Introduction to Transport Phenomena: <u>Solutions Module 1</u>

<u>Solution Exercise 1.1</u>

We start by writing the Bernoulli's equation between point 1 and point 2

$$p_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

We have P_1, h_1, h_2 ; we have to find a way to extract the velocities from the data we have.

Inlet velocity: $v_1 = \frac{Q}{A_1} = \frac{0.035\frac{m^3}{s}}{0.0314m^2} = 1.114\frac{m}{s}$

Inlet area: $A_1 = \frac{\pi}{4}(0.2m)^2 = 0.0314m^2$

Outlet velocity: $\vartheta_2 = \frac{Q_V}{A_2} = \frac{0.035\frac{m^3}{s}}{0.00785m^2} = 4.456\frac{m}{s}$

Outlet area: $A_2 = \frac{\pi}{4}(0.1m)^2 = 0.00785m^2$

Inlet pressure in Pa: $p_1 = 39.24\frac{N}{cm^2} = 39.24\frac{10^4\ Pa}{N/cm^2} = 39.24 * 10^4\ Pa$

Input all the numbers to Bernoulli equation and solve the equation for the outlet pressure:

$$p_2 = p_1 + \rho g(h_1 - h_2) + \frac{1}{2}\rho(v_1^2 - v_2^2) = \mathbf{4.027 bar}$$

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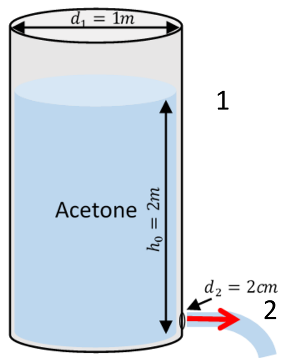
<u>Solution Exercise 1.2</u>

$$time = \frac{mass}{\dot{m}} \quad [1]$$



a) If we assume a constant average velocity, we can write:

$$\dot{m} = \rho A_2\, v_2$$

Since $A_1 \gg A_2$,

We may apply Torricelli's theorem,

$$v_2 = \sqrt{2gh_0}$$

As for the mass to be drained: $mass = \rho A_1 h_0$

Substituting in [1] $t = \frac{\rho A_1 h_0}{\rho A_2 v_2} = \frac{A_1 h_0}{\sqrt{2gh_0}.A_2} = 13.3\ min \quad [2]$

b) If we consider that the level of water drops over time, we need to consider *h* as a function of *t*, *h(t)*. The total mass of liquid to be drained does not change, instead the velocity becomes function of time

$$v_2 = -\frac{dh}{dt}\frac{A_1}{A_2} = \sqrt{2gh(t)}$$

The "minus" sign is there because the height of the liquid is decreasing over time.

As a result of this time dependence, we will integrate [2]

Integration limits for time (t): 0 to *t* (draining time)

Integration limits for height (h): h_o to 0.

$$\int_0^t dt = -\frac{A_1}{\sqrt{2g}.A_2}\int_{h_0}^{0}\frac{dh}{\sqrt{h}}$$

$$t = \frac{A_1}{\sqrt{2g}.A_2} \times 2\sqrt{h_o}$$

$$t = 26.6\,min$$

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Solution Exercise 1.3

$$Power\ output = Q\rho\, g\boldsymbol{H_p}$$

However, because we are given that the mechanical efficiency is 80%, we have to consider that the power input that we need to operate the pump will be given by

$$Power\ input\ = \frac{Q\rho\, g\boldsymbol{H_p}}{80} \times 100$$

We write the Bernoulli's equation in head terms:

$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + \boldsymbol{H_p} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g}$$

Velocities:

$$v_1 = \frac{Q}{A_1} = \frac{0.15}{0.3^2\frac{\pi}{4}} = 2.12\frac{m}{s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.15}{0.2^2\frac{\pi}{4}} = 4.78\frac{m}{s}$$

where $Q = \frac{9m^3}{min} = 0.15\ \frac{m^3}{s}$

Heights:

