ChE 204
Introduction to Transport Phenomena

Module 3

Microscopic and Molecular Transport of Heat

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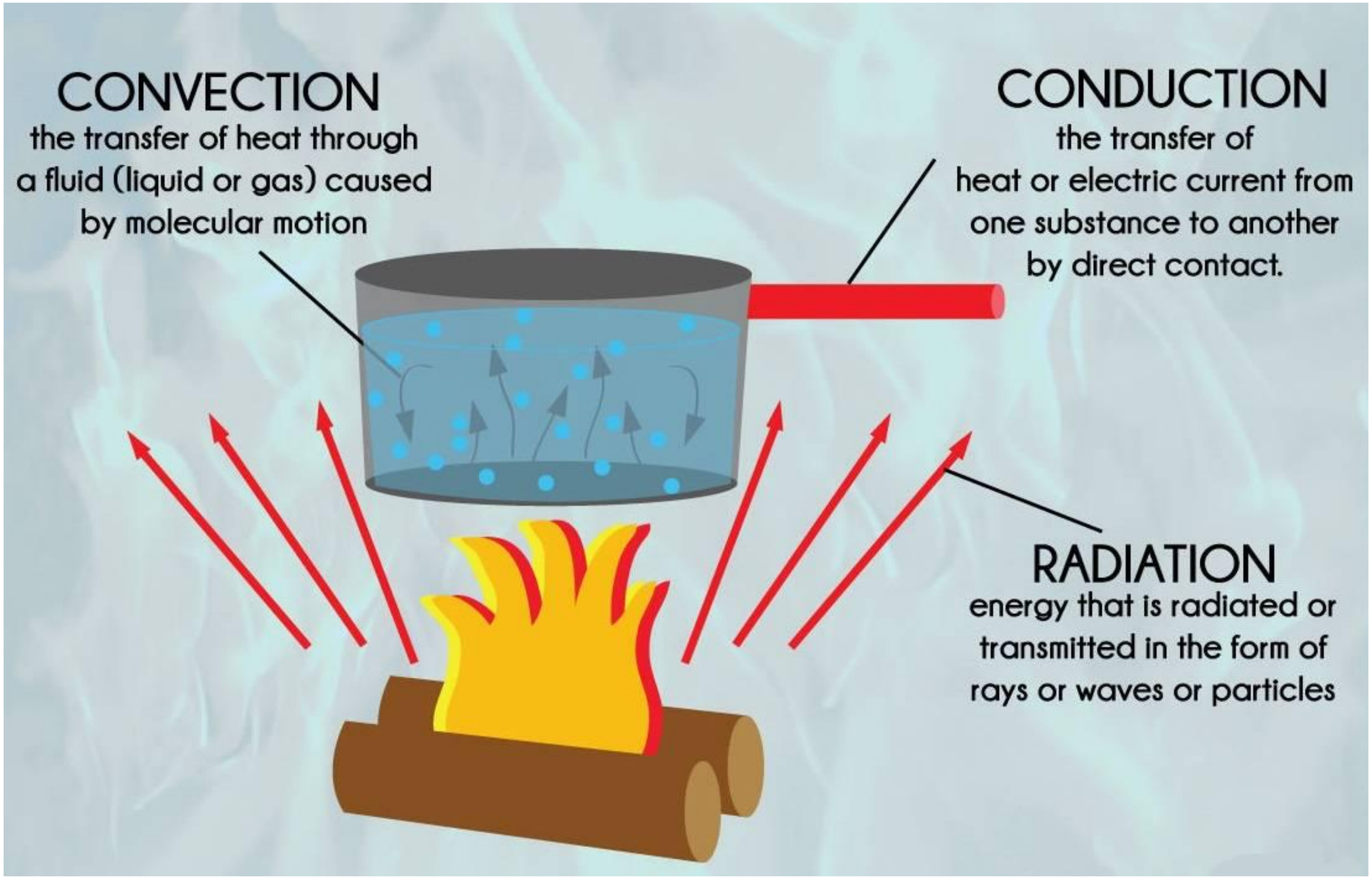
Microscopic and Molecular Transport of Heat

Objectives of this module:

- To describe the transport modes for heat
- To understand the meaning of thermal conductivity at the molecular scale
- To understand and to apply the Fourier's (1st) Law
- To understand and to apply the Newton's law of cooling for convective heat transport
- To solve a variety of problems combining different modes of heat transport

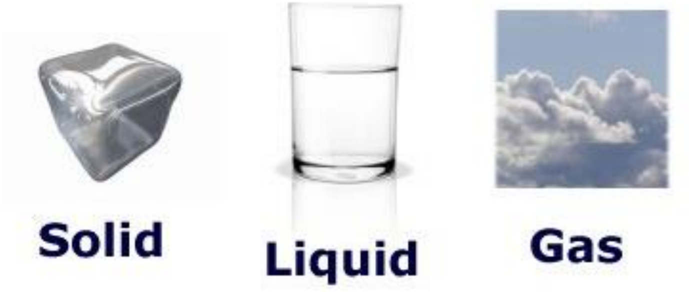
3.0. Modes of heat transport

Heat transport is the transport of energy from one area of higher temperature to one area of lower temperature

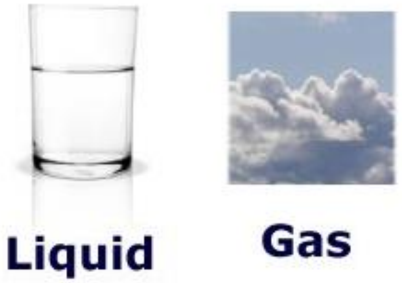


3.0. Modes of heat transport

CONDUCTION



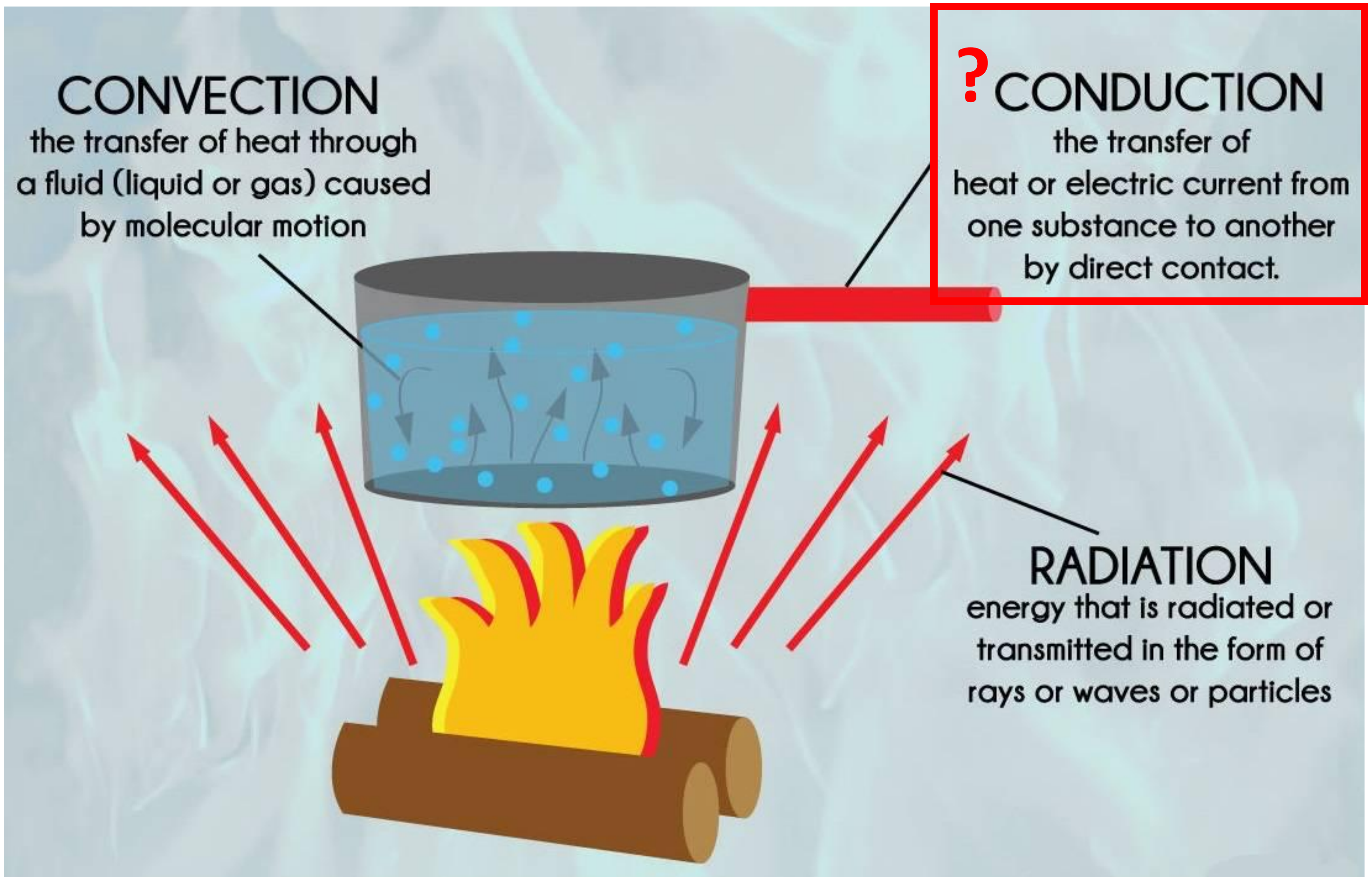
CONVECTION



RADIATION — does not require any material or medium

3.0. Modes of heat transport

Heat transport is the transport of energy from one area of higher temperature to one area of lower temperature

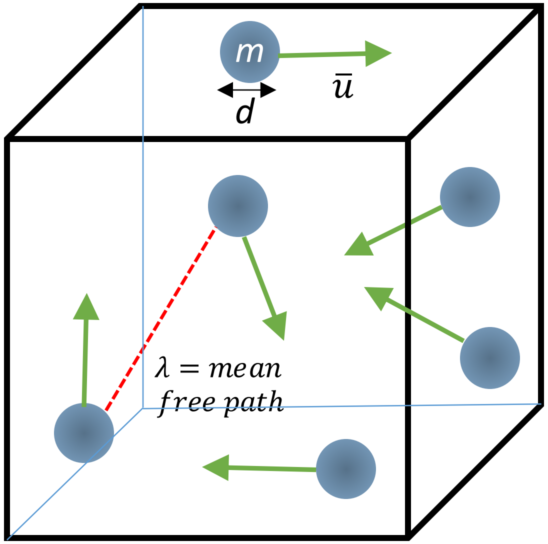


3.1. Heat and Kinetic Energy

$$\boldsymbol{KE_{avg}} = \frac{1}{2}\boldsymbol{m\bar{u}^2} = \frac{3}{2}\mathrm{k_B T}$$

The higher the temperature of the medium is, the faster molecules move.

From a diluted monoatomic gases (ideal gas):

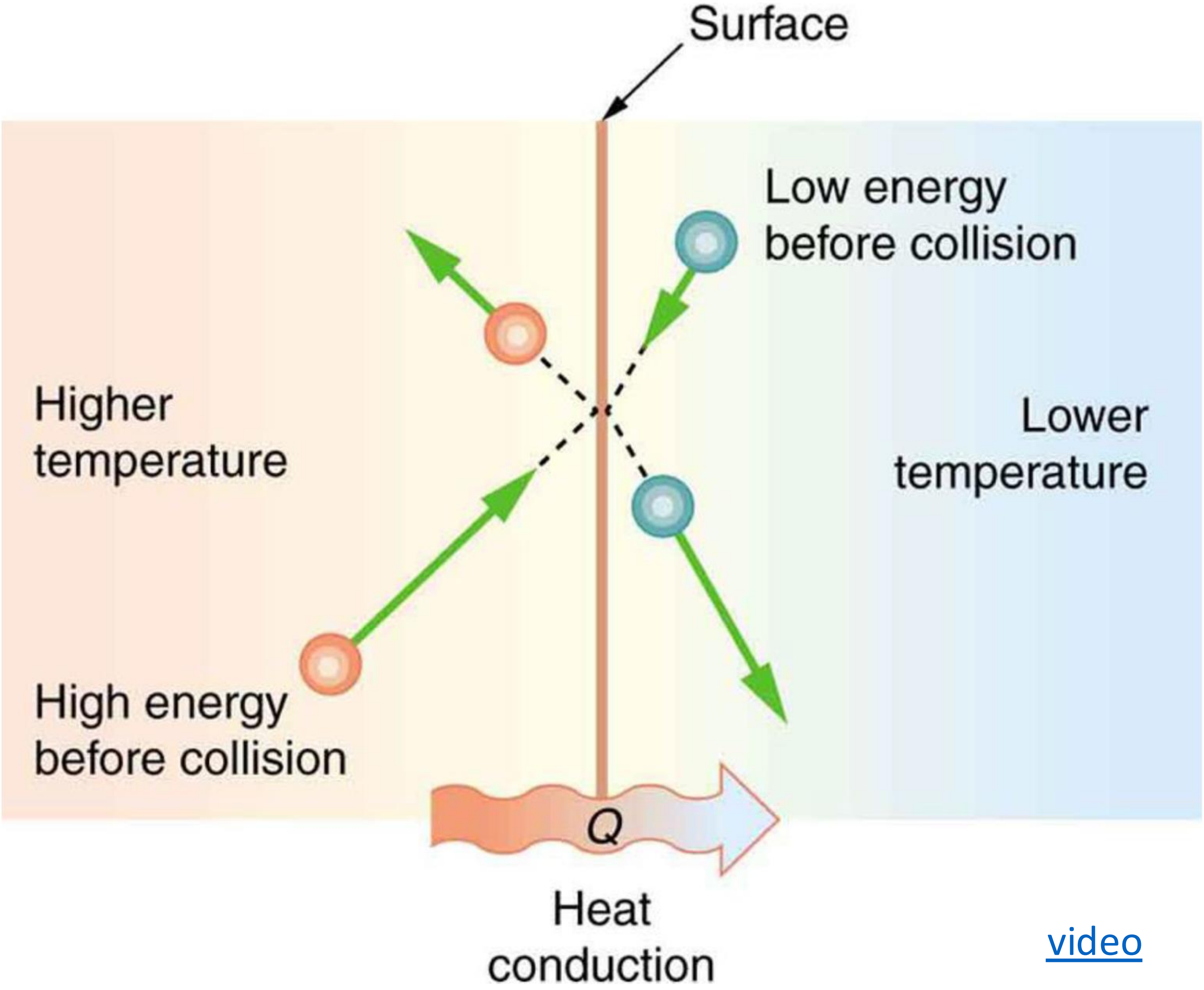


$$\bar{u} = average\ molecular\ speed = \sqrt{\frac{3k_BT}{m}}$$

$$\lambda = mean\ free\ path = \frac{1}{\sqrt{2}\pi d^2 n}$$

3.1. Heat and Kinetic Energy

HEAT CONDUCTION IN LIQUID AND GASES occurs by random collisions between molecules

