ChE 204
Introduction to Transport Phenomena

Module 5

Microscopic and Molecular Transport of Momentum:

Newton's law of viscosity

5.0. Modes of momentum transport

5.1. Viscosity at the molecular level

5.2 Newton's law of viscosity

5.3 Analogies in heat, mass and momentum transport

5.4 Non-dimensional numbers for simultaneous transport

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Module 5

Microscopic and Molecular Transport of Momentum:

Newton's law of viscosity

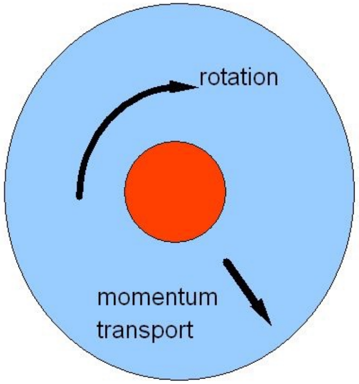
Objectives of this module:

- To understand the meaning of viscosity at the macro and at the molecular scale
- To describe the transport modes for momentum
- To understand and apply the Newton's law of viscosity

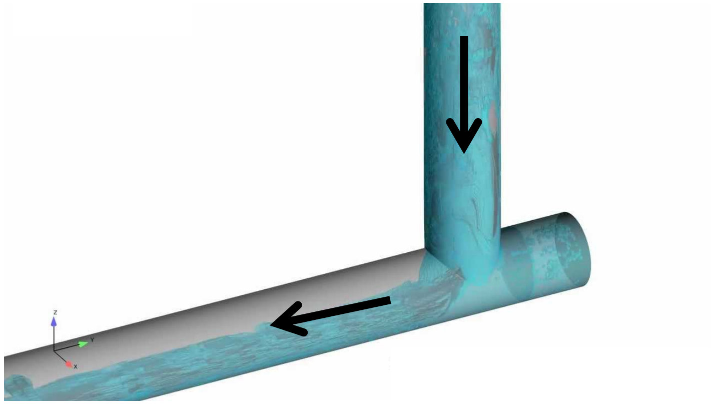
5.0. Modes of momentum transport

Momentum is transported every time there is a mass moving

- Radial transport of momentum



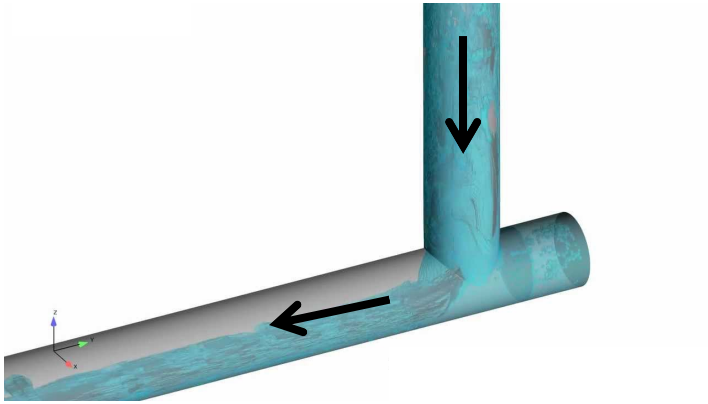
- Advective momentum transport (transport by bulk motion of the fluid)



5.0. Modes of momentum transport

Momentum is transported every time there is a mass moving

- Advective momentum transport (transport by bulk motion of the fluid)



See Module 2 for macroscopic description

advective momentum transport ⟶ fluid density ρ

molecular momentum transport ⟶ dynamic viscosity μ

5.0. Modes of momentum transport and viscosity

DEFINITION OF FLUID DENSITY

$$\rho = \frac{mass}{Volume} \quad \left[\frac{kg}{m^3}\right] \quad \left[\frac{g}{cm^3}\right] \quad \left[\frac{kg}{L}\right]$$

It is a macroscopic property!

DEFINITION OF DYNAMIC VISCOSITY

Viscosity is the property of the fluid which defines the interaction between the moving molecules within the fluid. It measures the resistance of the fluid to flow. Viscous forces are due to intermolecular forces acting in the fluid.

$$\mu \qquad [Pa \cdot s]$$

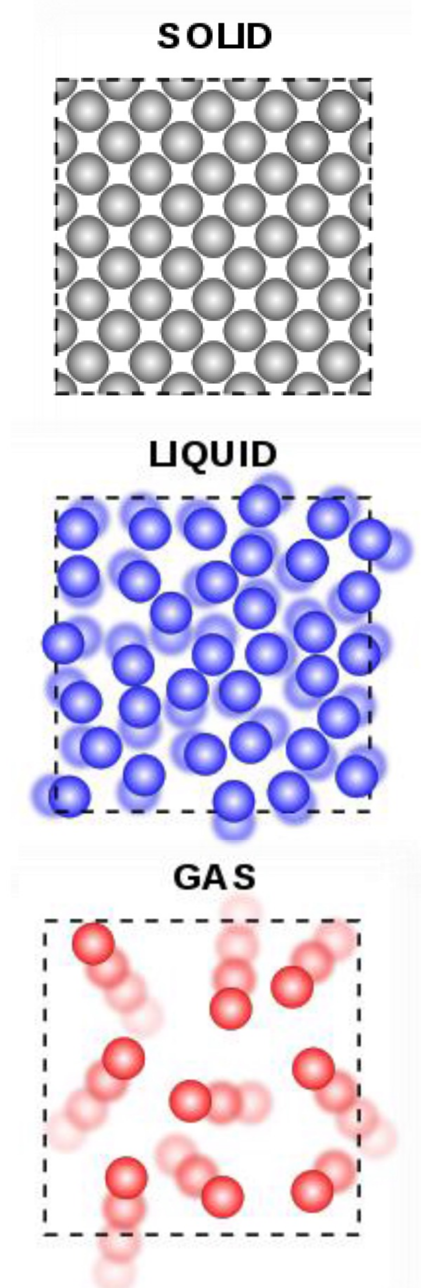
It is a microscopic property!

5.1. Viscosity at the molecular level

With an increase of temperature, there is typically an increase in the molecular interchange as molecules move faster at higher temperatures. $KE_{avg} = \frac{1}{2}m\bar{u}^2 = \frac{3}{2}k_B T$

The gas viscosity will increase with temperature. According to the kinetic theory of gases, viscosity should be proportional to the square root of the absolute temperature. In practice, it increases much more rapidly.