We take $h_1 = 0$ (reference), thus $h_2 = 1.22\ m$

Pressures:
$P_1 = p_{atm} - 2.63 * 10^4\ Pa$
$P_2 = p_{atm} + 0.7 * 10^5\ Pa$

We can then calculate $\boldsymbol{H_p}$

$$H_p = (h_2 - h_1) + \frac{P_2 - P_1}{\rho g} + \frac{(v_2^2 - v_1^2\ )}{2g} = 1.22 + \frac{9.63 * 10^4\ Pa}{10^3 \frac{kg}{m^3} * 9.81 \frac{m}{s^2}} + \frac{\left(4.78 \frac{m}{s}\right)^2 - \left(2.12 \frac{m}{s}\right)^2}{2 * 9.81 \frac{m}{s^2}} = 12m$$

We substitute in:

$$Power\ input\ = \frac{Q\rho\ g\boldsymbol{H_p}}{80} \times 100 = \frac{12m * 10^3 \frac{kg}{m^3} * 9.81 \frac{m}{s^2} * 0.15 \frac{m}{s}}{0.8} = 2.2 * 10^4 W = \boldsymbol{22kW}$$

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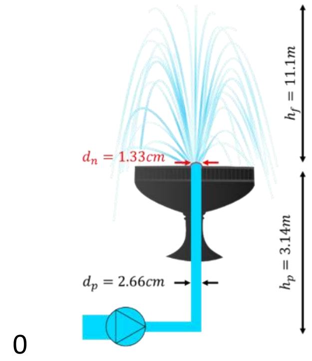
Solution Exercise 1.4



Applying Bernoulli's equation between point 0 and 1:

$$P_0 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 \quad [1]$$

Considering that the diameter before and after the pump is the same,
We get:

$$P_0 + P_{pump} = P_1$$

We apply the Bernoulli's equation between point 1 and point 2:

$$P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2 \quad [2]$$

Pressures:
$P_0 = P_2 = P_{atm} = 1atm$

Heights:
$h_1 = 0; h_2 = h_p = 3.14\, m$

Velocities:

From the continuity equation $A_1 v_1 = A_2 v_2$, we can write that $v_1 = \frac{A_2}{A_1} v_2 = \frac{d_n^2}{d_p^2} v_2$

We can now apply the Bernoulli's equation between point 2 and point 3

$$P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2 = P_3 + \rho g\, h_3 + \frac{1}{2}\rho v_3^2 \quad [3]$$

Pressures:
$P_2 = P_3 = P_{atm} = 1atm$

Heights:
$h_2 = 0; h_3 = h_f = 11.1\, m$

Velocities:
$v_3 = 0$! On top of the water jet the velocity is zero (TO REMEMBER)

Thus we can calculate

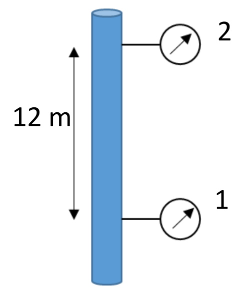
$$v_2 = \sqrt{2gh_f} = 14.757 \frac{m}{s}$$

Knowing v_2 we can calculate $v_1 = \frac{d_n^2}{d_p^2} v_2$ and insert the value in [2]

$$p_{pump} = \rho g\, h_p + \frac{1}{2}\rho v_2^2 \left(1 - \left(\frac{d_n}{d_p}\right)^4\right) = \mathbf{132.7kPa}$$

Solution Exercise 1.5

We apply Bernoulli's between point 1 and point 2, with $h_1 = 0$ and $h_2 = h = 12\ m$



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h + \frac{1}{2}\rho v_2^2$$

a) If the water is not flowing, it means $v = 0$ in both terms, thus

$$P_1 = P_2 + \rho g h$$

Implying that $P_1 > P_2$

However, we know both gauges read the same pressure. Hence, this hypothesis of stagnant water can be rejected. WATER IS FLOWING.

b) We go back to the Bernoulli's equation between point 1 and point 2, with $h_1 = 0$ and $h_2 = h = 12m$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h + \frac{1}{2}\rho v_2^2$$

Because the diameter of the pipe is constant, for the continuity equation, $v_1 = v_2$, so we obtain again

$$P_1 = P_2 + \rho g h$$

Once again implying that the pressure in the bottom gauge should be higher. But we know that in fact $P_1 = P_2$.