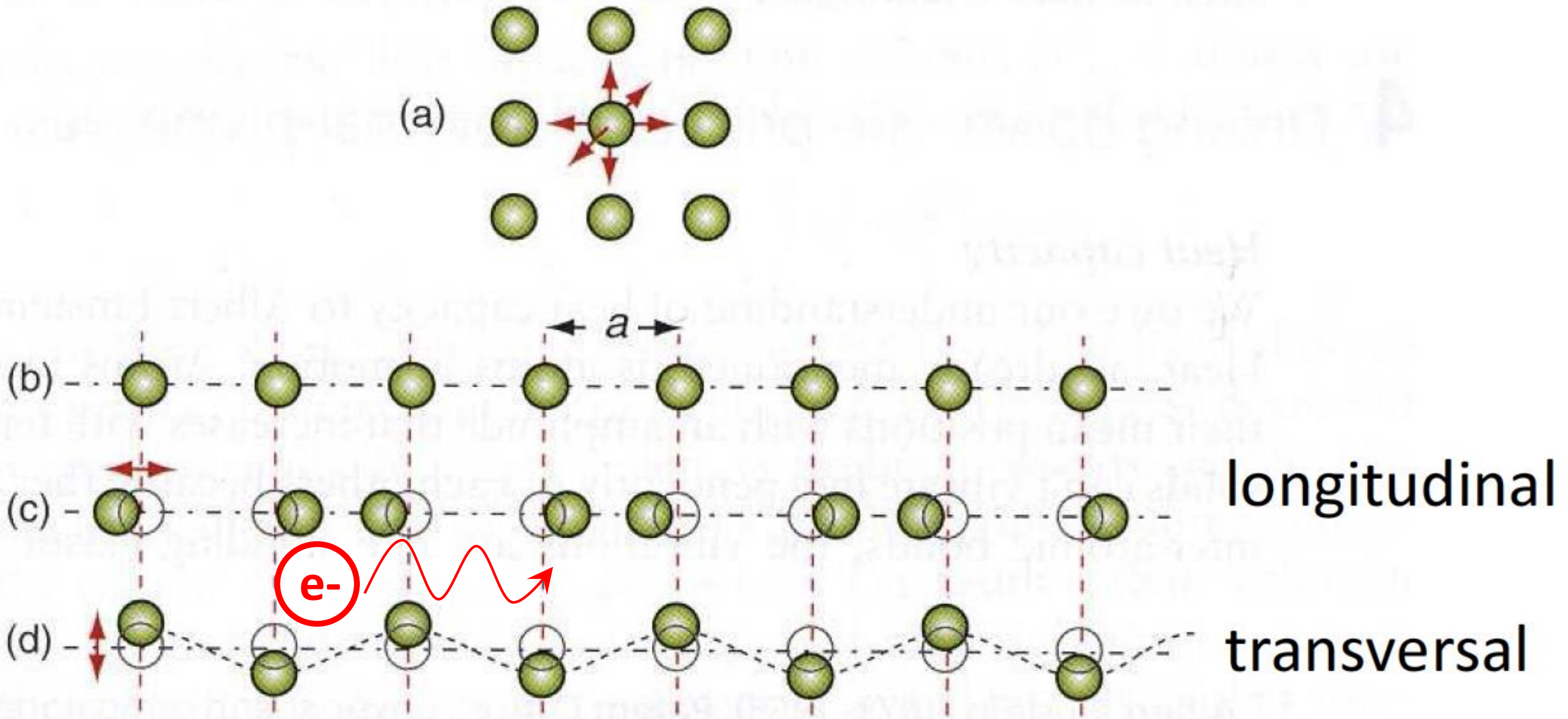


video

3.1. Heat and Kinetic Energy

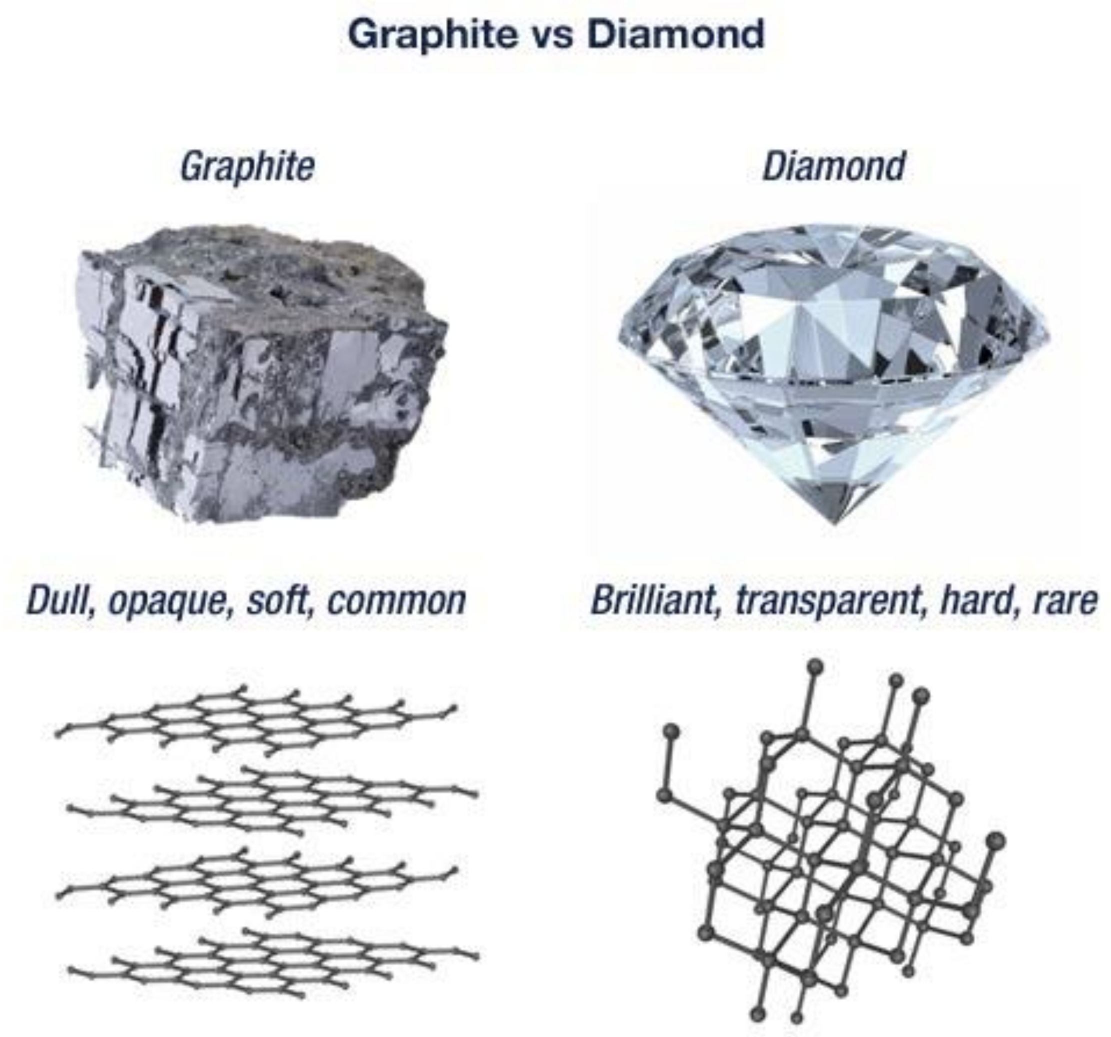
HEAT CONDUCTION IN SOLIDS occurs by lattice vibrations and by free electron energy transfer.

In solids, the position of the atoms is defined by the lattice structure. Thus, atoms can only vibrate around their position. Atoms in solids can't vibrate independently of each other because they are coupled by their inter-atomic bonds; the vibrations are like standing elastic waves and they are called phonons.

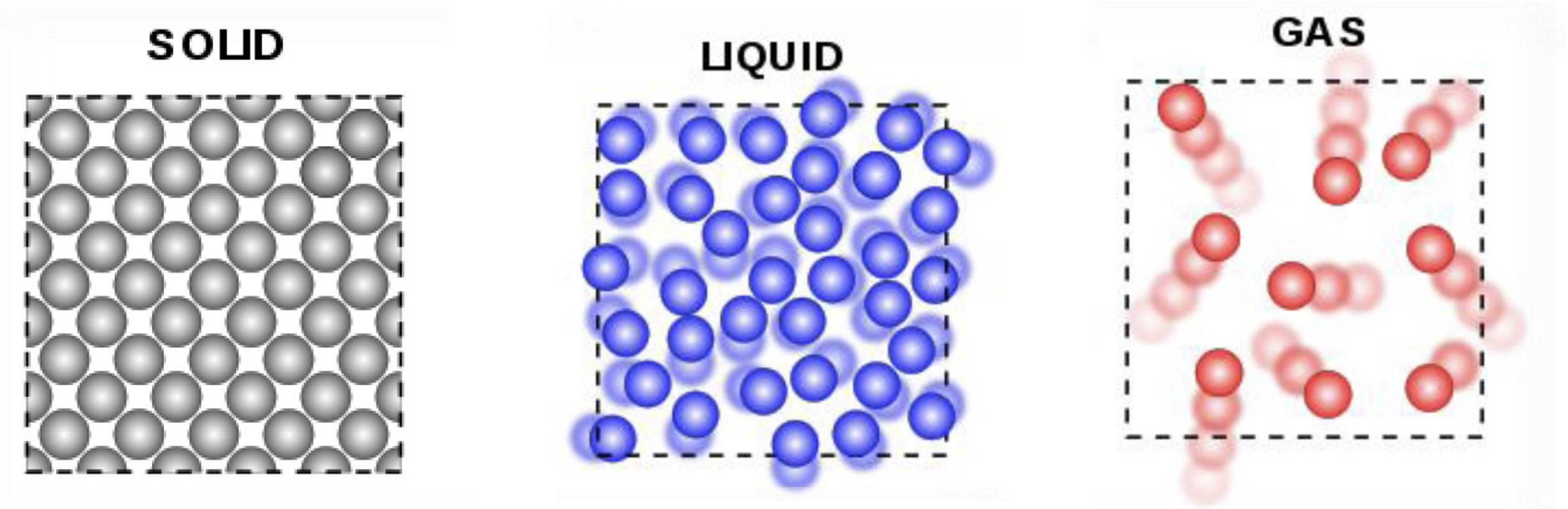


3.1. Heat and Kinetic Energy

HEAT CONDUCTION IN SOLIDS occurs by lattice vibrations and by free electron energy transfer.



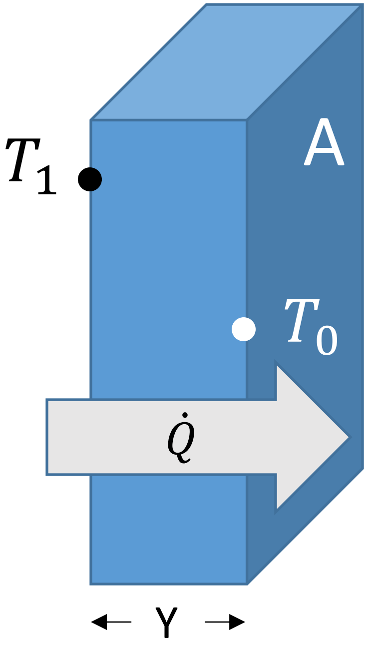
3.1. Heat and Kinetic Energy



The interaction between neighbouring atoms in solids are much stronger than in liquids and gases. Thus, conduction heat transfer is highest in solids.