In a liquid there will be *molecular interchange* similar to those developed in a gas, but there are *additional substantial attractive*, cohesive forces between the molecules of a liquid. Both cohesion and molecular interchange contribute to liquid viscosity. The impact of increasing the temperature of a liquid is to reduce the cohesive forces while simultaneously increasing the rate of molecular interchange. The former effect causes a decrease in the shear stress while the latter causes it to increase. The result is that **liquids show a reduction in viscosity with increasing temperature.**



GASES

momentum is transfered via random collisions

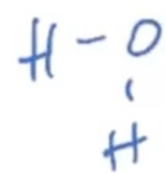
$\mu \uparrow \quad T \uparrow$

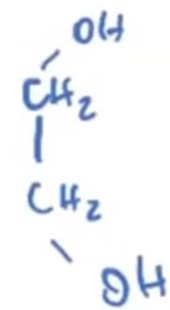
LIQUIDS

momentum is transfered by continuous breaking and reforming of chemical bonds / interaction

$T \uparrow \quad \mu \downarrow$

$\mu_{water} < \mu_{ethylene\ glycol}$





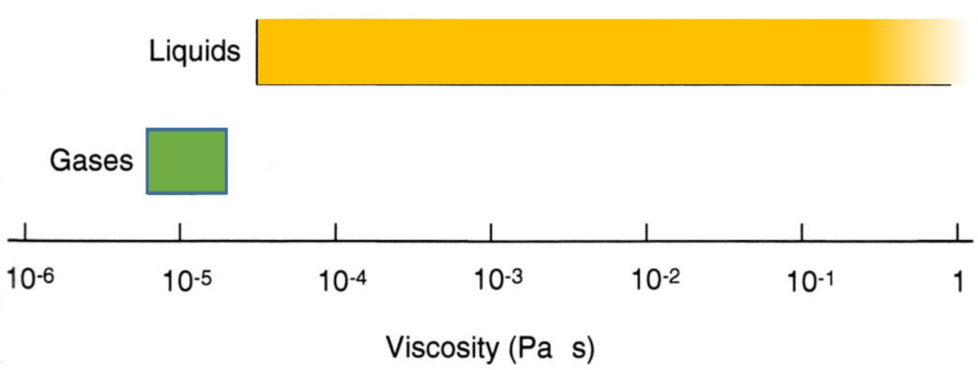
$\mu_{ethanol} < \mu_{ethylene\ glycol}$

5.1. Viscosity at the molecular level

With high temperatures, viscosity increases in gases and decreases in liquids.

Values of the Dynamic viscosity

$\mu = [\text{kg m}^{-1}\text{s}^{-1}] = [\text{N s m}^{-2}] = [\text{Pa s}]$



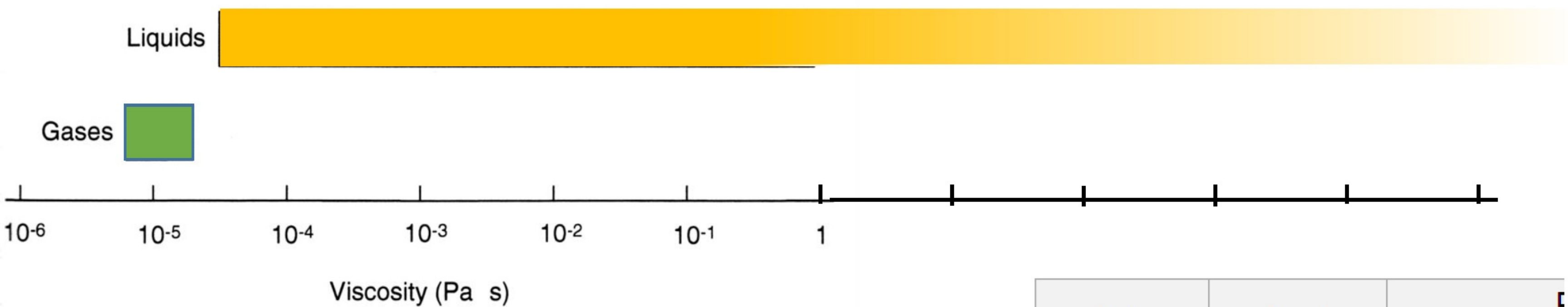
	water	air
Temperature T (°C)	Viscosity μ (mPa · s)	Viscosity μ (mPa · s)
0	1.787	0.01716
20	1.0019	0.01813
40	0.6530	0.01908
60	0.4665	0.01999
80	0.3548	0.02087
100	0.2821	0.02173

Gases	Temperature T (°C)	Viscosity μ (mPa · s)	Liquids	Temperature T (°C)	Viscosity μ (mPa · s)
i-C_4H_{10}	23	0.0076	$(C_2H_5)_2O$	0	0.283
SF_6	23	0.0153		25	0.224
CH_4	20	0.0109	C_6H_6	20	0.649
H_2O	100	0.01211	Br_2	25	0.744
CO_2	20	0.0146	Hg	20	1.552
N_2	20	0.0175	C_2H_5OH	0	1.786
O_2	20	0.0204		25	1.074
Hg	380	0.0654		50	0.694
			H_2SO_4	25	25.54
			Glycerol	25	934.

5.1. Viscosity at the molecular level

Values of the Dynamic viscosity

$$\mu = [\text{kg m}^{-1}\text{s}^{-1}] = [\text{N s m}^{-2}] = [\text{Pa s}]$$



Bitumen (pitch)

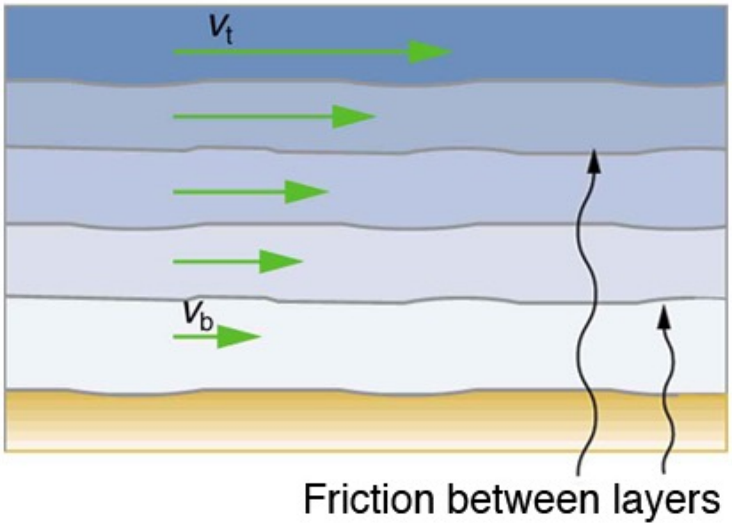
highly viscous liquid or semi-solid form of petroleum

$\mu \approx 2 \times 10^7$ Pa s (20°C)

Date	Event	D	
		Years	Months
1927	Hot pitch poured		
October 1930	Stem cut		
December 1938	1st drop fell	8.1	98
February 1947	2nd drop fell	8.2	99
April 1954	3rd drop fell	7.2	86
May 1962	4th drop fell	8.1	97
August 1970	5th drop fell	8.3	99
April 1979	6th drop fell	8.7	104
July 1988	7th drop fell	9.2	111
November 2000	8th drop fell[A]	12.3	148
April 2014	9th drop[B]	13.4	156

