Using a heat balance,

$$q = mc_p \Delta T$$
$$= 4.00(120)(505 - 500) = 2400 \text{ W} \qquad (4.5\text{-}16)$$

Substituting into Eq. (4.5-1),

$$\frac{q}{A} = \frac{2400}{A} = h_L(T_w - T) = 2512(30) = 75\,360 \text{ W/m}^2$$

Hence, $A = 2400/75\,360 = 3.185 \times 10^{-2}$ m^2. Then,

$$A = 3.185 \times 10^{-2} = \pi DL = \pi(0.05)(L)$$

Solving, $L = 0.203$ m.

4.5H Log Mean Temperature Difference and Varying Temperature Drop

Equations (4.5-1) and (4.3-12) as written apply only when the temperature drop $(T_i - T_o)$ is constant for all parts of the heating surface. Hence, the equation

$$q = U_i A_i (T_i - T_o) = U_o A_o (T_i - T_o) = UA(\Delta T) \qquad (4.5\text{-}17)$$

only holds at one point in the apparatus when the fluids are being heated or cooled. However, as the fluids travel through the heat exchanger, they become heated or cooled and both T_i and T_o or either T_i and T_o vary. Then $(T_i - T_o)$ or ΔT varies with position, and some mean ΔT_m must be used over the whole apparatus.

In a typical heat exchanger a hot fluid inside a pipe is cooled from T'_1 to T'_2 by a cold fluid which is flowing on the outside in a double pipe countercurrently (in the reverse direction) and is heated from T_2 to T_1 as shown in Fig. 4.5-3a. The ΔT shown is varying with distance. Hence, ΔT in Eq. (4.5-17) varies as the area A goes from 0 at the inlet to A at the outlet of the exchanger.

For countercurrent flow of the two fluids as in Fig. 4.5-3a, the heat-transfer rate is

$$q = UA\,\Delta T_m \qquad (4.5\text{-}18)$$

where ΔT_m is a suitable mean temperature difference to be determined. For a dA area, a heat balance on the hot and the cold fluids gives

$$dq = -m'c'_p\,dT' = mc_p\,dT \qquad (4.5\text{-}19)$$

where m is flow rate in kg/s. The values of m, m', c_p, c'_p, and U are assumed constant.

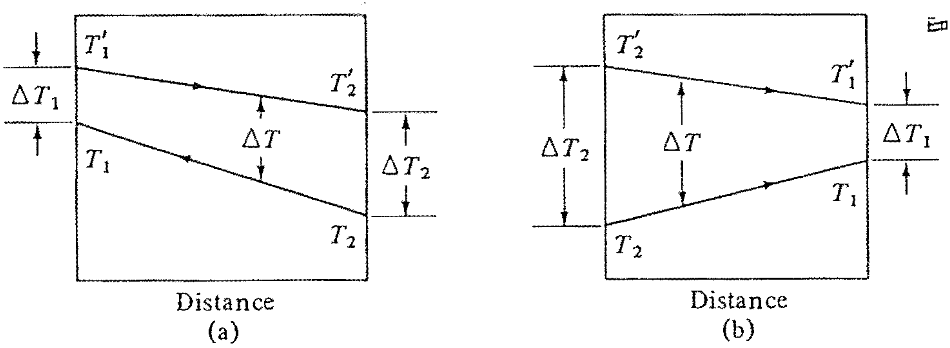

FIGURE 4.5-3. *Temperature profiles for one-pass double-pipe heat exchangers: (a) countercurrent flow; (b) cocurrent or parallel flow.*

Also,

$$dq = U(T' - T)\,dA \tag{4.5-20}$$

From Eq. (4.5-19), $dT' = -dq/m'c'_p$ and $dT = dq/mc_p$. Then,

$$dT' - dT = d(T' - T) = -dq\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right) \tag{4.5-21}$$

Substituting Eq. (4.5-20) into (4.5-21),

$$\frac{d(T' - T)}{T' - T} = -U\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right) dA \tag{4.5-22}$$

Integrating between points 1 and 2,

$$\ln\left(\frac{T'_2 - T_2}{T'_1 - T_1}\right) = -UA\left(\frac{1}{m'c'_p} + \frac{1}{mc_p}\right) \tag{4.5-23}$$

Making a heat balance between the inlet and outlet,

$$q = m'c'_p(T'_1 - T'_2) = mc_p(T_2 - T_1) \tag{4.5-24}$$

Solving for $m'c'_p$ and mc_p in Eq. (4.5-24) and substituting into Eq. (4.5-23),

$$q = \frac{UA[(T'_2 - T_2) - (T'_1 - T_1)]}{\ln[(T'_2 - T_2)/(T'_1 - T_1)]} \tag{4.5-25}$$

Comparing Eqs. (4.5-18) and (4.5-25), we see that ΔT_m is the log mean temperature difference ΔT_{lm}. Hence, in the case where the overall heat-transfer coefficient U is constant throughout the equipment and the heat capacity of each fluid is constant, the proper temperature driving force to use over the entire apparatus is the log mean driving force,

$$q = UA\Delta T_{lm} \tag{4.5-26}$$

where,

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} \tag{4.5-27}$$

It can be also shown that for parallel flow as pictured in Fig. 4.5-3b, the log mean temperature difference should be used. In some cases where steam is condensing, T'_1 and T'_2 may be the same. The equations still hold for this case. When U varies with distance or other complicating factors occur, other references should be consulted (B2, P3, W1).