3.2. Fourier's law of heat conduction

$T_1 > T_0$



The **heat flow rate** is the energy, in form of heat, transferred by conduction per unit time

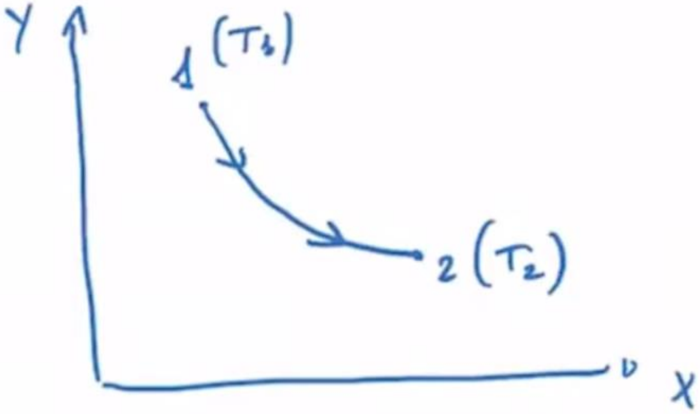
$$\dot{Q} = \frac{heat}{time} \quad \left[\frac{J}{s}\right] = [W]$$

The **heat flux** is the heat flow per unit area

$$q = \frac{\dot{Q}}{A} = \frac{heat}{Area \times time} \qquad \left[\frac{W}{m^2}\right]$$

$$\dot{\boldsymbol{Q}} = ? \qquad \boldsymbol{T} = \boldsymbol{f}(\boldsymbol{Y})$$

Discussion on Thermodynamics versus Transport Phenomena



thermodynamics

→ magnitude of heat transfer

→ $(T_2 - T_1)$

transport phenomena

→ rate of heat transfer $\dot{Q} = \frac{heat}{time}$

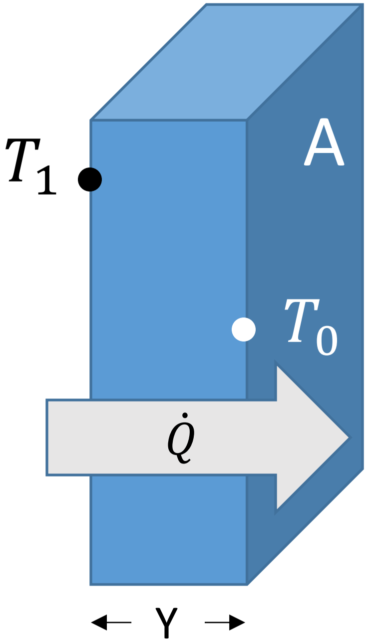
→ $T = f(time)$

↓

temperature profile in the slab

Derivation of the formula for thermal conductivity based on “intuition”

$T_1 > T_0$



We want to establish the relationships between

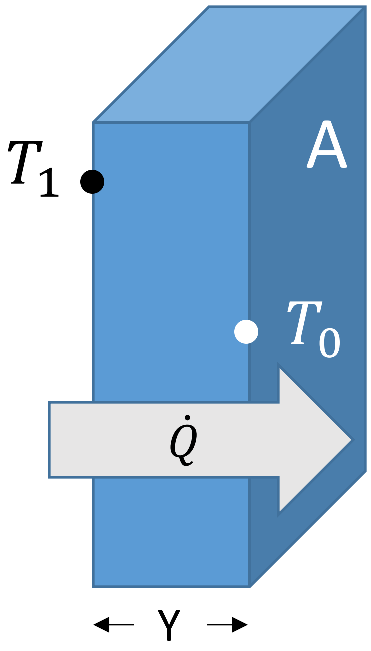
$$\dot{Q}, \mathrm{A}, \Delta\mathrm{T}, \mathrm{Y}$$

$$\dot{Q} = \frac{heat}{time}$$

3.2. Fourier's law of heat conduction

HOW DID FOURIER DO IT?

$T_1 > T_0$



He measured how the heat flow changes with the thickness Y of the slab, which can be done using an heat flux sensor. What he found is the following relation:

$$\boxed{\dot{Q} = k\,A\,\frac{(T_1 - T_0)}{Y}}$$

$\dot{\mathrm{Q}} = [\mathrm{W}]$

$T = [\mathrm{K}]$ Temperature

$\mathrm{A} = [\mathrm{m}^2]$ Surface Area

$\mathrm{q} = \frac{\dot{\mathrm{Q}}}{\mathrm{A}} = [\mathrm{W\ \ m}^{-2}]$

The constant of proportionality is <u>*k*</u>, which is the <u>*thermal conductivity*</u> of the slab.

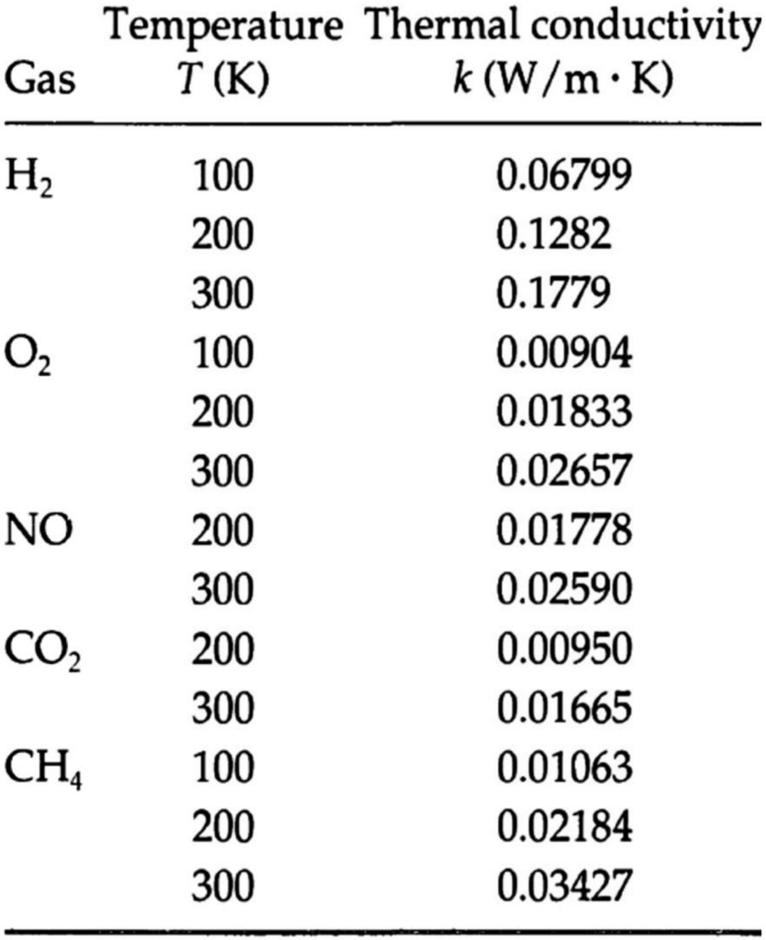
$$k = [\mathrm{W\ \ m}^{-1}\,\mathrm{K}^{-1}]$$

3.2. Fourier's law of heat conduction

Values of the thermal conductivity, *k*

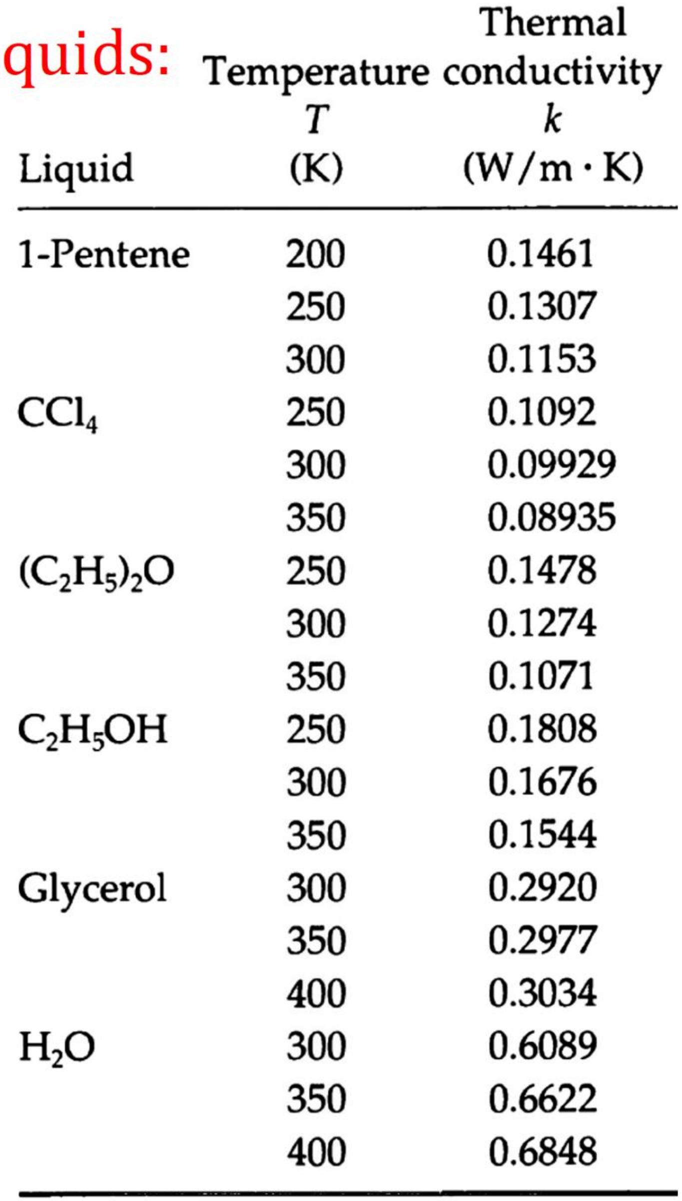
$k = f(T)$

in gases:



Gas	Temperature T (K)	Thermal conductivity k (W/m · K)
H_2	100	0.06799
	200	0.1282
	300	0.1779
O_2	100	0.00904
	200	0.01833
	300	0.02657
NO	200	0.01778
	300	0.02590
CO_2	200	0.00950
	300	0.01665
CH_4	100	0.01063
	200	0.02184
	300	0.03427

in liquids:



Liquid	Temperature T (K)	Thermal conductivity k (W/m · K)
1-Pentene	200	0.1461
	250	0.1307
	300	0.1153
CCl_4	250	0.1092
	300	0.09929
	350	0.08935
$(C_2H_5)_2O$	250	0.1478
	300	0.1274
	350	0.1071
C_2H_5OH	250	0.1808
	300	0.1676
	350	0.1544
Glycerol	300	0.2920
	350	0.2977
	400	0.3034
H_2O	300	0.6089
	350	0.6622
	400	0.6848

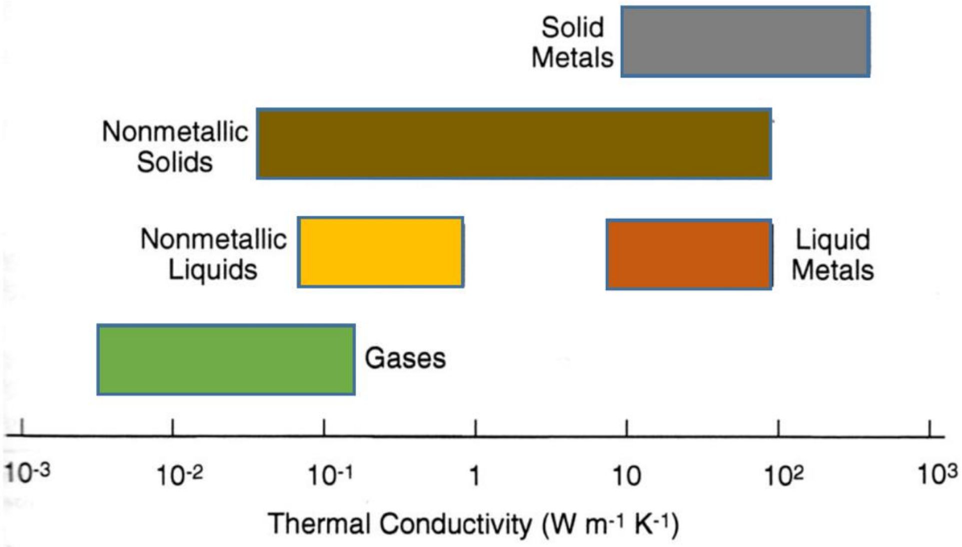
3.2. Fourier's law of heat conduction

Values of the thermal conductivity, *k*

$k = f(T)$

in solids:

Substance	Temperature T (K)	Thermal conductivity k (W/m · K)
Aluminum	373.2	205.9
	573.2	268
	873.2	423
Cadmium	273.2	93.0
	373.2	90.4
Copper	291.2	384.1
	373.2	379.9
Steel	291.2	46.9
	373.2	44.8
Tin	273.2	63.93
	373.2	59.8
Brick (common red)	—	63
Concrete (stone)	—	92
Earth's crust (average)	—	1.7
Glass (soda)	473.2	0.71
Graphite	—	5.0
Sand (dry)	—	0.389
Wood (fir)		
parallel to axis	—	0.126
normal to axis	—	0.038



Note: K will generally be given and assumed to be constant in your exercizes

3.2. Fourier's law of heat conduction

$$\boxed{q = \frac{\dot{Q}}{A} = k\,\frac{(T_1 - T_0)}{Y}}$$

- Constant temperature ⇨ no heat energy flow
- Heat flows from warm regions to cold regions
- The greater the difference in temperature, the faster the flow of energy
- The flow of heat energy is different for different materials

"The heat flux is proportional to the negative gradient in the temperature"

We have not answer yet to our question regarding the temperature profile T(y)!