5.2. Newton's law of viscosity

For **laminar** flow

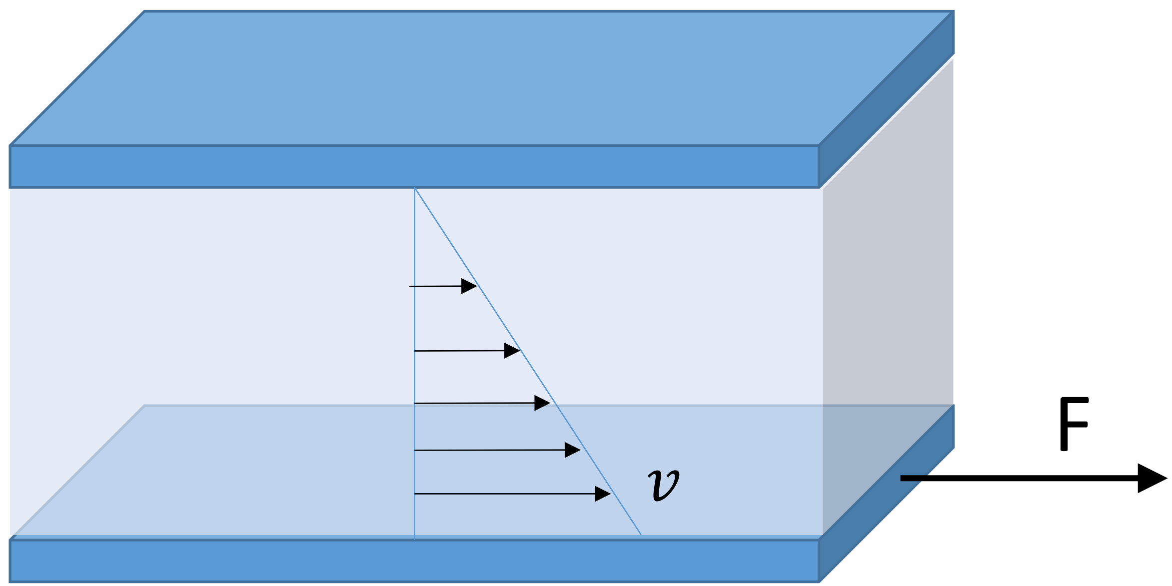


The fluid flows in parallel layers, with no disruption or mixing between layers

Friction= resistance to motion
Viscosity=resistance to flow, thus it is special case of friction

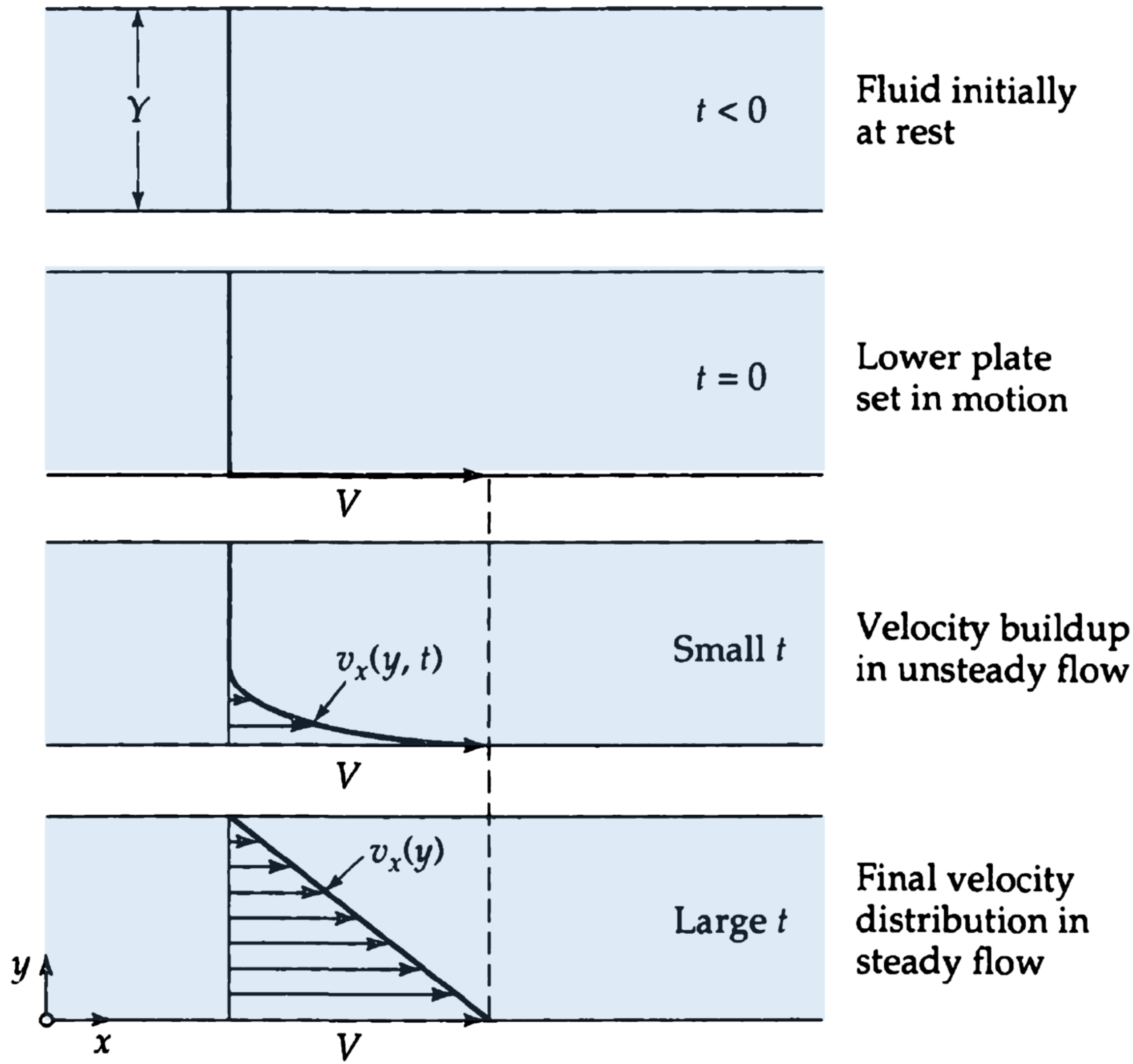
5.2. Newton's law of viscosity

Laminar flow between two plates



Let's assume we put the bottom plate in motion by applying the force F. Because of the friction between the plate and the fluid and the fluid layers, **momentum will be transferred perpendicularly to the direction of flow**.

5.2. Newton's law of viscosity



5.2. Newton's law of viscosity

At t large (steady state)

$$\frac{F}{A} \propto \frac{V}{Y}$$

$F = [\mathrm{kg\ m\ s^{-2}}]$ Force

$V = [\mathrm{m\ s^{-1}}]$ velocity

$A = [\mathrm{m^2}]$ Surface Area

F is the amount of force required to move the plate at uniform velocity

At steady state this force will be equal to the viscosity of the fluid, the area of the plate, inversely proportional to the thickness of the the fluid

$$F \uparrow \quad A \uparrow \quad \frac{v}{Y}$$

$$F \propto A \frac{v}{Y}$$

$$\frac{F}{A} = \mu \frac{V}{Y} \quad (1.5)$$

$\mu = [\mathrm{kg\ m^{-1} s^{-1}}]$

Dynamic viscosity

5.2. Newton's law of viscosity

At t large (steady state)

$$\frac{F}{A} \propto \frac{V}{Y}$$

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Dynamic viscosity

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$$\frac{F}{A} = \mu \frac{V}{Y} \quad (1.5)$$

$\mu = [\mathrm{kg\ m^{-1} s^{-1}}]$ Dynamic viscosity

The force is the momentum flow rate, thus $\frac{F}{A}$ is the momentum flux!

