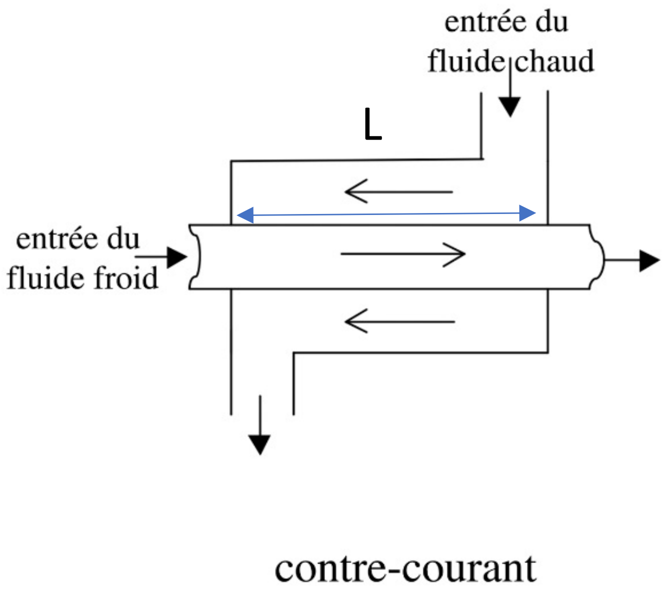
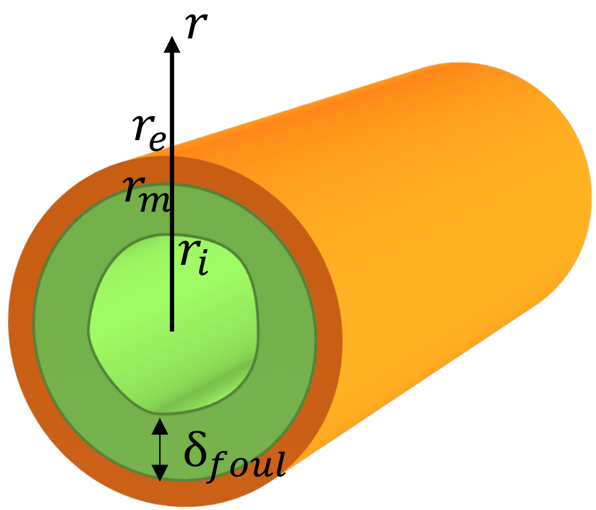
3.6. Heat transfer in composite systems: fouling



Example: A counter-current concentric tube heat exchanger made from copper is used to heat a stream of biological waste (before feeding to a digestion reactor). The inner tube initially has an external diameter of 10 mm and an internal diameter of 9 mm, however, the proteins in the biological waste slowly foul the inside of the copper pipe, building up a layer.

Write an expression for the overall heat transfer coefficient U_e (based on the external area of the internal tube) as a function of the thickness of the fouling layer.
Plot your result using reasonable values for the required parameters. Compare this with a linear approximation.



$$\frac{1}{r_e U_e} = \frac{1}{r_i h_i} + \frac{\ln(r_e/r_m)}{k_{Cu}} + \frac{\ln(r_m/r_i)}{k_{foul}} + \frac{1}{r_e h_e}$$

thickness of the fouling layer, $\delta_{foul} = r_m - r_i$ $\quad (r_i = r_m - \delta_{foul})$

$$\frac{1}{r_e U_e} = \frac{1}{(r_m - \delta_{foul})h_i} + \frac{\ln(r_e/r_m)}{k_{Cu}} + \frac{\ln\left(r_m/(r_m - \delta_{foul})\right)}{k_{foul}} + \frac{1}{r_e h_e}$$

$$\frac{1}{r_e U_e} = \frac{1}{(r_m - \delta_{foul})h_i} + \frac{\ln(r_e/r_m)}{k_{Cu}} + \frac{\ln\left(r_m/(r_m - \delta_{foul})\right)}{k_{foul}} + \frac{1}{r_e h_e}$$