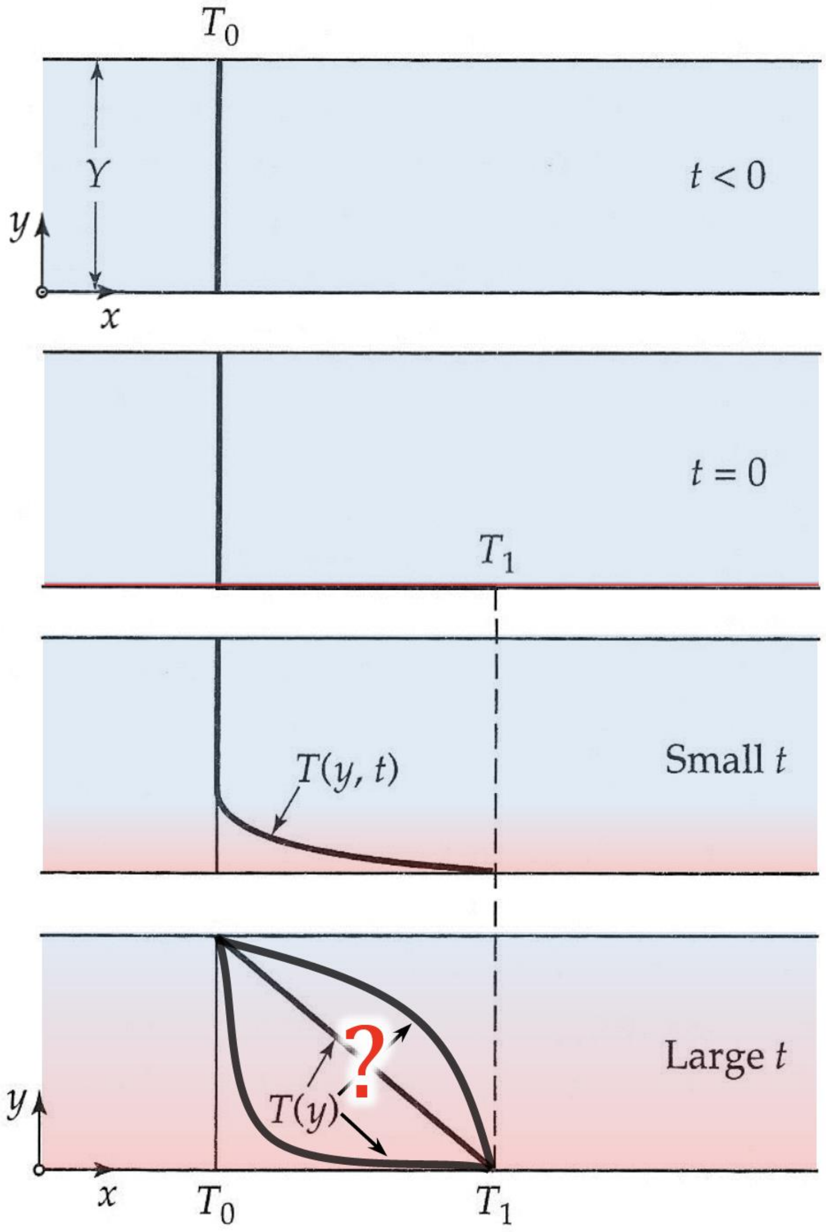
3.2. Fourier's law of heat conduction

1) The two blocks are separated

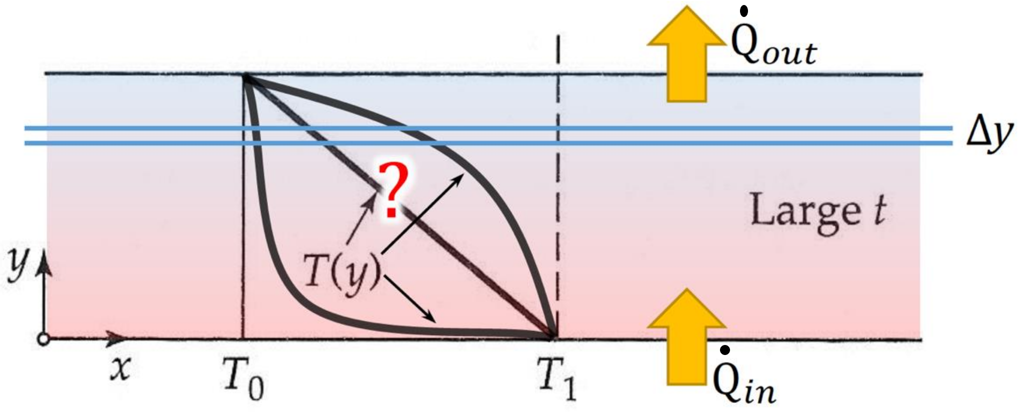
2) We position the two blocks in contact

3) Heat will start to transfer

4) How is the temperature profile going to look like at steady state ($T \neq f(time)$)?



3.2. Fourier’s law of heat conduction

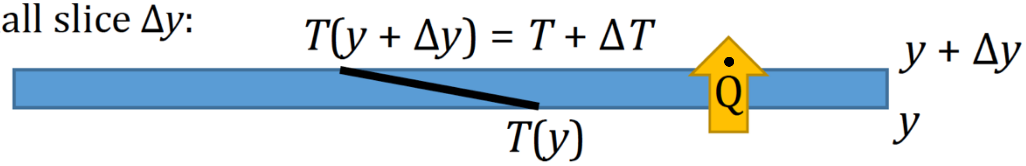


At large t (steady state):

$$\dot{Q}_{in} = \dot{Q}_{out} = k\ A\ \frac{(T_1 - T_0)}{Y}$$

What is T(y)?

Small slice Δy:



$$\frac{\dot{Q}}{A} = k\frac{T - (T + \Delta T)}{\Delta y} \Rightarrow \frac{\dot{Q}}{A} = k\frac{-\Delta T}{\Delta y} \Rightarrow \frac{\dot{Q}}{A} = -k\frac{dT}{dy} \Rightarrow \boxed{q = -k\frac{dT}{dy}}$$

The Fourier’s Law is this differential form!

$$q\ dy = -k\ dT$$

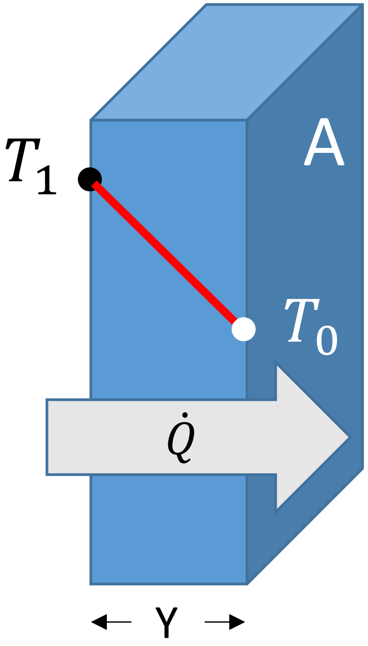
integrate

We know that q is constant (steady state), if k is constant:

$$\int_0^y q\ dy = \int_{T_1}^{T} -k\ dT \Rightarrow q\ y = -k\ (T - T_1) \Rightarrow T(y) = T_1 - \left(\frac{q}{k}\right) y$$

3.2. Fourier's law of heat conduction

$T_1 > T_0$



$$T(y) = T_1 - \left(\frac{q}{k}\right) y$$

This relation states that the <u>temperature profile</u> inside the slab is <u>linear.</u>

3.2. Fourier's law of heat conduction

$$\boxed{q = -k\frac{dT}{dy}}$$

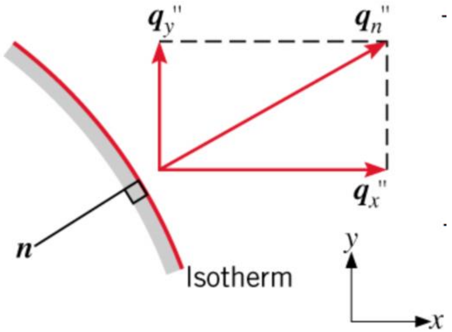
"The rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area"

Assumptions:

- Steady state heat conduction
- One directional heat flow
- Bounding surfaces are isothermal (uniform temperature at the two faces)
- k is constant
- No internal heat generation

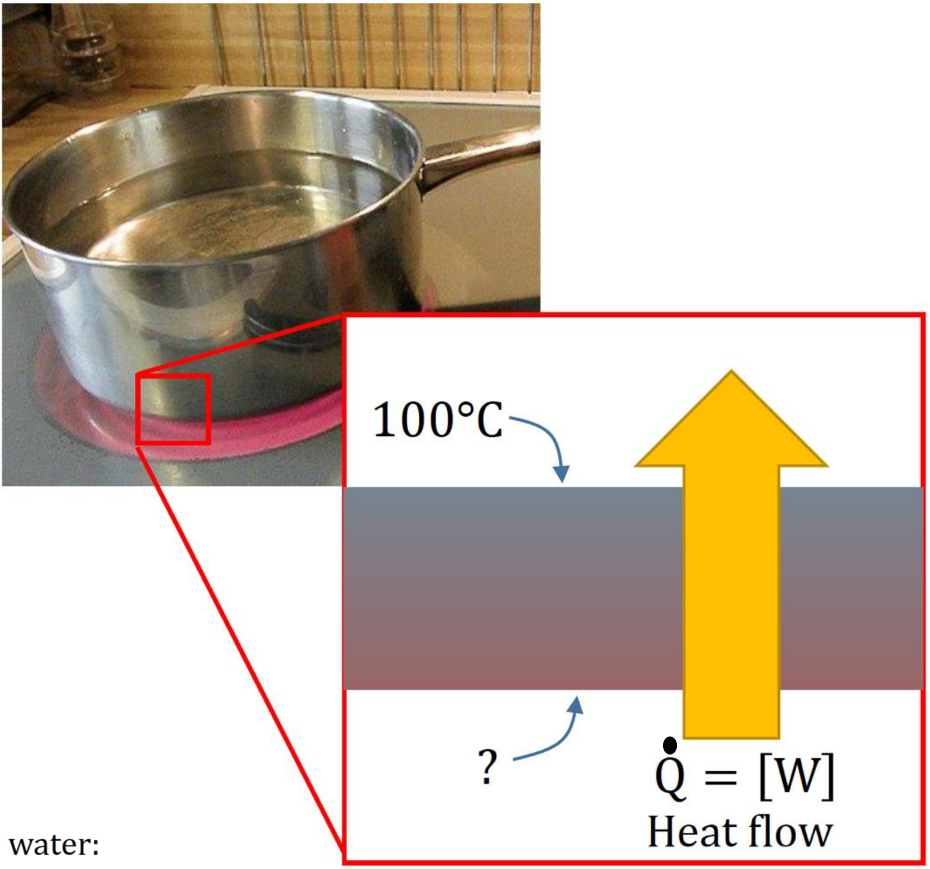
Features:

- Fourier equation is valid for all matter of solid, liquid and gas
- The vector expression indicating that heat flow rate is normal to an isotherm and is in the direction of decreasing temperature
- It cannot be derived from first principles



3.2. Fourier's law of heat conduction

Example: An aluminum saucepan (casserole) with a surface area of 0.02m^2 and thickness of 7mm is placed on a hot stove to boil 1Kg of water (at sea level). The maximum heat output of the stove is 1000 W. After the water reaches 100C, can you estimate how long it should take to boil all of the water? What is the temperature at the bottom of the pan? What would be the bottom temperature be if the pan was made of steel?



For aluminum:

$k = 206 \text{ W m}^{-1} \text{K}^{-1}$

For steel:

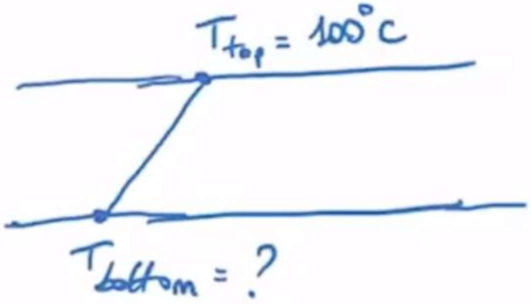
$k = 44.8 \text{ W m}^{-1} \text{K}^{-1}$

Latent heat of vaporization for water:

$\Delta H^{vap} = 2260 \text{ kJ kg}^{-1}$

$$\Delta H_{ev} = 2260 \, \frac{kJ}{kg}$$ energy needed to evaporate 1 kg water

$$\underset{\overset{\uparrow}{1000\,W}}{\dot{Q}} = \frac{m\,\Delta H_{ev}}{\underset{\nwarrow \text{time}}{t}} \qquad t = \frac{m\,\Delta H_{ev}}{\dot{Q}} = 2260\,s$$



$$\dot{Q} = \frac{KA\,(T_{bottom} - T_{top})}{Y} \qquad 1000\,W = \frac{(206\,W/m\cdot K)(0.02\,m^2)(x - 100°C)}{0.007\,m}$$

$$T_{bottom,Al} \underline{\underline{= 101.7°C}} \qquad T_{bottom,\underline{\underline{steel}}} = 108\,°C$$

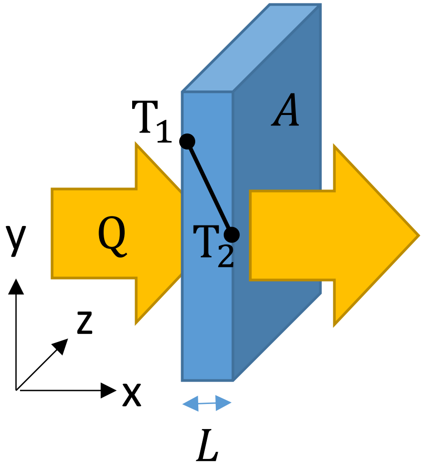
3.2. Fourier's law of heat conduction

DEFINITION OF THERMAL RESISTANCE

$$R_T = \frac{(\mathrm{T}_1 - \mathrm{T}_2)}{Q}$$

$$\boxed{\dot{\mathrm{Q}} = \frac{\Delta T}{\sum R_{th}}}$$

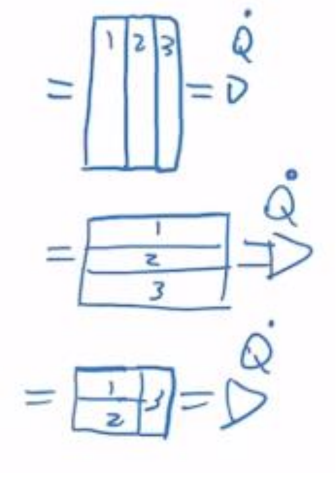
The thermal resistance of a wall (**conductive**):