$$\overrightarrow{M} = m\,\vec{v}$$

$$\vec{F} = m\vec{a} = m\,\frac{\delta \vec{v}}{\delta t} = \frac{\delta m \vec{v}}{\delta t}$$

5.2. Newton's law of viscosity

At t large (steady state)

$$\frac{F}{A} \propto \frac{V}{Y}$$

$\mathrm{F} = [\mathrm{kg\ m\ s^{-2}}]$ Force

$V = [\mathrm{m\ s^{-1}}]$ velocity

$\mathrm{A} = [\mathrm{m^2}]$ Surface Area

$$\frac{F}{A} = \mu \frac{V}{Y} \quad (1.5)$$

$\mu = [\mathrm{kg\ m^{-1} s^{-1}}]$

Dynamic viscosity

Flux of x-momentum in the y-direction

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

The first subscription gives the transport direction while the second subscription gives the component of the momentum being transported.

$$\tau_{yx} = \frac{\mathrm{F}_{yx}}{\mathrm{A}} = [\mathrm{kg\ m^{-1} s^{-2}}]$$

Plate moving along –x and momentum being transported along -y

Note. The Newton's law of viscosity applies to Newtonian fluid, which are incompressible fluids with constant viscosity.

5.2. Newton's law of viscosity

The dual interpretation of τ_{yx}

$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y}$$

Viscous momentum flux

$$\tau_{yx} = \frac{F_{yx}}{A}$$

Viscous shear stress

Both are generated by the molecular forces during shear flow and are interchangeable.

5.2. Newton's law of viscosity

The shear stress τ is a tensor, which means that it will have 3 components per each direction

Flux of x-momentum

$$\tau_x = -\mu\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial z}\right) = \hat{x}\tau_{xx} + \hat{y}\tau_{yx} + \hat{z}\tau_{zx}$$

Flux of y-momentum

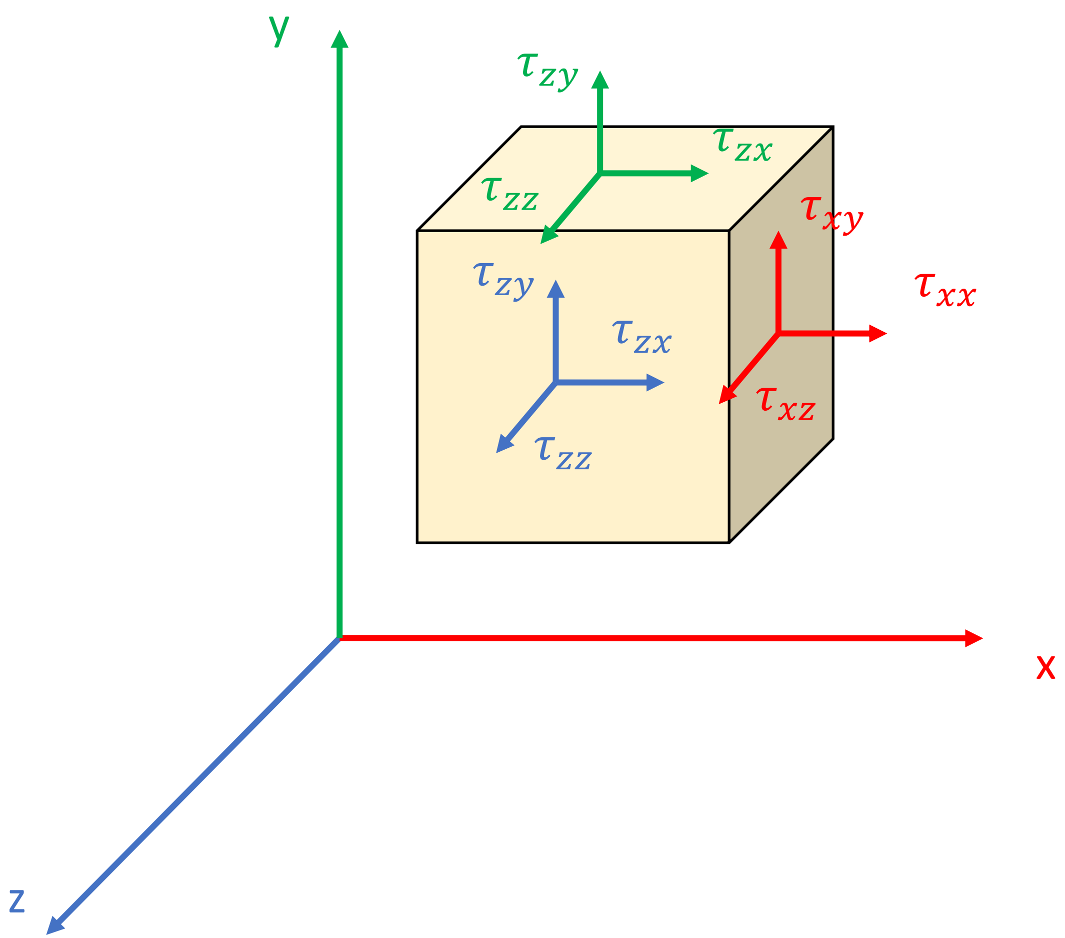
$$\tau_y = -\mu\left(\frac{\partial v_y}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial z}\right) = \hat{x}\tau_{xy} + \hat{y}\tau_{yy} + \hat{z}\tau_{zy}$$

Flux of z-momentum

$$\tau_z = -\mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_z}{\partial y} + \frac{\partial v_z}{\partial z}\right) = \hat{x}\tau_{xz} + \hat{y}\tau_{yz} + \hat{z}\tau_{zz}$$

5.2. Newton's law of viscosity

If we consider a cube ΔxΔyΔz:



Complete stress tensor

$$\boldsymbol{\tau} = \hat{\mathbf{x}}\boldsymbol{\tau}_x + \hat{\mathbf{y}}\boldsymbol{\tau}_y + \hat{\mathbf{z}}\boldsymbol{\tau}_z$$

$$= \hat{\mathbf{x}}\hat{\mathbf{x}}\tau_{xx} + \hat{\mathbf{x}}\hat{\mathbf{y}}\tau_{xy} + \hat{\mathbf{x}}\hat{\mathbf{z}}\tau_{xz}$$

$$+\hat{\mathbf{y}}\hat{\mathbf{x}}\tau_{yx} + \hat{\mathbf{y}}\hat{\mathbf{y}}\tau_{yy} + \hat{\mathbf{y}}\hat{\mathbf{z}}\tau_{yz}$$