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$$U_e = \frac{1}{r_e\left(\frac{1}{(r_m - \delta_{foul})h_i} + \frac{\ln(r_e/r_m)}{k_{Cu}} + \frac{\ln(r_m/(r_m - \delta_{foul}))}{k_{foul}} + \frac{1}{r_e h_e}\right)}$$

$$U_e = \frac{1}{0.010\left(\frac{1}{500(0.009 - \delta_{foul})} + \frac{\ln(0.010/0.009)}{385} + \frac{\ln(0.009/(0.009 - \delta_{foul}))}{0.2} + \frac{1}{10}\right)}$$

$r_e = 0.010$ m

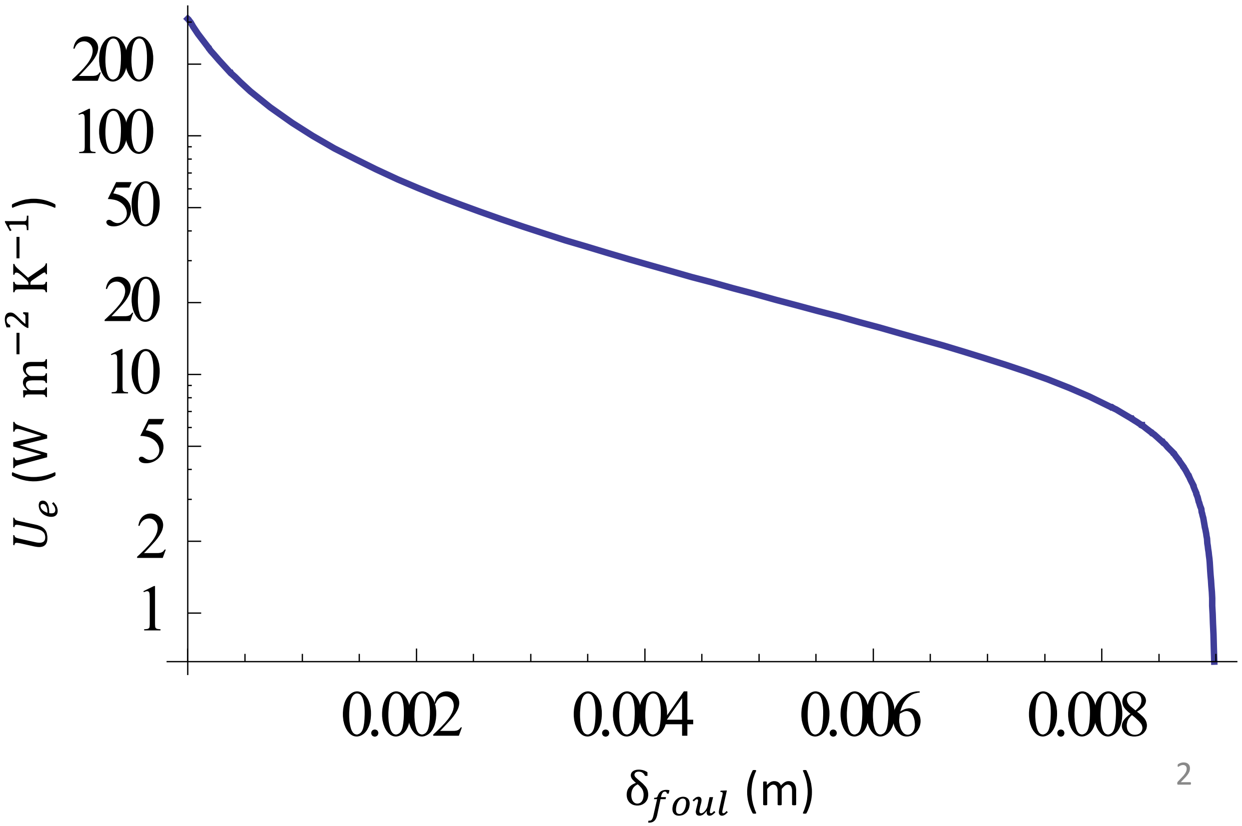