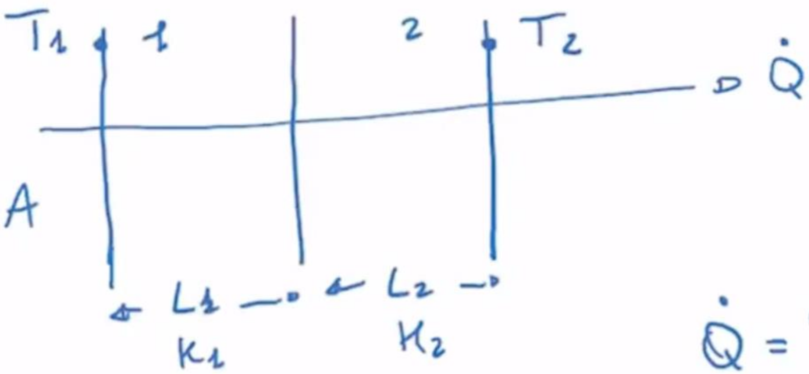
$$\dot{\mathrm{Q}} = A\,k\,\frac{(T_1 - T_2)}{L} = \frac{\Delta T}{R_{cond}}$$

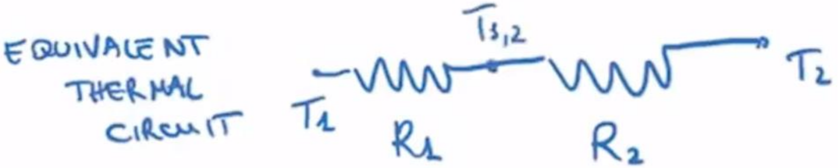
$$R_{cond} = \frac{L}{kA}$$

Examples of more complex systems



Thermal resistance circuits: series thermal resistance



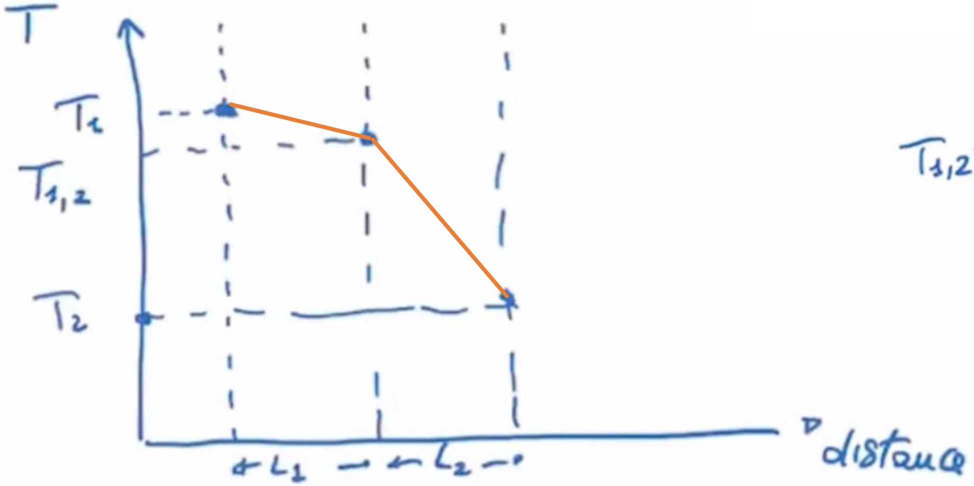


$$\dot{Q} = \dot{Q}_1 = \dot{Q}_2$$

$$R_{TOT} = R_1 + R_2$$

$$\dot{Q} = \frac{\Delta T}{R_{TOT}} = \frac{T_2 - T_1}{R_1 + R_2}$$

Temperature profile of the system

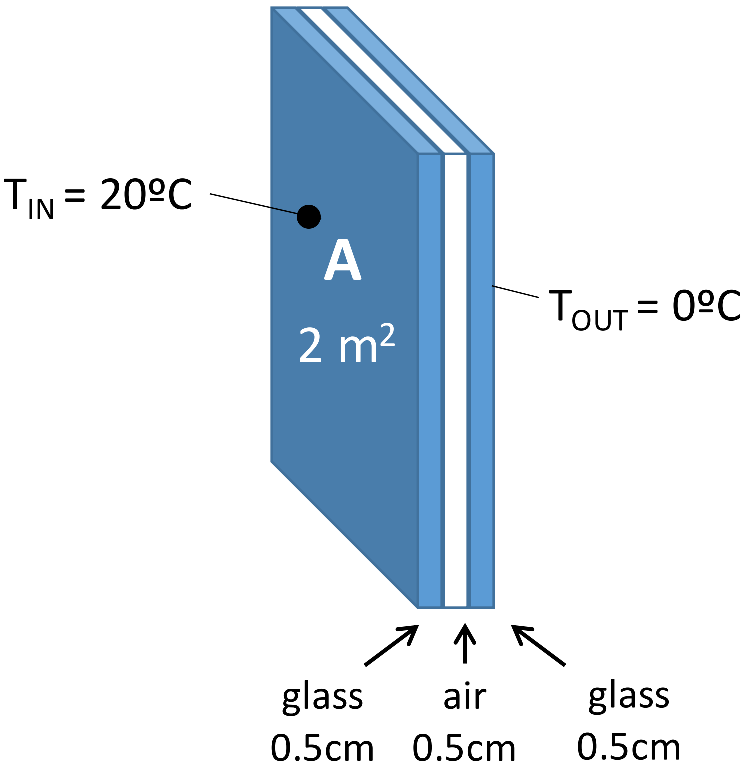


$T_{1,2} = ?$

$$\dot{Q} = \dot{Q}_1 = \frac{T_1 - T_{1,2}}{R_1}$$

3.2. Fourier's law of heat conduction

Example: Double pane window



$$k_{glass} = 0.8 \frac{J}{m\ sec\ ℃}$$

$$k_{air} = 0.024 \frac{J}{m\ sec\ ℃}$$

Determine the rate of heat transfer.

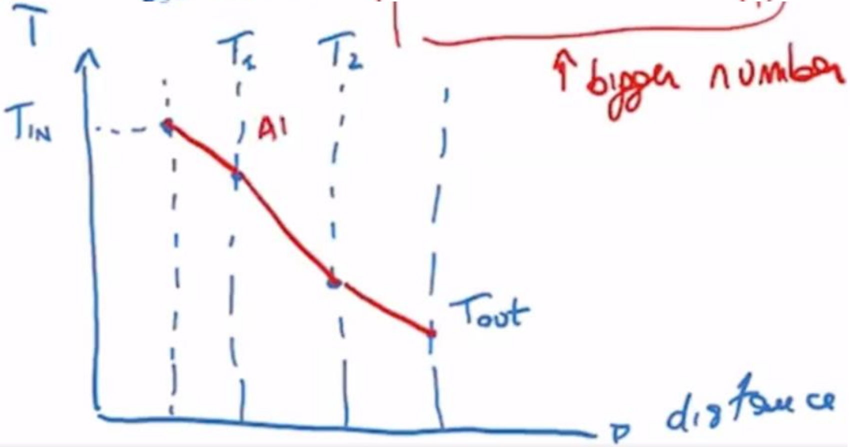
Thermal resistance circuits: parallel thermal resistance

equivalent circuit
of our double pane window

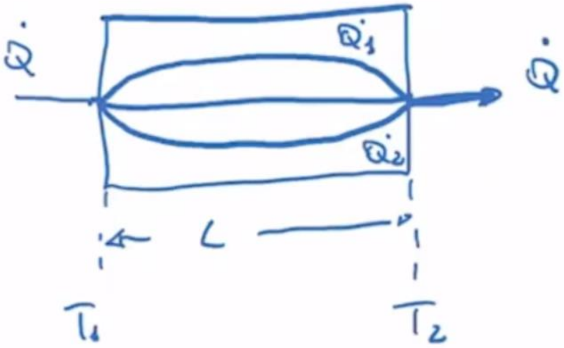
$$\dot{Q} = \frac{\Delta T}{R_{tot}} = \frac{\Delta T}{R_1+R_2+R_3} = \frac{\Delta T}{\frac{L_1}{k_1 A}+\frac{L_2}{k_2 A}+\frac{L_3}{k_3 A}} = \frac{A\ \Delta T}{\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}} =$$

$$= \frac{(2\, m^2)(20°C)}{\frac{0.005\, m}{0.8\, \frac{J}{sec \cdot m\, °C}} + \underbrace{\frac{0.005\, m}{0.024\, \frac{J}{sec \cdot m\, °C}}}_{} + \frac{0.005}{0.8\, \frac{J}{sec \cdot m\, °C}}} = 181\ W \left(\frac{J}{S}\right)$$

↑ bigger number → limiting resistance in the circuit

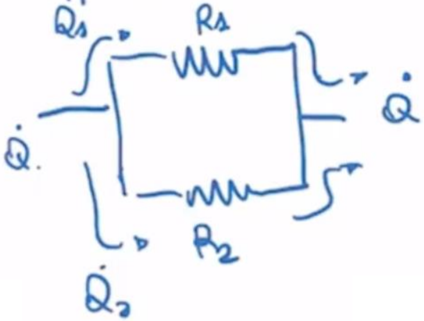


Thermal resistance circuits: parallel thermal resistance



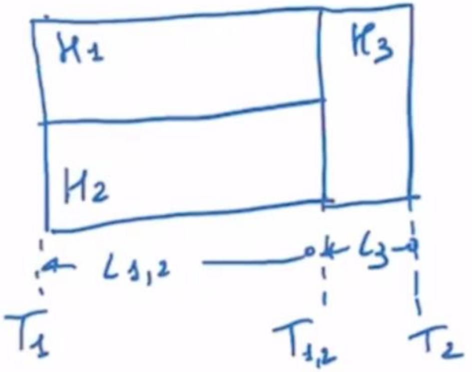
$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

equivalent circuit

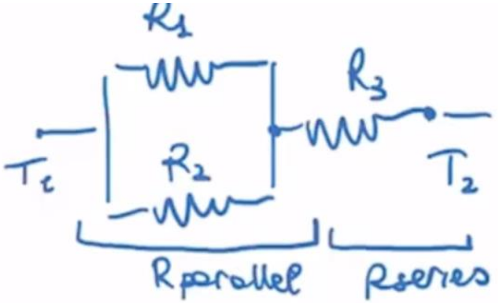


$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Thermal resistance networks



equivalent circuit

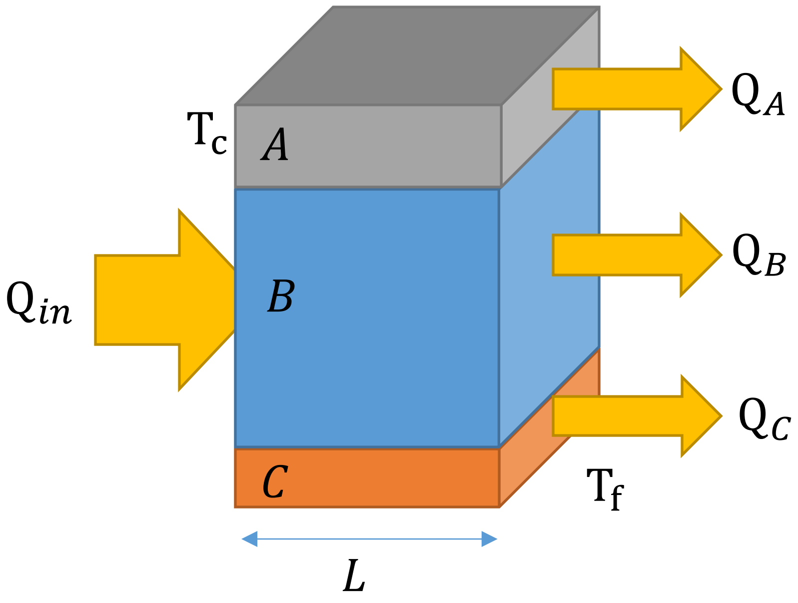


$$\dot{Q} = \frac{T_2 - T_1}{R_{tot}} \qquad R_{tot} = \frac{R_1 R_2}{R_1 + R_2} + R_3$$

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$$

3.2. Fourier's law of heat conduction

Example: The case of parallel conduction



They have common length L

$$\dot{Q}_{in} = \dot{Q}_A + \dot{Q}_B + \dot{Q}_C \quad \text{(heat balance)}$$

$$\frac{1}{\sum R_{cond}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$$

$$\frac{1}{\sum R_{cond}} = \frac{1}{\frac{L}{k_A A_A}} + \frac{1}{\frac{L}{k_B A_B}} + \frac{1}{\frac{L}{k_C A_C}}$$

$$\frac{1}{\sum R_{cond}} = \frac{k_A A_A}{L} + \frac{k_B A_B}{L} + \frac{k_C A_C}{L}$$

$$\sum R_{cond} = \frac{L}{(k_A A_A + k_B A_B + k_C A_C)}$$

$$Q_{in} = \frac{(T_c - T_f)}{\frac{L}{(k_A A_A + k_B A_B + k_C A_C)}}$$

$$Q_{in} = \frac{(T_c - T_f)(k_A A_A + k_B A_B + k_C A_C)}{L}$$

Recap Heat Conduction

We simplify problems, heat transfer is more complex in reality!