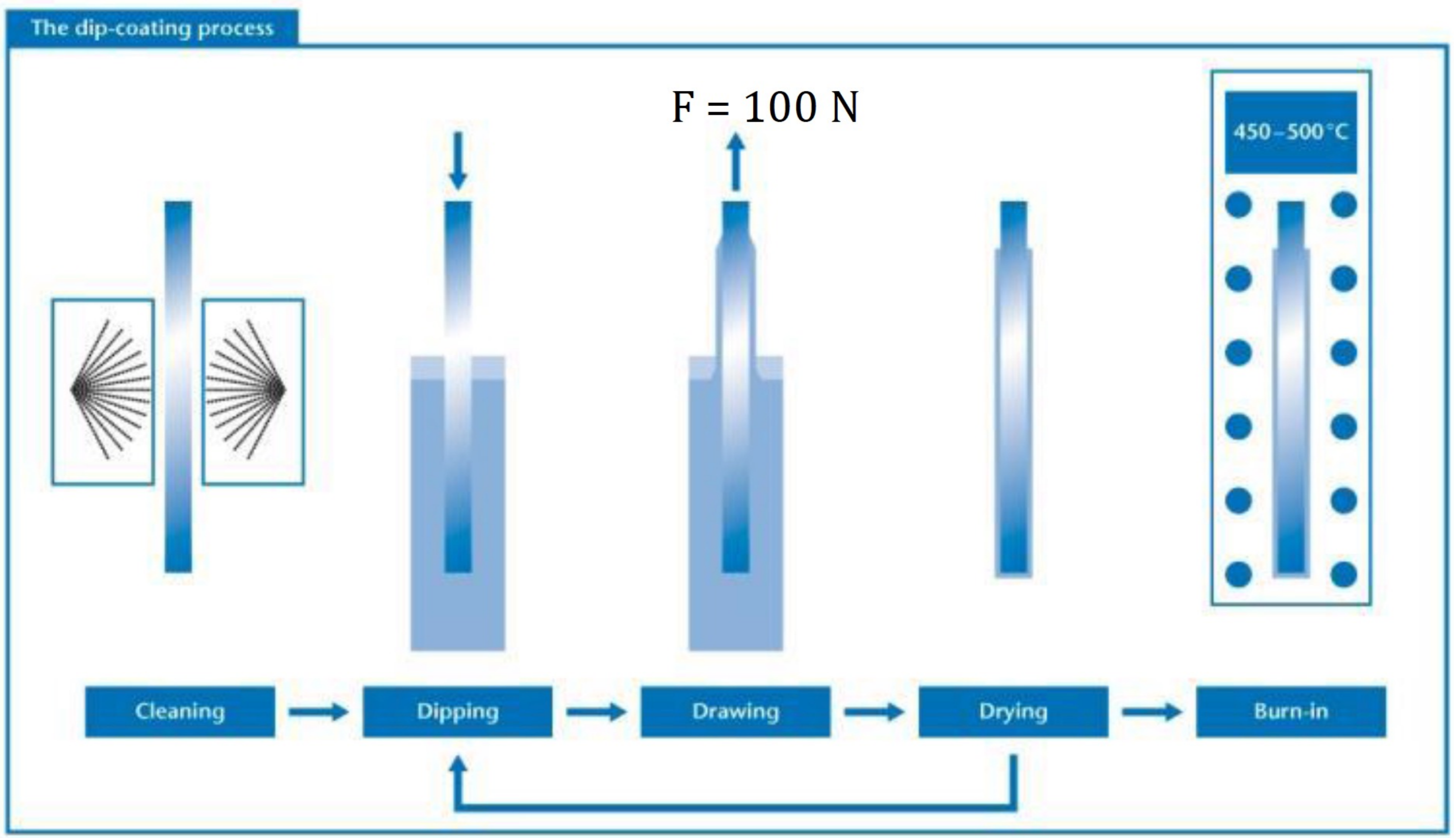
$$+\hat{\mathbf{z}}\hat{\mathbf{x}}\tau_{zx} + \hat{\mathbf{z}}\hat{\mathbf{y}}\tau_{zy} + \hat{\mathbf{z}}\hat{\mathbf{z}}\tau_{zz}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

More in Transport Phenomena I!

5.2. Newton's law of viscosity

Example: An industrial process for coating of concrete panels with glycerol at 25°C (density 920 $Kg\ m^{-3}$) uses a dip coating procedure. The panels (with dimensions of 1 m × 1 m × 0.005 m) have a density of 1260 $Kg\ m^{-3}$ and are removed from the glycerol container (which has a width of 0.025 m) with a force of 100 N. Can you estimate the maximum speed the panel can be removed (upward) while it is still submerged ?



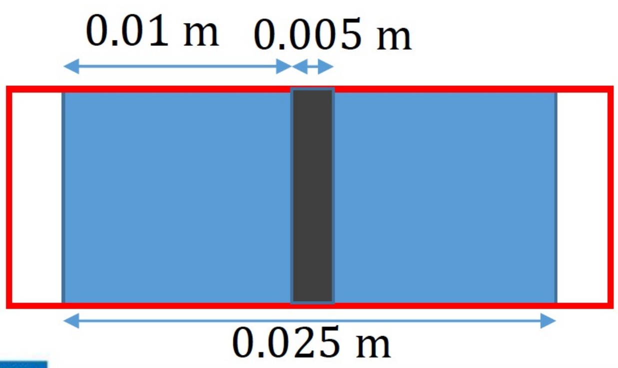
5.2. Newton's law of viscosity

Example: An industrial process for coating of concrete panels with glycerol at 25°C (density 920 kg m^{-3}) uses a dip coating procedure. The panels (with dimensions of 1 m × 1 m × 0.005 m) have a density of 1260kg m^{-3} and are removed from the glycerol container (which has a width of 0.025 m) with a force of 100 N. Can you estimate the maximum speed the panel can be removed (upward) while it is still submerged ?

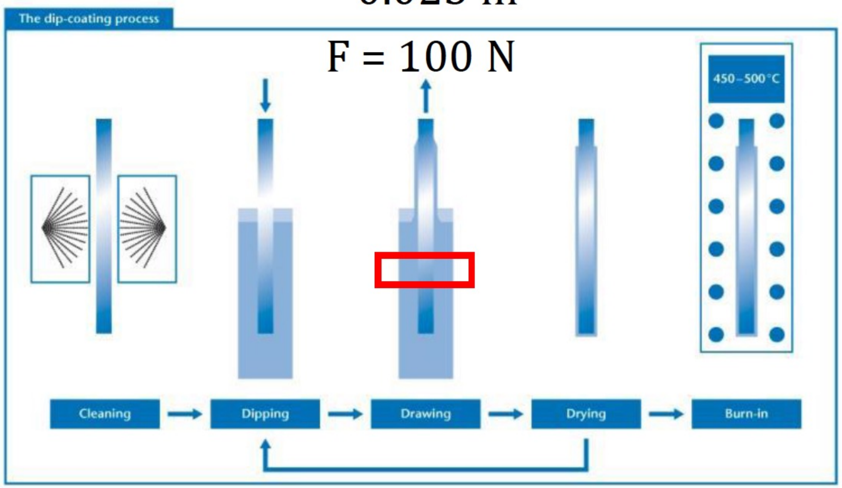
$$F_{total} = F_{gravity} - F_{buoyant} + F_{shear}$$ balance of forces

$$= mg - V\rho_{glycerol}g + A\,\mu\,\frac{v}{Y}$$

$$= V(\rho_{panel} - \rho_{glycerol})g + A\,\mu\,\frac{v}{Y}$$



$$100\text{ N} = (0.005\text{ m}^3)((1260\text{-}920)\text{ kg m}^{-3})(9.81\text{ m s}^{-2}) + (2\text{ m}^2)(0.934\text{ kg m}^{-1}\text{s}^{-1})V/(0.01\text{m})$$



