in an isothermal system, we can write that :

$$p_1 V_1 = p_2 V_2 (= nRT)$$

$$p_1 \frac{m_1}{\rho_1} = p_2 \frac{m_2}{\rho_2}$$

because $m_1 = m_2$ $$\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$$

thus : $$\rho_2 = \rho_1 \frac{p_2}{p_1}$$

If we substitute in the equation above, we obtain

$$\boldsymbol{v_{2,avg} = \frac{p_1 Q}{p_2 A_2} = 9.37\ m/s}$$

Solution 2.2

a) To know the pressure we apply Bernoulli's along a stream line between point 1 and point 2. We consider $h_1 = h_2$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

The velocities can be derived from the flow rate

$$v_1 = \frac{Q}{A_1} = 7.96\frac{m}{s}$$

$$v_2 = \frac{Q}{A_2} = 56.6\frac{m}{s}$$

It is given that $P_2 = 1\ atm$

Substituting the values $\boldsymbol{p_1} = p_2 + \frac{1}{2}\rho\ (v_2^2 - v_1^2) = 10^5 + \frac{1}{2}1000\ (56.6^2 - 7.96^2) = \mathbf{16.7\ bar}$

Because one of the two pressures is atmospheric, we can decide to express the pressure values as gauge pressure. This is a good idea everytime one of the two pressures is atmospheric.

$$P_{2,gauge} = 0$$

$$\boldsymbol{P_{1,gauge}} = \mathbf{15.7\ bar}$$

b) To calculate the force applied by the fluid on the control volume we apply the momentum balance

$$\overrightarrow{F_{rxn}} - p_1 A_1 \overrightarrow{n_1} - p_2 A_2 \overrightarrow{n_2} = \int_{A_1} \rho \overrightarrow{v_1}(\overrightarrow{v_1} \cdot \overrightarrow{n_1})\, dA_1 + \int_{A_2} \rho \overrightarrow{v_2}(\overrightarrow{v_2} \cdot \overrightarrow{n_2})\, dA_2$$

Projecting along the flow direction x (Remember the sign convention of the vector normal to the surface ! Module 2, slide 5):

$$F_{rxn} + p_1 A_1 - p_2 A_2 = -\rho v_1^2 A_1 + \rho v_2^2 A_2$$

If we use gauge pressure, we can eliminate p_2

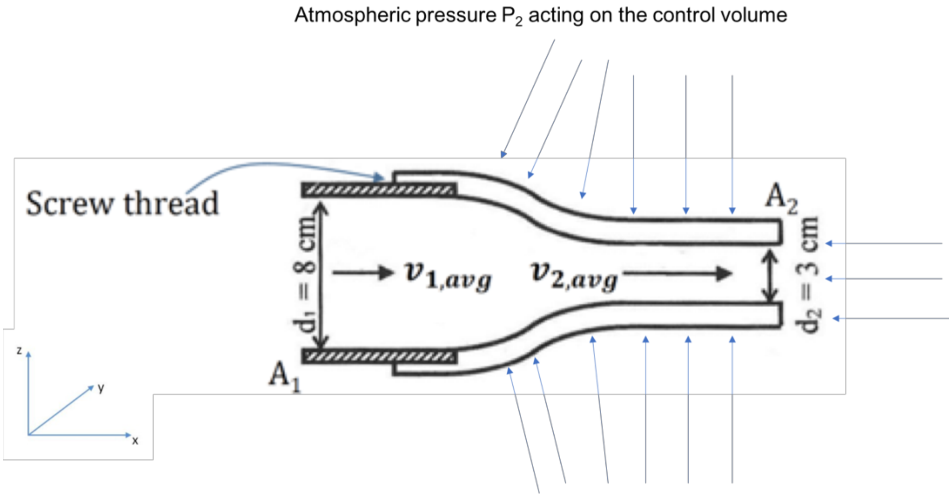
$$F_{rxn} = \rho(v_2^2 A_2 - v_1^2 A_1) - p_{1,gauge} A_1$$
$$= 1000 \times (56.6^2 \times 7.07 \times 10^{-4} - 7.96^2 \times 5.03 \times 10^{-3})$$
$$- 15.7 \times 10^5 \times 5.03 \times 10^{-3} = -5956\ N$$

This is the force of the screw on the water. Thus, the force from the water on the screw is $-F_{rxn} = 5956\ N$

Some of you may wonder, what if we were using the absolute pressure instead of the gauge pressure ? Here the answer!

$$F_{rxn} = \rho(v_2^2 A_2 - v_1^2 A_1) - p_1 A_1 + p_2 A_2^*$$

Which area A_2^* should we consider ? The interesting phenomenon here, is that the atmospheric pressure p_2 acts on the entire control volume as shown in the figure below.



Hence we need to consider an area A_2^* for the atmospheric pressure P_2 and not just A_2. Due to the irregular shape of the control volume, P_2 is acting on every elemental area in a different direction.

So the total contribution to the force from p_2 =

$$\int p_2 \, . \, dA_2^*$$

Despite the shape of the control volume being complicated, we can see that the *y* and *z* components of the atmospheric pressure on the control volume will cancel out. The *x* component is the only one we need to calculate.