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Composite Structures

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Heat transfer analysis in multi-layered materials with interfacial thermal resistance

Wei-bin Yuan[a], Nanting Yu[b], Long-yuan Li[b], Yuan Fang[c]

Recap Heat Conduction

$$\dot{Q} = H \frac{A \, \Delta T}{L} \, .$$

FOURIER'S LAW

$\uparrow$ thermal conductivity

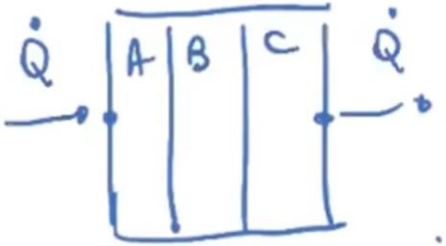
temperature profile is lineare

$$T = f(L)$$

THERMAL CIRCUITS

$$\dot{Q} = \frac{\Delta T}{\Sigma R_{TOT}}$$

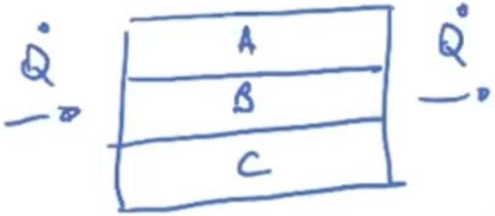
$$R_{COND} = \frac{L}{HA}$$



series

$$\dot{Q}_A = \dot{Q}_B = \dot{Q}_C$$

$$R_{TOT} = R_A + R_B + R_C$$

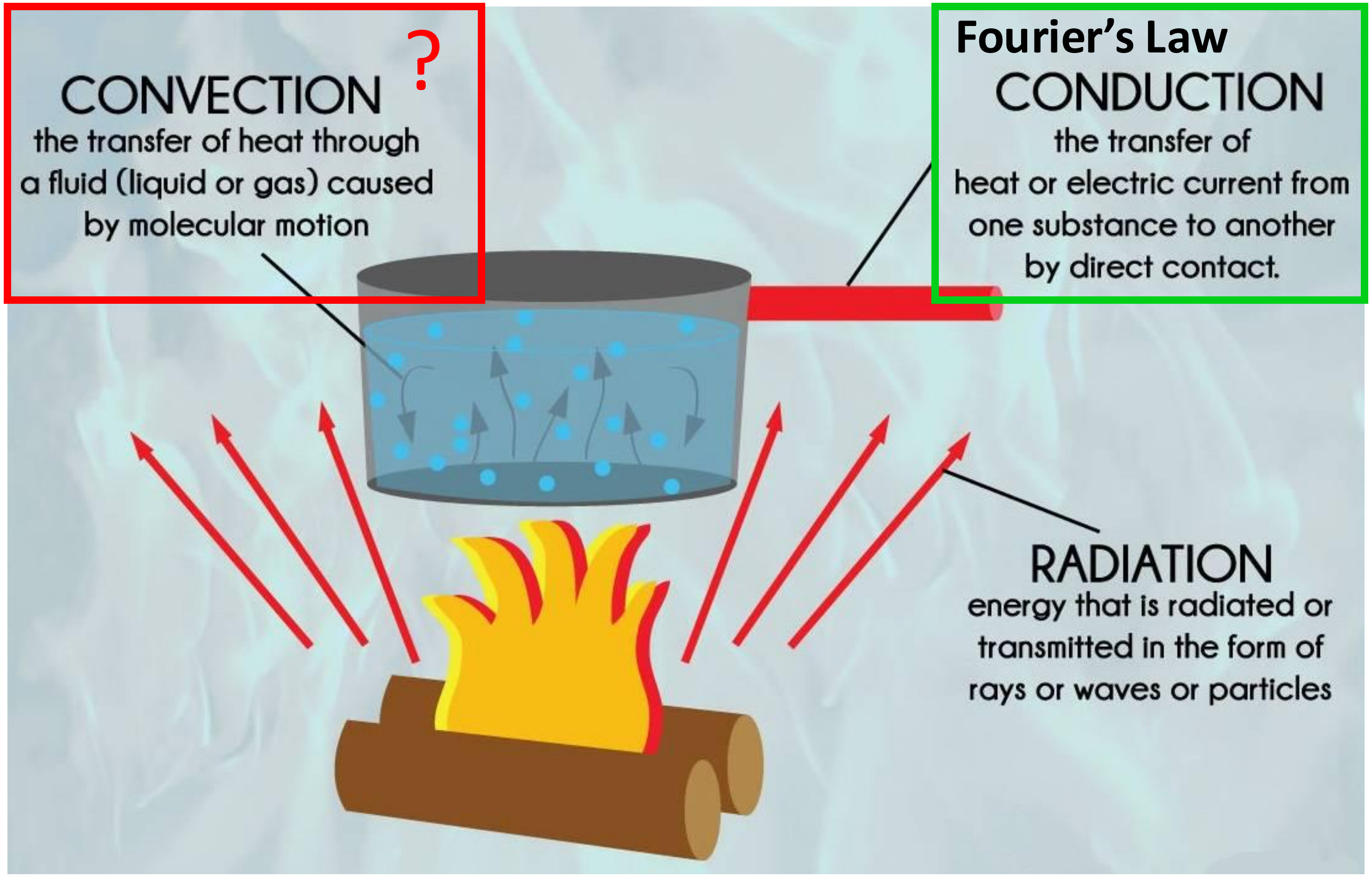


$$\dot{Q} = \dot{Q}_A + \dot{Q}_B + \dot{Q}_C$$

$$\frac{1}{R_{TOT}} = \frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}$$

3.0. Modes of heat transport

Heat transport is the movement of heat energy from one area of higher temperature to one area of lower temperature



?

CONVECTION

the transfer of heat through a fluid (liquid or gas) caused by molecular motion

Fourier's Law

CONDUCTION

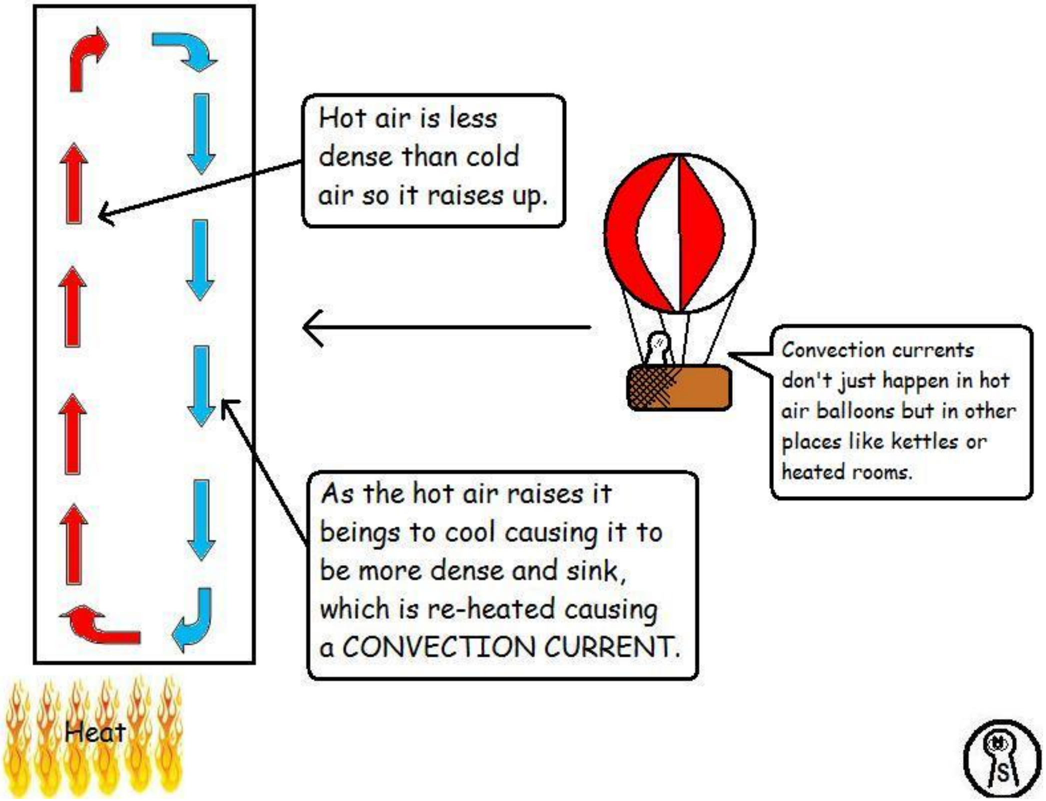
the transfer of heat or electric current from one substance to another by direct contact.

RADIATION

energy that is radiated or transmitted in the form of rays or waves or particles

3.3. Newton’s law of cooling for convective heat transfer

Convection is the heat transfer between a surface and a fluid in movement, which are at two different temperatures. Convection is the result of two phenomena: the energy transported by the movement of molecules (diffusion) and the energy transported by the fluid flow (advection).



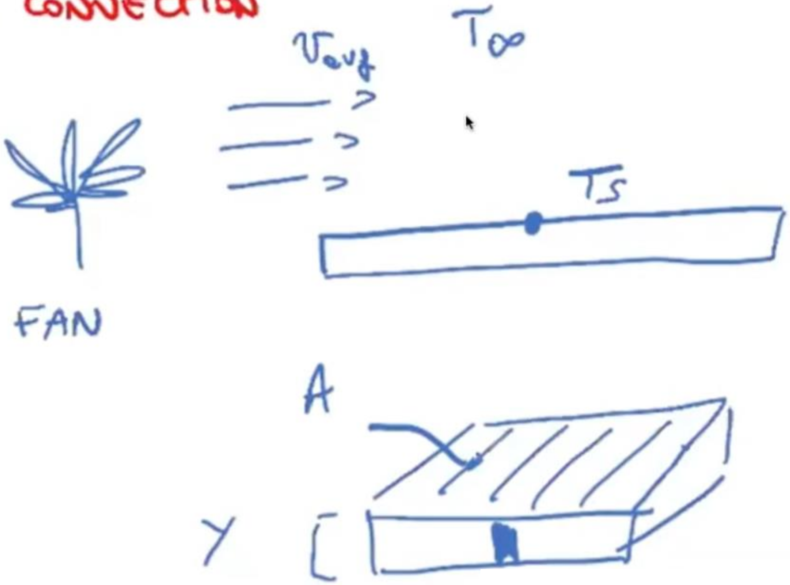
Hot air is less dense than cold air so it raises up.

As the hot air raises it beings to cool causing it to be more dense and sink, which is re-heated causing a CONVECTION CURRENT.

Convection currents don't just happen in hot air balloons but in other places like kettles or heated rooms.

Newton's Law by intuition

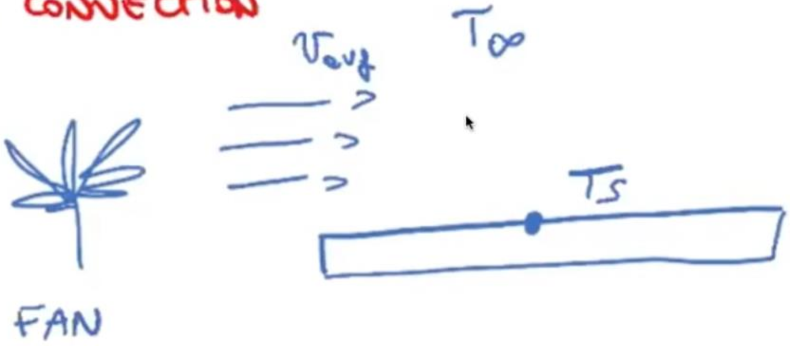
FORCED CONVECTION



$T_s > T_\infty$

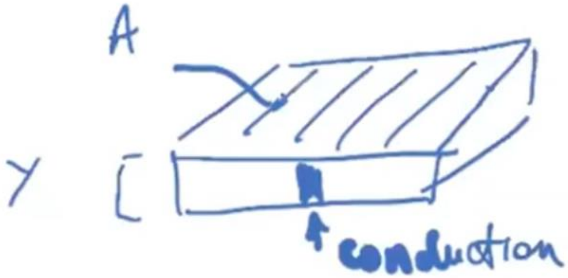
Newton's Law by intuition

FORCED CONVECTION



$T_s > T_\infty$

$\dot{Q}_{conv} \propto A(T_s - T_\infty)$



$A \uparrow \quad \dot{Q}_{conv} \uparrow$

$A \uparrow \quad Q_{conv} \downarrow$

$Y \uparrow \quad \dot{Q} \uparrow$

$\boxed{Y \uparrow \quad Q \downarrow}$

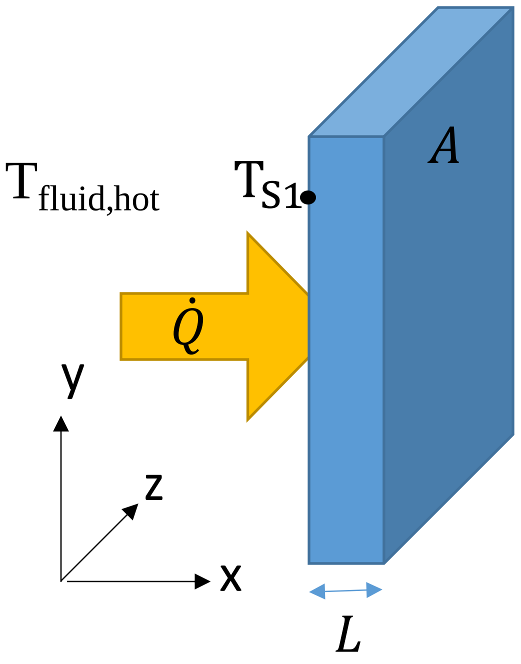
$\boxed{Q \not\propto Y}$

$\dot{Q}_{conv} \propto A_s (T_s - T_\infty)$

$\dot{Q}_{conv} = h \, A_s \, \Delta T$

h = heat transfer coefficient

3.3. Newton's law of cooling for convective heat transfer



$$\boxed{\dot{Q}_{conv} = A\,h\,\left(\mathrm{T}_{fluid,hot} - \mathrm{T}_{S1}\right)}$$