EXAMPLE 4.5-4. Heat-Transfer Area and Log Mean Temperature Difference

A heavy hydrocarbon oil which has a $c_{pm} = 2.30$ kJ/kg · K is being cooled in a heat exchanger from 371.9 K to 349.7 K and flows inside the tube at a rate of 3630 kg/h. A flow of 1450 kg water/h enters at 288.6 K for cooling and flows outside the tube.

(a) Calculate the water outlet temperature and heat-transfer area if the overall $U_i = 340$ W/m^2 · K and the streams are countercurrent.

(b) Repeat for parallel flow.

Solution: Assume a $c_{pm} = 4.187$ kJ/kg · K for water. The water inlet $T_2 =$ 288.6 K, outlet $= T_1$; oil inlet $T'_1 = 371.9$, outlet $T'_2 = 349.7$ K. Calculating the heat lost by the oil,

$$q = \left(3630\,\frac{\text{kg}}{\text{h}}\right)\left(2.30\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(371.9 - 349.7)\text{K}$$

$$= 185\,400 \text{ kJ/h} \quad \text{or} \quad 51\,490 \text{ W } (175\,700 \text{ btu/h})$$

By a heat balance, the q must also equal the heat gained by the water.

$$q = 185\,400 \text{ kJ/h} = \left(1450 \frac{\text{kg}}{\text{h}}\right)\left(4.187 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(T_1 - 288.6) \text{ K}$$

Solving, $T_1 = 319.1$ K.

To solve for the log mean temperature difference, $\Delta T_2 = T_2' - T_2 = 349.7 - 288.6 = 61.1$ K, $\Delta T_1 = T_1' - T_1 = 371.9 - 319.1 = 52.8$ K. Substituting into Eq. (4.5-27),

$$\Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \frac{61.1 - 52.8}{\ln(61.1/52.8)} = 56.9 \text{ K}$$

Using Eq. (4.5-26),

$$q = U_i A_i \Delta T_{\text{lm}}$$

$$51\,490 = 340(A_i)(56.9)$$

Solving, $A_i = 2.66 \text{ m}^2$.

For part (b), the water outlet is still $T_1 = 319.1$ K. Referring to Fig. 4.5-3b, $\Delta T_2 = 371.9 - 288.6 = 83.3$ K and $\Delta T_1 = 349.7 - 319.1 = 30.6$ K. Again, using Eq. (4.5-27) and solving, $\Delta T_{\text{lm}} = 52.7$ K. Substituting into Eq. (4.5-26), $A_i = 2.87 \text{ m}^2$. This is a larger area than for counterflow. This occurs because counterflow gives larger temperature driving forces and is usually preferred over parallel flow for this reason.

EXAMPLE 4.5-5. Laminar Heat Transfer and Trial and Error

A hydrocarbon oil at 150°F enters inside a pipe with an inside diameter of 0.0303 ft and a length of 15 ft with a flow rate of 80 lb_m/h. The inside pipe surface is assumed constant at 350°F since steam is condensing outside the pipe wall and has a very large heat-transfer coefficient. The properties of the oil are $c_{pm} = 0.50$ btu/$\text{lb}_m \cdot$°F and $k_m = 0.083$ btu/h · ft · °F. The viscosity of the oil varies with temperature as follows: 150°F, 6.50 cp; 200°F, 5.05 cp; 250°F, 3.80 cp; 300°F, 2.82 cp; 350°F, 1.95 cp. Predict the heat-transfer coefficient and the oil outlet temperature, T_{bo}.

Solution: This is a trial-and-error solution since the outlet temperature of the oil T_{bo} is unknown. The value of $T_{bo} = 250$°F will be assumed and checked later. The bulk mean temperature of the oil to use for the physical properties is (150 + 250)/2 or 200°F. The viscosity at 200°F is

$$\mu_b = 5.05(2.4191) = 12.23 \frac{\text{lb}_m}{\text{ft}\cdot\text{h}}$$

At the wall temperature of 350°F,

$$\mu_w = 1.95(2.4191) = 4.72 \frac{\text{lb}_m}{\text{ft}\cdot\text{h}}$$

The cross-section area of the pipe A is

$$A = \frac{\pi D_i^2}{4} = \frac{\pi(0.0303)^2}{4} = 0.000722 \text{ ft}^2$$

$$G = \frac{m}{A} = \frac{80 \text{ lb}_m/\text{h}}{0.000722 \text{ ft}^2} = 111\,000 \frac{\text{lb}_m}{\text{ft}^2\cdot\text{h}}$$

The Reynolds number at the bulk mean temperature is

$$N_{\text{Re}} = \frac{D_i v \rho}{\mu} = \frac{D_i G}{\mu} = \frac{0.0303(111\,000)}{12.23} = 275.5$$