Hence, the integration turns out to be

$$p_2 \int_x dA_2^*$$

Interestingly the x component of A_2^* happens to be A_1! So we get exactly the same expression as using the gauge pressure.

$$\begin{aligned} F_{rxn} &= \rho(v_2^2 A_2 - v_1^2 A_1) - p_1 A_1 + p_2 A_1 = (v_2^2 A_2 - v_1^2 A_1) - p_1 A_1 + p_2 A_1 \\ &= (v_2^2 A_2 - v_1^2 A_1) - (p_1 - p_2) A_1 \\ &= 1000 \times (56.6^2 \times 7.07 \times 10^{-4} - 7.96^2 \times 5.03 \times 10^{-3}) \\ &\quad - 15.7 \times 10^5 \times 5.03 \times 10^{-3} = -5956\ N \end{aligned}$$

Solution 2.3

Based on the definition of pressure head, we start by converting the pressure drop in standard units :

$$\Delta P = \rho_{huile} g h_{huile}$$

$$\Delta P = 850 * 9.81 * 1$$

$$\boldsymbol{\Delta P = p_1 - p_2 = 8338\ Pa}$$

If we neglet the gravitational, and friction forces, the momentum balance gives :

$$\overrightarrow{F_{rxn}} - p_1 A_1 \overrightarrow{n_1} - p_2 A_2 \overrightarrow{n_2} = \int_{A_1} \rho \overrightarrow{v_1} (\overrightarrow{v_1} \cdot \overrightarrow{n_1})\, dA_1 + \int_{A_2} \rho \overrightarrow{v_2} (\overrightarrow{v_2} \cdot \overrightarrow{n_2})\, dA_2$$

Projecting along x :

$$F_{rxn,x} + p_1 A_1 = -\rho v_1^2 A_1$$

$$F_{rxn,x} = -\frac{\rho \dot{V}^2}{A_1} - p_1 A_1$$

$$F_{rxn,x} = -850 * \frac{0.9^2}{\pi * 0.3^2} - 300 * 10^3 * \pi * 0.3^2$$

$$\boldsymbol{F_{rxn,x} = -87.26\ kN}$$

Projecting along y :

$$F_{rxn,y} - p_2 A_2 = \rho v_2^2 A_2$$

$$F_{rxn,y} = \frac{\rho \dot{V}^2}{A_2} + p_2 A_2$$

$$F_{rxn,y} = 850 * \frac{0.9^2}{\pi * 0.3^2} + 291660 * \pi * 0.3^2$$

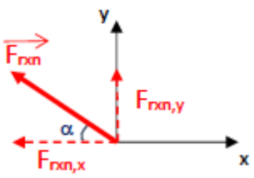
$$\boldsymbol{F_{rxn,y} = 84.9\ kN}$$

Therefore, the inensity of the reaction force is:

$$\|\overrightarrow{F_{rxn}}\| = \sqrt{(F_{rxn,x})^2 + (F_{rxn,y})^2}$$

$$\|\overrightarrow{F_{rxn}}\| = \sqrt{87.26^2 + 84.9^2}$$

$$\boxed{\|\overrightarrow{F_{rxn}}\| = 121.7\ kN}$$



And it is directed along the angle :

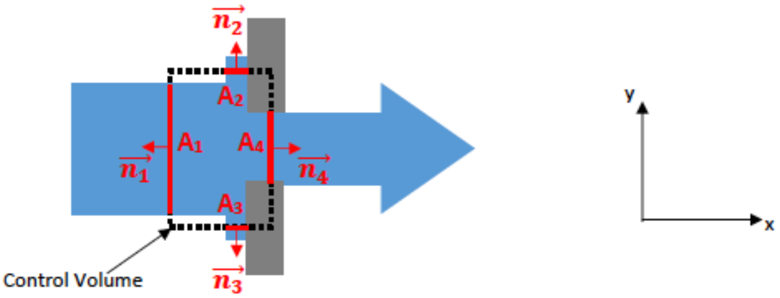
$$\tan \alpha = \frac{F_{rxn,y}}{-F_{rxn,x}}$$

$$\boxed{\alpha = \arctan\left(\frac{84.9}{87.26}\right) = 44.2°}$$

Note : We neglected the friction forces, while they are the very cause of the pressure drop (!)... To justify this, we can estimate these forces :

$$F_{friction} = \Delta P * A_1 = 8338 * \pi * 0.3^2 = \boldsymbol{2.36\ kN} \ll \boldsymbol{F_{rxn}}$$

Solution 2.4



In this system, all the fluid is at atmospheric pressure, therefore the sum of the forces applied on the control volume is equal to 0. Moreover, if we neglect the gravitational and friction forces, the momentum blance gives :

$$\overrightarrow{F_{rxn}} = \int_{A_1} \rho\overrightarrow{v_1}(\overrightarrow{v_1}\cdot\overrightarrow{n_1})\,dA_1 + \int_{A_2} \rho\overrightarrow{v_2}(\overrightarrow{v_2}\cdot\overrightarrow{n_2})\,dA_2 + \int_{A_3} \rho\overrightarrow{v_3}(\overrightarrow{v_3}\cdot\overrightarrow{n_3})\,dA_3 + \int_{A_4} \rho\overrightarrow{v_4}(\overrightarrow{v_4}\cdot\overrightarrow{n_4})\,dA_4$$

Projecting along x (horizontal) :

$$F_{rxn,x} = -\rho v_1^2 A_1 + \rho v_4^2 A_4$$

And $v_4 = v_1$(from Bernouilli equation), so :

$$F_{rxn,x} = \rho v_1^2 (A_4 - A_1)$$

$$F_{rxn,x} = 1000 * 25^2 * (\pi * 0.02^2 - \pi * 0.03^2)$$

$$\boldsymbol{F_{rxn,x} = -981\ N}$$

Moreover, the symmetry of the problem implies that the momentum through A_2 and A_3 are compensating, so that the total force along y is zero.

<u>Therefore, we need to apply 981 N to maintain the plate in position</u>