$\dot{Q}$ = heat flow $[W]$

***h* = heat transfer coefficient** $[W \cdot m^{-2} \cdot K^{-1}]$

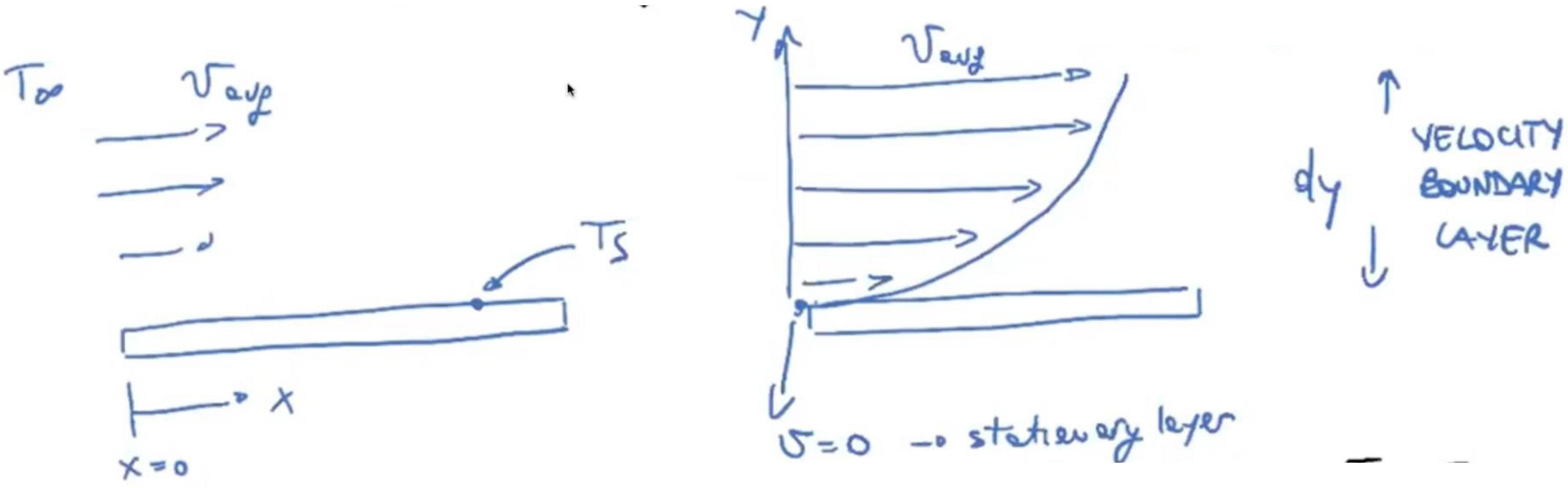
h can be local, h_{loc}, or a global average, $\bar{h}$

3.3. Newton’s law of cooling for convective heat transfer

HEAT TRANSFER COEFFICIENTS

Conditions of the convection	h $[\mathrm{W\ m^{-2}\ K^{-1}}]$
Natural (free) convection of a gas	5-37
Natural (free) convection of water	100-1200
Natural (free) convection of an oil	50-350
Gas flow in tubes or between tubes	10-350
Water flow in tubes	500-1200
Oil flow in tubes	300-1700
Flow of molten metal in the tubes	2000-45000
Water under nucleate boiling	3000-60000
Water under film-type boiling	100-300
Film-type condensation of water vapor	4000-17000
Condensation of water vapor with drops	30000-140000
Condensation of organic liquids	500-2300

Mechanism of convection and the boundary layers (microscopic view of convection)



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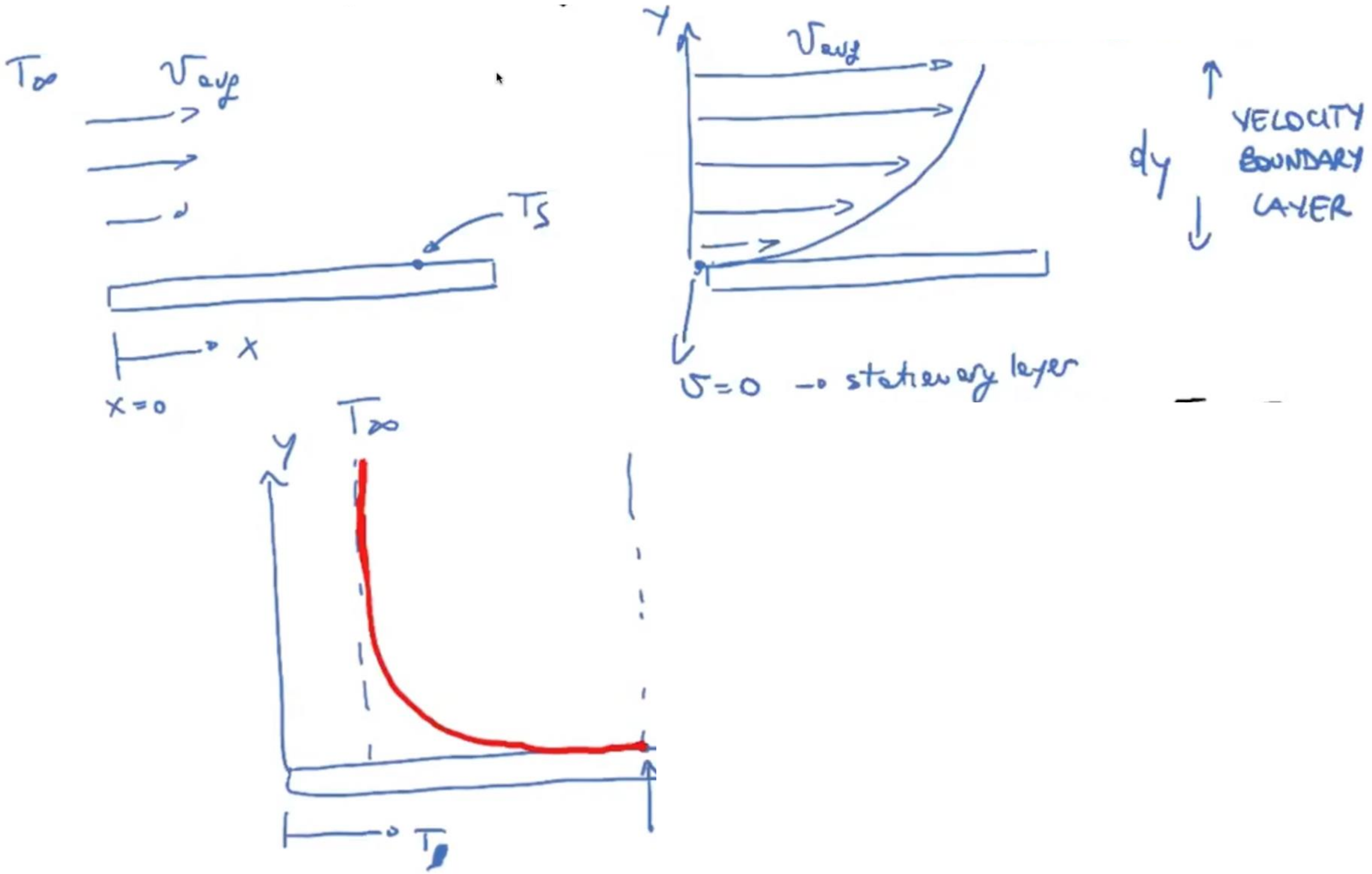

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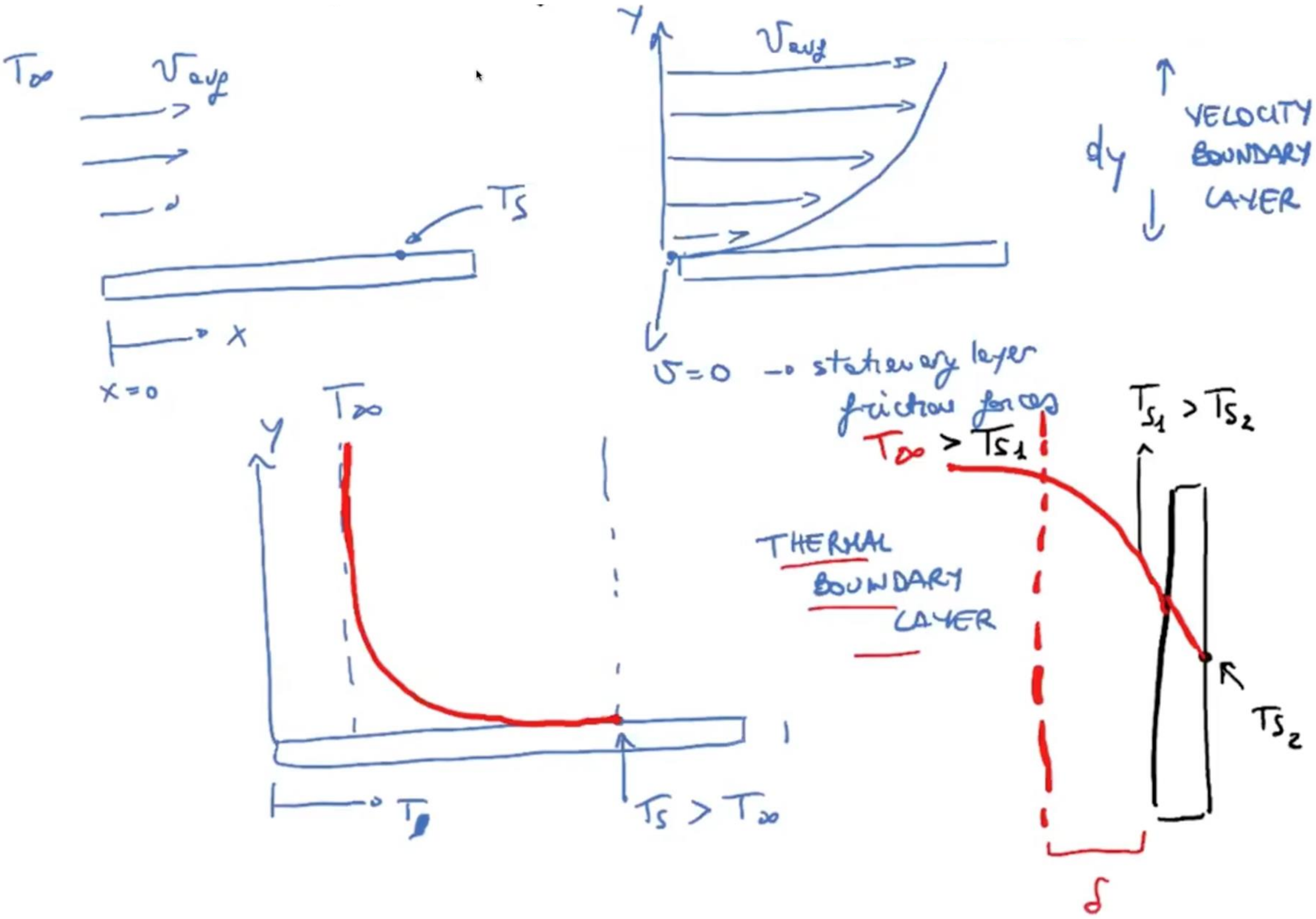
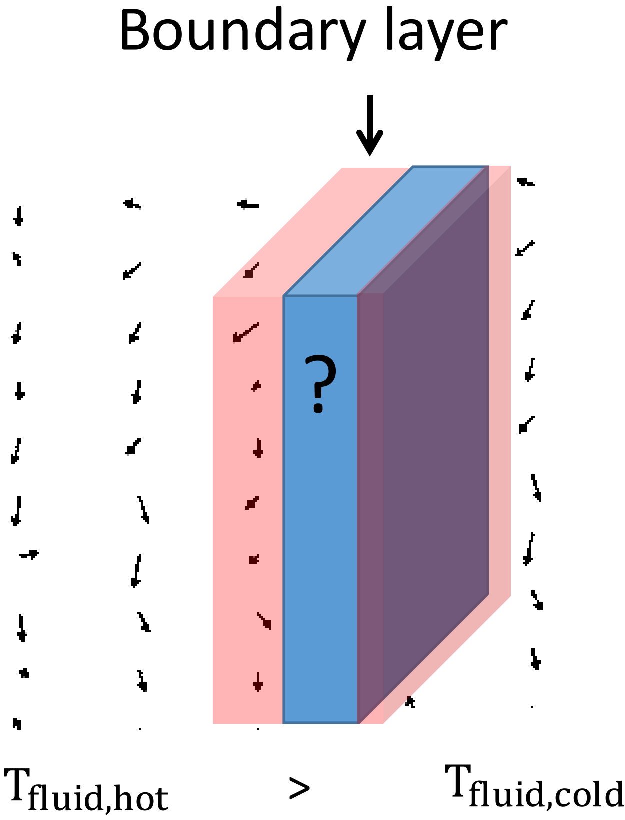