$r_m = 0.009$ m

$h_i = 500$ W m^{-2} K^{-1}

$k_{Cu} = 385$ W m^{-1} K^{-1}

$k_{foul} = 0.2$ W m^{-1} K^{-1}

$h_e = 1000$ W m^{-2} K^{-1}



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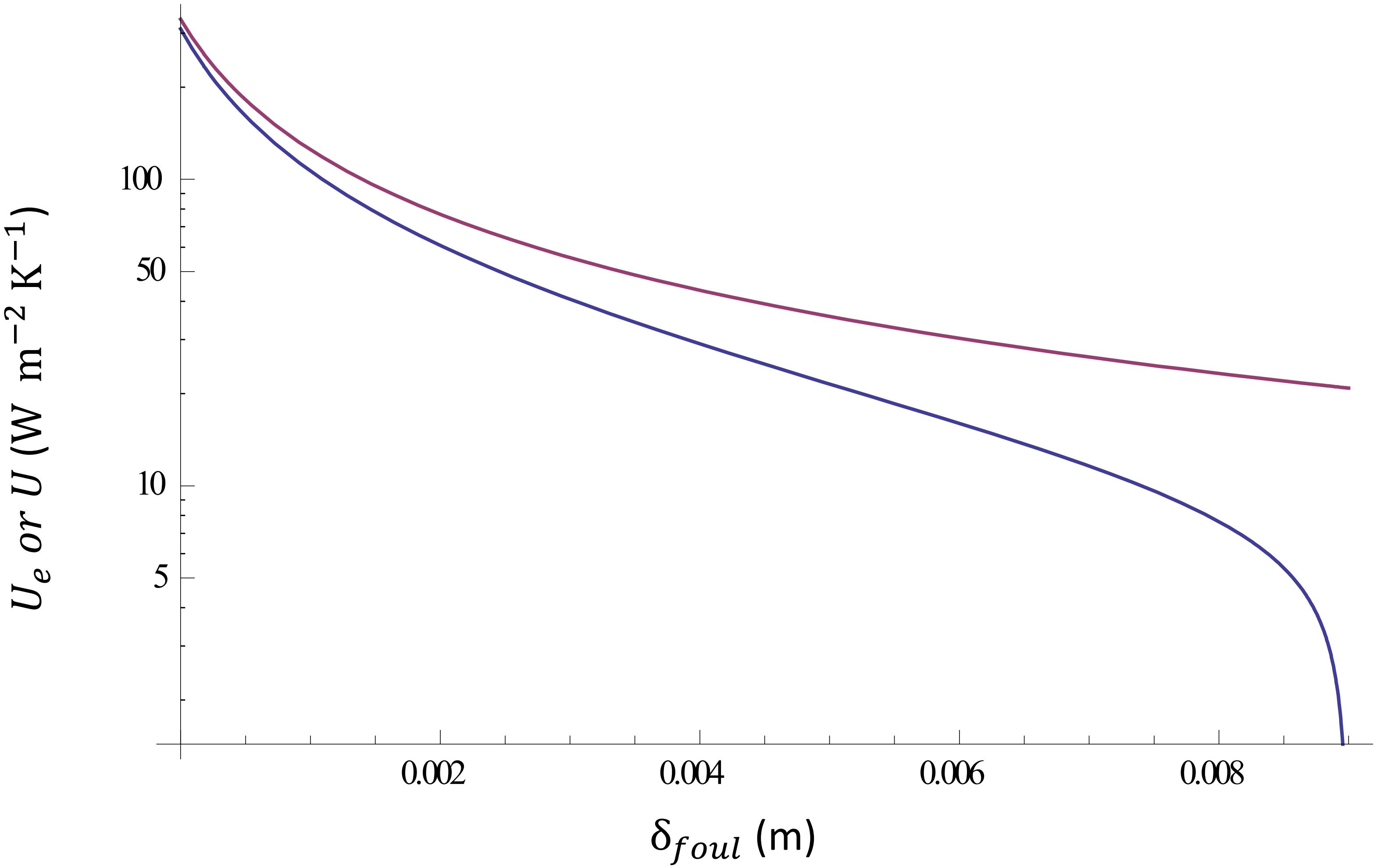
Linear approximation:

$$U = \frac{1}{\frac{1}{h_i} + \frac{(r_e - r_m)}{k_{Cu}} + \frac{\delta_{foul}}{k_{foul}} + \frac{1}{h_e}} \quad = \frac{1}{\frac{1}{500} + \frac{0.001}{385} + \frac{\delta_{foul}}{0.2} + \frac{1}{1000}}$$