What are we missing? FRICTION LOSSES

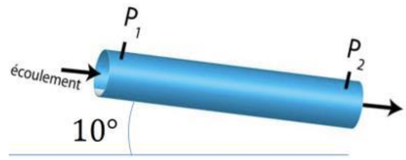
$$P_1 = P_2 + \rho g h + \Delta P_f$$

The condition $P_1 = P_2$ requires that

$$-\rho g h = \Delta P_f$$

Remember that friction opposes the movement of a fluid, friction having the opposite sign of gravity means that the fluid is flowing downwards

Solution 1.6



We apply the Bernoulli's equation between point 1 and 2, including friction:

$p_1 + \frac{\rho v^2}{2} + \rho g h_1 = p_2 + \frac{\rho v^2}{2} + \rho g h_2 + \frac{1}{2}\rho v_{avg}^2 4 \frac{f_f}{d} L$

Applying continuity equation at point 1 and 2, $A_1 v_1 = A_2 v_2$
Since $A_1 = A_2$, $v_1 = v_2$

$$p_1 + \frac{\rho v^2}{2} + \rho g h_1 = p_2 + \frac{\rho v^2}{2} + \rho g h_2 + \frac{1}{2}\rho v_{avg}^2 4 \frac{f_f}{d} L$$

The pressure drop can be expressed as:

$$\boldsymbol{p_1 - p_2 = \frac{1}{2}\rho v_{avg}^2 4 \frac{f_f}{d} L + \rho g (h_2 - h_1).}$$

In this equation, everything is known except the term f_f. To get this value from Moody's chart, we need to know if the flow is turbulent or laminar.

To know this, we need the Re number.
The flowrate through the pipe is given as Q

$$v_{avg} = \frac{Q}{A_2} = \frac{4Q}{\pi D^2} = 6.36 \frac{m}{s}$$

$$Re = \frac{\rho v_{avg} D}{\mu} = \frac{v_{avg} D}{v} = 1.3 * 10^5$$

From the Reynolds number it follows that we are in the turbulent regime (Re>2000)

The pipe is from cast-iron and the roughness factor $\varepsilon \approx 0.5 * 10^{-3} m$. Therefore $\frac{\varepsilon}{D} = 0.0025$.

From the Moody diagram we get:

$$f_f \approx 0.0065$$

The height h_1 can be calculated from:

$$h_1 = L \sin\alpha = 86.8m$$

$$h_2 - h_1 = -86.8m$$

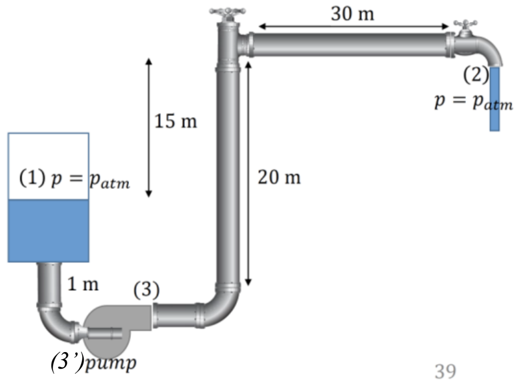
This gives a pressure drop of:

$$\rho g(h_2 - h_1) = -765.5\ kPa$$

$$\frac{1}{2}\rho v_{avg}^2 4\frac{f_f}{d}L = 1183.15\ kPa$$

$$\boldsymbol{p_1 - p_2 = 417.65\ kPa}$$

Solution 1.7



a) To calculate the pressure p_3 at the pump outlet we apply the Bernoulli's equation between point (3) and point (2)

Please note that (3) is after the pump inlet and (3') is before the pump inlet.

$$p_3 + \rho g h_3 + \frac{1}{2}\rho v_{avg}^2 = p_2 + \rho g h_2 + \frac{1}{2}\rho v_{avg}^2 + \frac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D} \sum_{i=1}^{2} L_i + \sum_{j=1}^{3} K_{Lj} \right)$$

$$p_3 - p_2 = \frac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D} \sum_{i=1}^{2} L_i + \sum_{j=1}^{3} K_{Lj} \right) + \rho g (h_2 - h_3)$$

We have

$$\sum_{i=1}^{2} L_i = 20m + 30m = 50m$$

$$\sum_{j=1}^{3} K_{Lj} = 0.3 + 2 + 0.05 + 0.3 = 2.65$$

f_f can be estimated from the Moody diagram, taking coefficient of kinematic viscosity as $\frac{\mu}{\rho} = 0.9 * 10^{-6} m^2/s$:

$$v_{avg} = \frac{4Q}{\pi D^2} = 5.3 \frac{m}{s}$$

$$Re = \frac{\rho v_{avg} D}{\mu} = 1.17 * 10^5$$

We are in turbulent regime, $\varepsilon = 0.015 * 10^{-3} m$ for the stainless steel and therefore $\frac{\varepsilon}{D} = 0.00075$.