$$v = 0.44\text{ m s}^{-1}$$

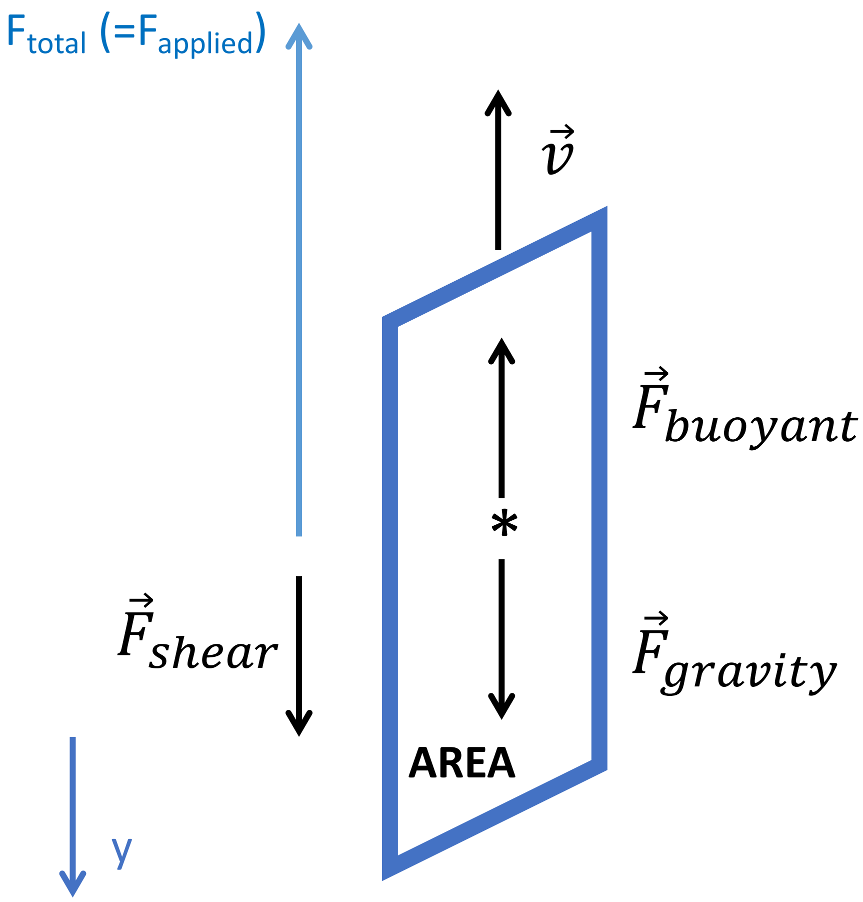
V= volume

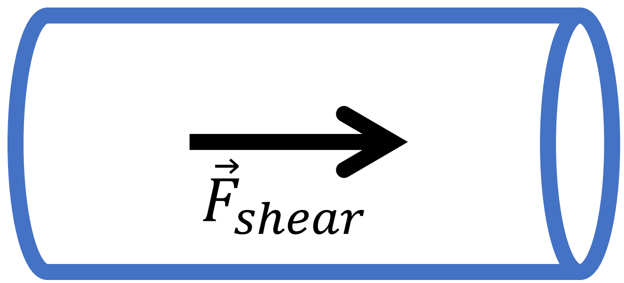
v = velocity

Note: 2m^2 because you consider the two sides of the block

5.2. Newton's law of viscosity

NOTE: The Area indicates the AREA PARALLEL TO THE DIRECTION OF THE SHEAR FORCE





AREA =
longitudinal area of the cylinder = 2πr $\times$ L

5.3. Analogies in heat, mass and momentum transport

Fourier's (1st) law of heat conduction

Flux density

[W m^{-2}]

$$q = -k\frac{dT}{dy}$$

→ ?

→ ?

Fick's (1st) law of molecular diffusion

[mol s^{-1}m^{-2}]

$$j_{A,y} = -\mathcal{D}_{AB}\frac{dc_A}{dy}$$

→ Concentration gradient [(mol m^{-3}) m^{-1}]

→ Diffusivity [m^{2}s^{-1}]

Newton's law of viscosity

[kg m^{-1}s^{-2}]

$$\tau_{yx} = -\mu\frac{dv_x}{dy}$$

→ ?

→ ?

General transport relation for "entity"

["entity" s^{-1}m^{-2}] →

$$\psi = -\delta\frac{d\Gamma}{dy}$$

→ Concentration gradient [("entity" m^{-3}) m^{-1}]

Flux density

→ Diffusivity [m^{2}s^{-1}]

5.3. Analogies in heat, mass and momentum transport

Flux density

$[\text{W m}^{-2}] = [\text{J s}^{-1}\text{m}^{-2}]$

Our "entity" is heat

Fourier's (1st) law of heat conduction

$$q = -k\frac{dT}{dy}$$

$$q = -\frac{k}{\rho c_p}\frac{d(\rho c_p T)}{dy}$$

$$q = -\alpha\frac{d(\rho c_p T)}{dy}$$

Concentration gradient of heat $[(\text{J m}^{-3})\ \text{m}^{-1}]$

$$\alpha = \frac{k}{\rho c_p}$$

Thermal diffusivity $[\text{m}^2\text{s}^{-1}]$

General transport relation for "entity"

$$\psi = -\delta\frac{d\Gamma}{dy}$$

$["\text{entity}"\ \text{s}^{-1}\text{m}^{-2}]$ → ψ

Concentration gradient $[("\text{entity}"\ \text{m}^{-3})\ \text{m}^{-1}]$

Diffusivity $[\text{m}^2\text{s}^{-1}]$

5.3. Analogies in heat, mass and momentum transport

Flux density

Newton's law of viscosity

$[\mathrm{kg\ m^{-1}s^{-2}}] = [(\mathrm{kg\ m\ s^{-1}})\ \mathrm{m^{-2}s^{-1}}]$

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

Our "entity" is momentum

$$\tau_{yx} = -\frac{\mu}{\rho}\frac{d(\rho v_x)}{dy}$$

$$\tau_{yx} = -\nu \frac{d(\rho v_x)}{dy}$$

Concentration gradient of momentum

Momentum Diffusivity $[\mathrm{m^2 s^{-1}}]$ OR Kinematic viscosity

General transport relation for "entity"

$["\mathrm{entity}"\ \mathrm{s^{-1}m^{-2}}]$

$$\psi = -\delta \frac{d\Gamma}{dy}$$

Concentration gradient $[("\mathrm{entity}"\ \mathrm{m^{-3}})\ \mathrm{m^{-1}}]$

Diffusivity $[\mathrm{m^2 s^{-1}}]$

5.3. Analogies in heat, mass and momentum transport

"entity"	Heat transport	Mass transport	Momentum transport
Flux	$q = [\mathrm{W\ m^{-2}}]$ $= [\mathrm{J\,s^{-1}m^{-2}}]$	$j = [\mathrm{mol\,s^{-1}m^{-2}}]$	$\tau = [\mathrm{kg\,m^{-1}s^{-2}}]$ $= [(\mathrm{kg\,m\,s^{-1}})\,\mathrm{m^{-2}s^{-1}}]$
Law	$q = -k\dfrac{dT}{dy}$	$j = -\mathcal{D}\dfrac{dc}{dy}$	$\tau = -\mu\dfrac{dv}{dy}$
Coefficient of proportionality	k	$\mathcal{D}$	μ
Driving force	$\dfrac{dT}{dy}$	$\dfrac{dc}{dy}$	$\dfrac{dv}{dy}$
Diffusivity	$\alpha = \dfrac{k}{\rho c_p}$	$\mathcal{D}$	$\nu = \dfrac{\mu}{\rho}$
Concentration	$\rho c_p T$	c	ρv
Concentration Gradient	$\dfrac{d(\rho c_p T)}{dy}$	$\dfrac{dc}{dy}$	$\dfrac{d\rho v}{dy}$