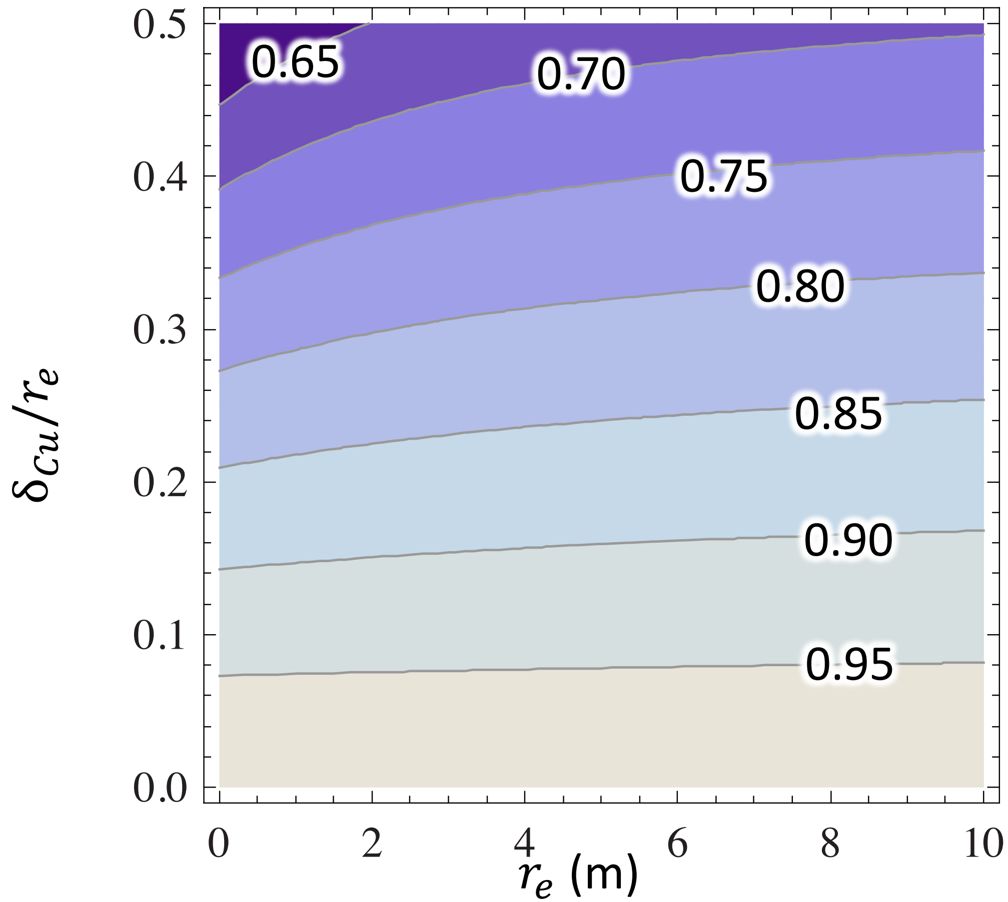


3.6. Heat transfer in composite systems

When can we use the Linear approximation?

Consider only the copper layer of different thickness, δ_{Cu} (no fouling layer)

$$\frac{U_e}{U} = \frac{\frac{1}{r_e h_i} + \frac{\delta_{Cu}/r_e}{k_{Cu}} + \frac{1}{r_e h_e}}{\left(\frac{1}{(1-\delta_{Cu}/r_e)h_i r_e} + \frac{\ln(1/(1-\delta_{Cu}/r_e))}{k_{Cu}} + \frac{1}{r_e h_e}\right)}$$



If δ_{Cu}/r_e < 0.1 the there is <~10% error on the linear approximation