Then from the diagram:

$$f_f = 0.0053$$

This gives:

$$p_3 - p_2 = \frac{1}{2} * 1000 * 5.3^2 * \left(4 * \frac{0.0053}{0.02} * 50 + 2.65\right) + 1000 * 9.81 * 20$$

$$\boldsymbol{p_3 - p_2 = 9.8 * 10^5 Pa}$$

Since the outlet is at atmospheric pressure:

$$p_2 = 1.013 * 10^5 Pa$$

And therefore:

$$\boldsymbol{p_3 = 1.08 * 10^6 Pa}$$

b)

We are given that the mechanical efficiency of the pump is 65 %, thus the power input that we need to operate the pump will be given by

$$Power\ input\ = \frac{Q\rho\ g\boldsymbol{H_p}}{0.65}$$

We have two ways to answer this question

(1)

$$\rho\ g\boldsymbol{H_p} = \Delta P_p = (p_3 -\ p_3')$$

which corresponds to the pressure drop that the pump has to compensate to keep the water flowing.

We just calculated p_3 , we need p_3'

We apply Bernoulli's between (1) and (3')

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_{avg}^2 = p_{3'} + \rho g h_{3'} + \frac{1}{2}\rho v_{avg}^2 + \frac{1}{2}\rho v_{avg}^2 \left(\frac{4 f_f}{D}\sum_{i=1}^{1} L_i + \sum_{j=1}^{1} K_{Lj}\right)$$

$$p_1 - p_{3'} = \frac{1}{2}\rho v_{avg}^2 * \left(\frac{4 f_f}{D}\sum_{i=1}^{1} L_i + \sum_{j=1}^{1} K_{Lj}\right) + \rho g(h_{3'} - h_1)$$

Worst case scenario, the tank is empty, and the water level corresponds to the bottom of the tank:

$$h_{3\prime} - h_1 = 1m$$

In this case:

$$\sum_{i=1}^{1} L_i = 1m$$

$$\sum_{j=1}^{1} K_{Lj} = 0.3$$

$$p_1 - p_{3'} = \frac{1}{2} * 1000 * 5.3^2 * \left(4 * \frac{0.021}{0.02} * 1 + 0.3\right) + 1000 * 9.81 * (-1)$$

$$p_1 - p_{3'} = 9291Pa$$

And with $p_1 = 1atm = 1.013 * 10^5 Pa$

$$\boldsymbol{p_{3'} = 9.2 * 10^4 Pa}$$

$$p_3 - p_{3'} = 9.88 * 10^5 Pa$$

$$\boldsymbol{Power\ input} = \frac{Q\rho\, g\boldsymbol{H_p}}{0.65} = \frac{Q*(p_3 - p_{3'})}{0.65} = \frac{\frac{100}{(1000*60)}*9.88*10^5}{0.65} = \boldsymbol{2432W}$$

(2)

The second approach to solve this problem is to apply the Bernoulli's equation between point 1 and point 2 and include all the pressure drop due to all the pipe elements. This gives very similar results.

$$p_1 + \rho g h_1 + \frac{1}{2}\rho v_{avg}^2 + \rho g H_{pump} = p_2 + \rho g h_2 + \frac{1}{2}\rho v_{avg}^2 + \frac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D}\sum_{i=1}^{3} L_i + \sum_{j=1}^{4} K_{Lj}\right)$$

$10^5 + 10^3 * 9.8 * 1 + 10^3 * 9.8 * H_{pump} = 10^5 + 10^3 * 9.8 * 20 + \frac{1}{2} * 10^3 * 5.3^2 \left(\frac{4*0.0053}{0.02}(51) + (0.3 + 0.3 + 2 + 0.05 + 0.3)\right)$

$$\boldsymbol{Power\ input} = \frac{Q\rho\, g\boldsymbol{H_p}}{0.65} = \frac{\frac{100}{(1000*60)}*1000*9.8*100.7}{0.65} = \boldsymbol{2531W}$$