5.4. Non dimensional numbers for simultaneous transport

The **Reynold Number** $\boldsymbol{Re}$ quantifies the relative importance of inertial forces and viscous forces for given flow conditons

$$Re = \frac{inertial\ forces}{viscous\ forces} = \frac{\rho v_{avg} d}{\mu} = \frac{v_{avg} d}{\mathrm{v}}$$

$$\mathrm{v} = kinematic\ viscosity = \frac{\mu}{\rho}$$

ρ density of the fluid [Kg/m^3]

v_{avg} average velocity [m/s]

d tube diameter [m]

μ dynamic (or absolute) viscosity [Kg · m^{-1} · s^{-1} or Pa · s]

High Re (> 3500) Turbulent flow, inertia dominates over friction

Low Re (< 2000) Laminar flow, friction dominates over inertia

5.4. Non dimensional numbers for simultaneous transport

The ***Prandtl* Number *Pr*** defines the importance of momentum over thermal diffusivity

$$Pr = \frac{momentum\ diffusion\ rate}{heat\ diffusion\ rate} = \frac{\nu}{\alpha} = \frac{C_p \mu}{\mathrm{k}}$$

ν kinematic viscosity [m^2/s]

α thermal diffusivity [m^2/s]

μ dynamic viscosity [Kg · m^{-1} · s^{-1}]

C_p specific heat [J/Kg · K]

k thermal conductivity [W/m · K]

$Pr_{gas} \approx 0.7 - 1$ Momentum diffusion is more or less equal to the heat diffusion

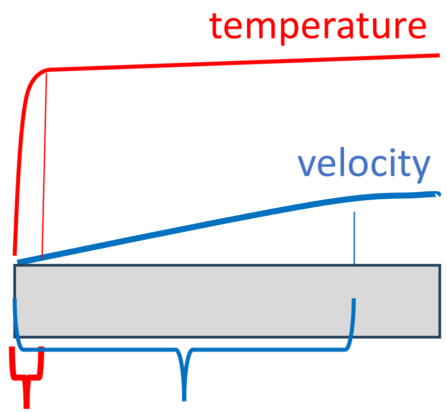
$Pr_{oil} \approx 100 - 100000$ Heat diffusion is extremely slow compared to the momentum diffusion

$Pr_{mercury} \approx 0.015$ Heat diffusion is extremely fast compared to the momentum diffusion

5.4. Non dimensional numbers for simultaneous transport

Relationship with boundary layers

Pr<<1

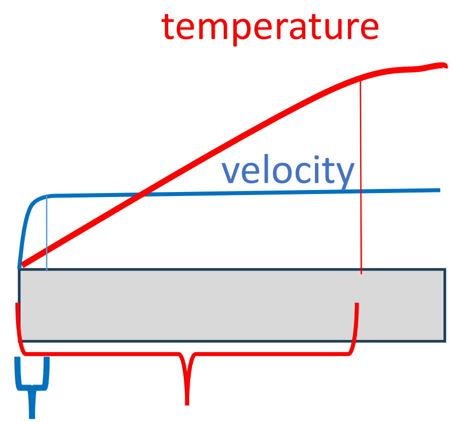


δT=thickness thermal boudary layer

δv =thickness velocity boudary layer

$\delta v > \delta T$

Pr>>1



δv=thickness velocity boudary layer

δT=thickness thermal boudary layer

$\delta v < \delta T$

A thinner boundary layer means that the variable takes less time to reach steady state, thus it is dominant mode of transport

5.4. Non dimensional numbers for simultaneous transport

The ***Schmidt* Number Sc** compares the momentum over mass diffusivity

$$Sc = \frac{momentum\ diffusion\ rate}{mass\ diffusion\ rate} = \frac{\nu}{D} = \frac{\mu}{\rho D}$$

ν kinematic viscosity [m^2/s]

D mass diffusivity [m^2/s]

μ dynamic viscosity [Kg · m^{-1} · s^{-1}]

ρ density of the fluid [Kg/m^3]

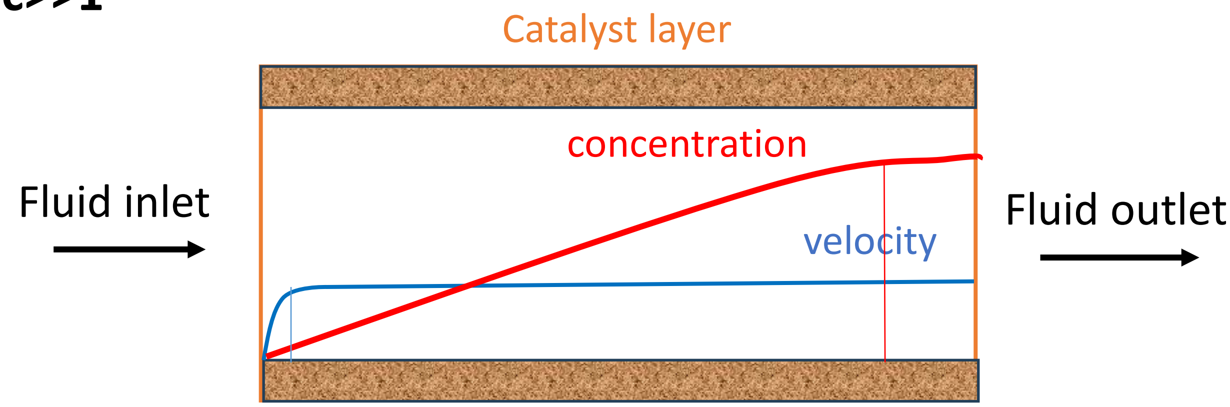
$Sc_{gas} \approx 0.7 - 1$ Momentum diffusion is more or less equal to the mass diffusion

$Sc_{liquid} \approx 100 - 1000$ Momentum diffusion is faster than mass diffusion

5.4. Non dimensional numbers for simultaneous transport

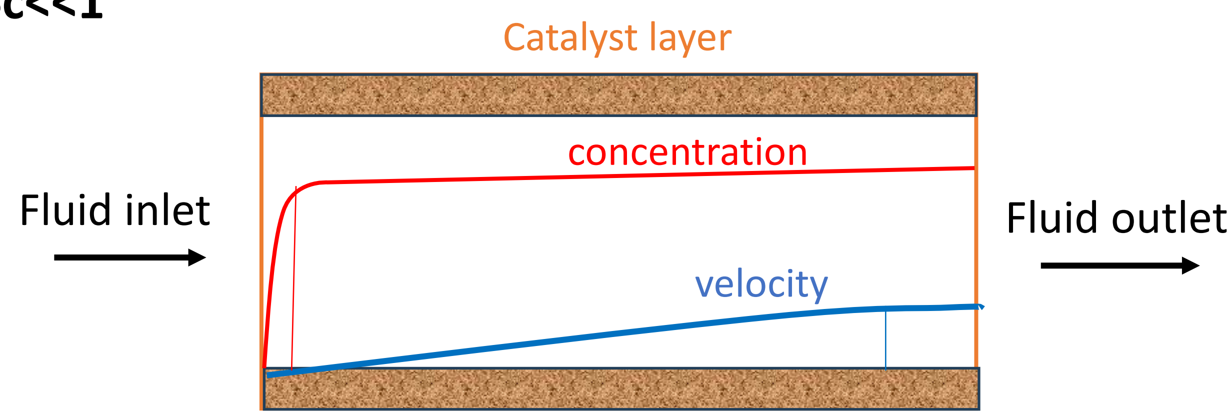
Exemple: Fluid bed catalytic reactor

Sc>>1



HIGHER AMOUNT OF UNREACTED PRODUCT

Sc<<1



LOWER AMOUNT OF UNREACTED PRODUCT

5.4. Non dimensional numbers for simultaneous transport

The ***Lewis* Number *Le*** compares the heat over mass transfer by convection

$$Le = \frac{heat\ diffusion\ rate}{mass\ diffusion\ rate} = \frac{\alpha}{D}$$

α thermal diffusivity [m²/s]

D mass diffusivity [m²/s]

$Sc_{gas} \approx 0.7 - 1$ — Heat diffusion is more or less equal to the mass diffusion

$Sc_{liquid} \approx 100$ — Heat diffusion is faster than mass diffusion

5.4. Non dimensional numbers for simultaneous transport

- Prandtl Number $$Pr = \frac{\nu}{\alpha}$$

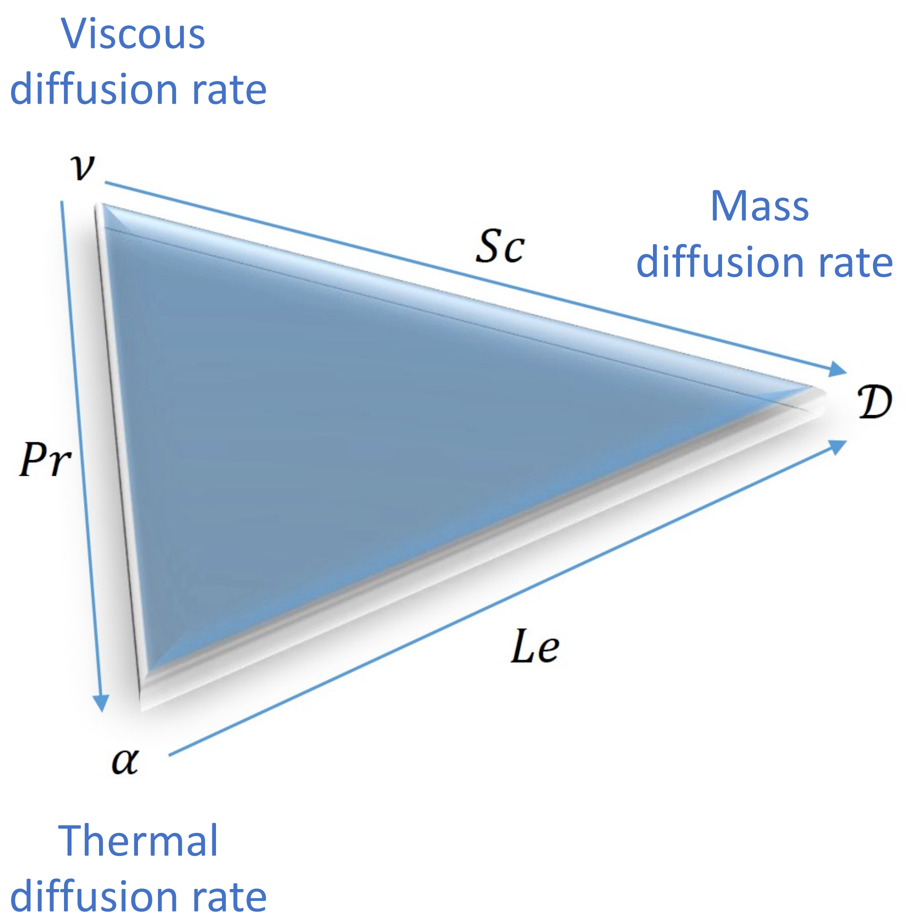
characterizes the simultaneous transfer of heat and viscous momentum

- Schmidt Number $$Sc = \frac{\nu}{\mathcal{D}}$$

gives the relative importance of the transfer of momentum and matter

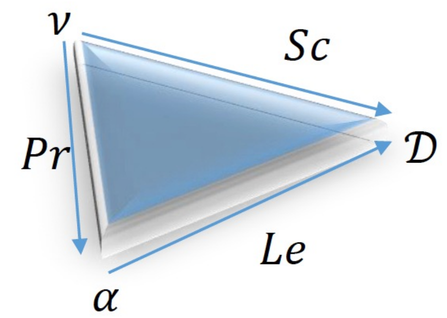
- Lewis Number $$Le = \frac{\alpha}{\mathcal{D}}$$

characterizes combined heat transfer and mass transport by diffusion



5.4. Non dimensional numbers for simultaneous transport

$$Pr = \frac{\nu}{\alpha} \qquad Sc = \frac{\nu}{\mathcal{D}} \qquad Le = \frac{\alpha}{\mathcal{D}}$$



Prandtl Numbers for Some Common Fluids

Substance	Temperature K	Temperature °C	Prandtl Number $Pr = \nu/\alpha$
Mercury	300	27	2.72×10^{-2}
Air	300	27	7.12×10^{-1}
Water	300	27	5.65
Ethyl alcohol	293	20	1.70×10
Glycerin	293	20	1.16×10^{4}

Schmidt and Lewis Numbers for Dilute gases and Dilute Solutions

Substance	Temperature K	Temperature °C	Schmidt Number $Sc = \nu/D$	Lewis Number $Le = \alpha/D$
O_2–N_2	273	0	7.3×10^{-1}	1.0
Dilute gases	293	20	~ 1	~ 1
NaCl aqueous	293	20	7.0×10^{2}	1.0×10^{2}
Dilute solutions	293	20	$\sim 10^{3}$	$\sim 10^{2}$

5.4. Non dimensional numbers for simultaneous transport

Why are dimensionless numbers useful in transport phenomena?

1) Dimensionless numbers allow for comparisons between very different systems

Let's say you are designing a stirrer for a batch reactor and you want to test a prototype. It would make sense to test a miniature but you now that the size of the reactor makes a difference. Yet, if you also decrease the viscosity of the liquid you are using , so that the Reynolds number is the same in both processes, you can take your conclusions from the miniature and apply them to larger scale.

2) Dimensionless numbers tell you how the system will behave.

The classic example involves again the Reynold number to predict the onset of turbolence in a system. Critical values for the Reynold number for many different systems are tabulated and so you can easily predict the onset of turbolence.