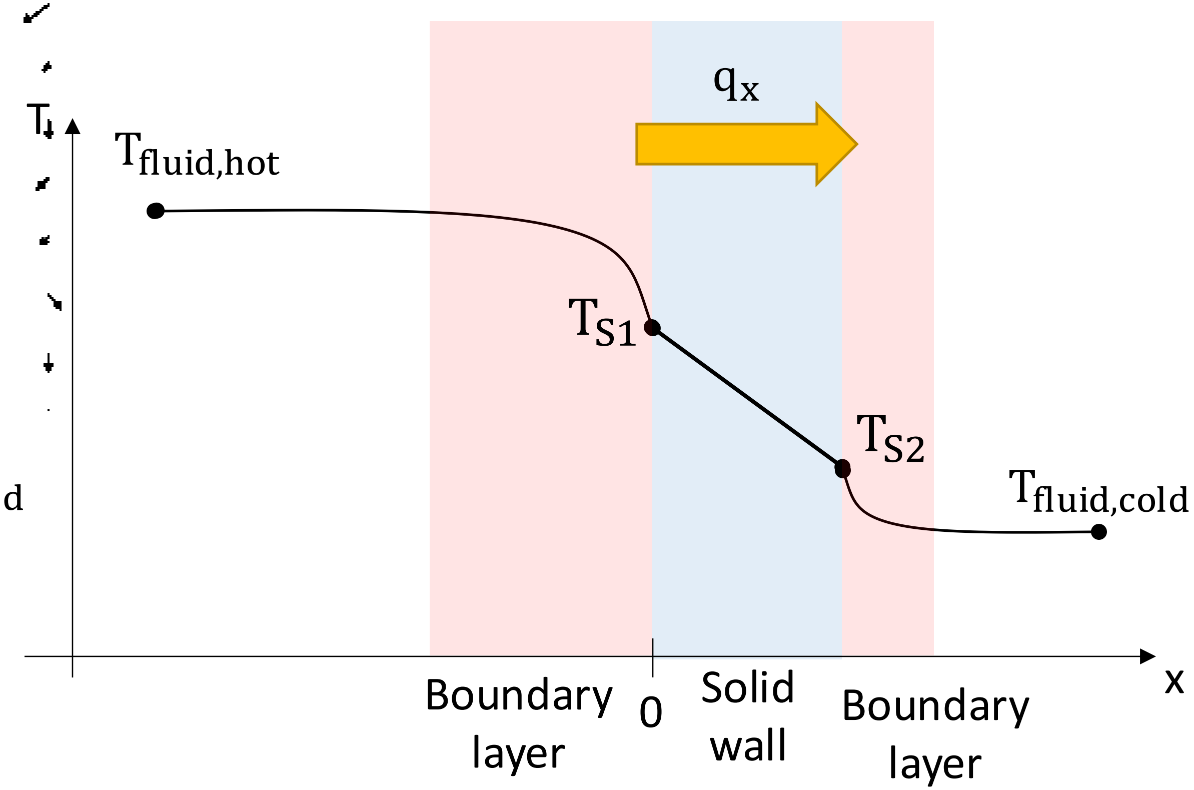
3.3. Newton's law of cooling for convective heat transfer

If the fluid flow is sufficiently turbulent we can consider there to be two regions:

Well mixed fluid all of the same temperature

Boundary layer near the interface





(at steady state)

3.3. Newton's law of cooling for convective heat transfer

At steady state (when temperatures are effectively constant):

$$q_x = h\left(T_{\mathrm{fluid,hot}} - T_{S1}\right)$$

$$[q_x] = [\mathrm{W\ m^{-2}}]$$

If the fluid flow is sufficiently turbulent we can consider there to be two regions:

Well mixed fluid all of the same temperature

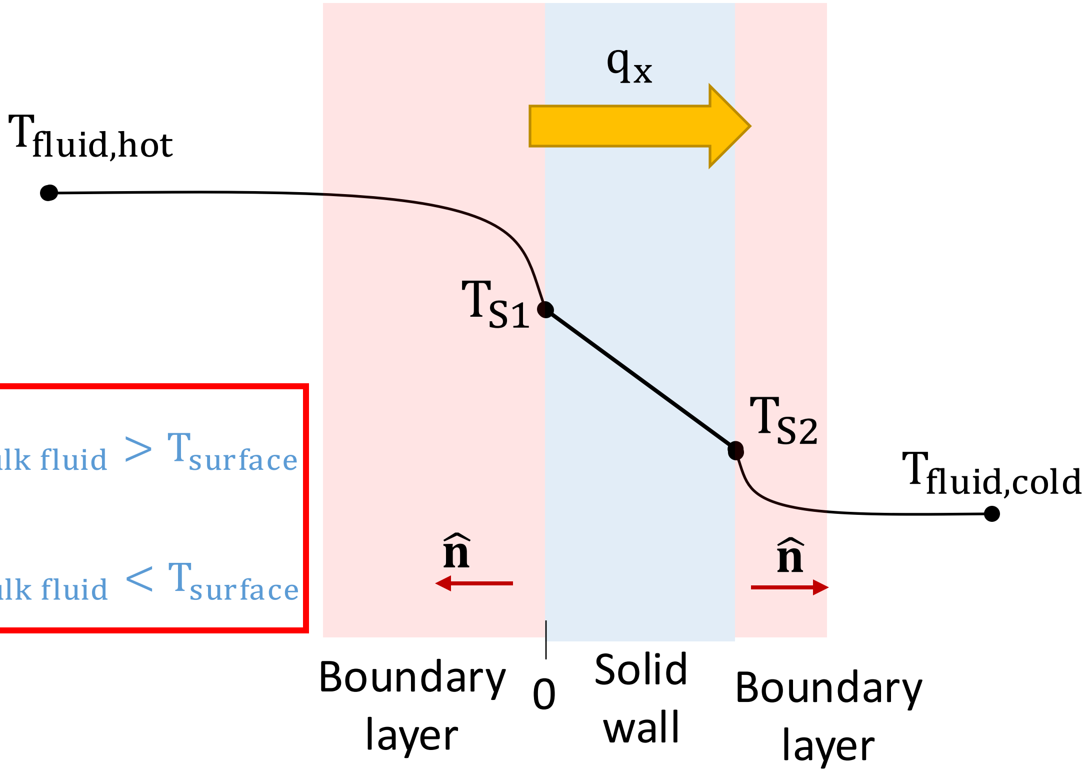
Boundary layer near the interface

In general over a convective boundary layer:

$$\mathbf{q} = -\hat{\mathbf{n}}\, h\,(T_{\mathrm{bulk\ fluid}} - T_{\mathrm{surface}}) \quad T_{\mathrm{bulk\ fluid}} > T_{\mathrm{surface}}$$

-or-

$$\mathbf{q} = \hat{\mathbf{n}}\, h\,(T_{\mathrm{surface}} - T_{\mathrm{bulk\ fluid}}) \quad T_{\mathrm{bulk\ fluid}} < T_{\mathrm{surface}}$$



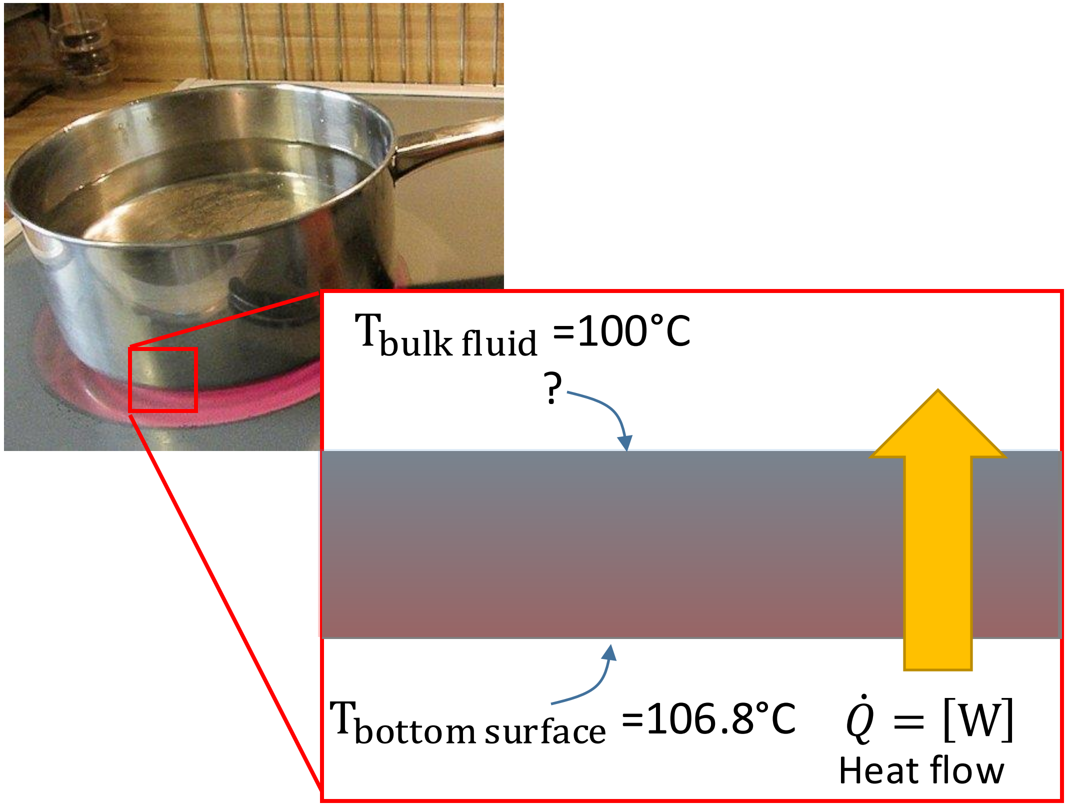
(at steady state)

3.3. Newton's law of cooling for convective heat transfer

Example: An aluminum saucepan (casserole) with a surface area of 0.02 m^2 and thickness, Y, of 7 mm is placed on a hot stove to boil 1 kg of water (at sea level). After the bulk of the water reaches 100°C, you measure the temperature of the bottom of the pan and it is steady at 106.8°C. After 3600 seconds at these conditions the water is completely gone. Can you estimate the average heat transfer coefficient, *h*, between the water and the saucepan? Can you estimate the temperature of the top surface of the pan?

For water: $\Delta H^{vap} = 2260\ \mathrm{kJ\ \ kg^{-1}}$

For aluminum: $k = 206\ \mathrm{W\ \ m^{-1}\ K^{-1}}$



3.3. Newton's law of cooling for convective heat transfer

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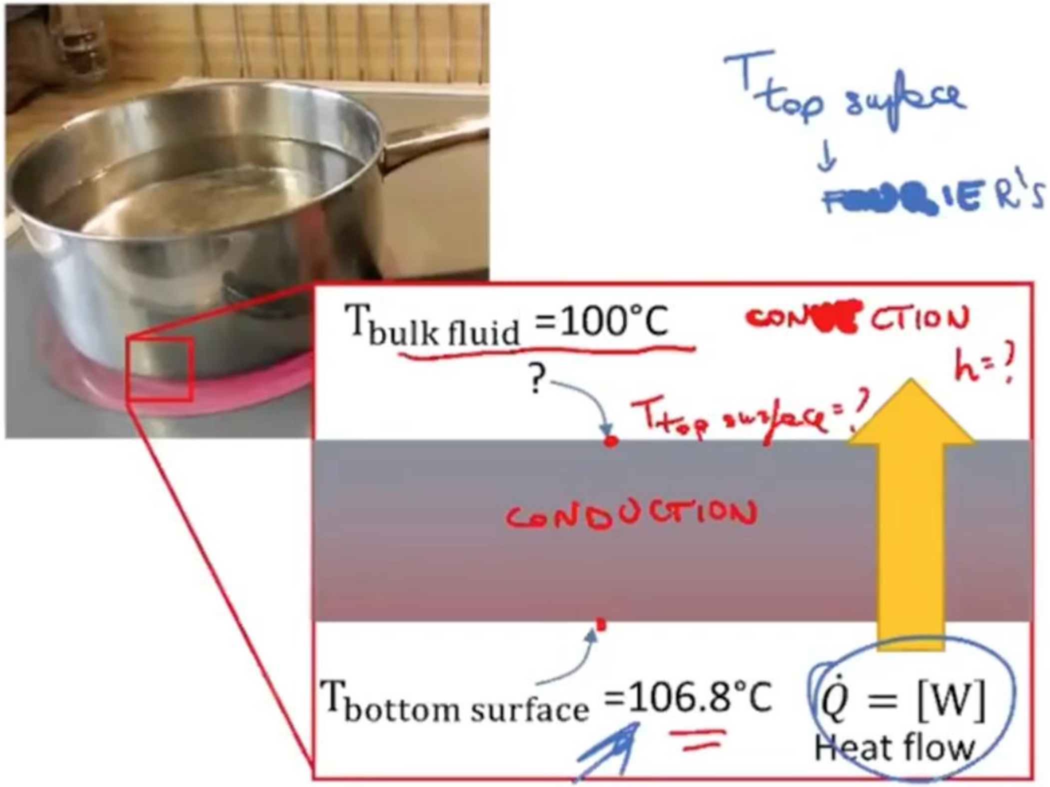
For water: $\Delta H^{vap} = 2260\ \mathrm{kJ\ kg^{-1}}$ → $\dot{Q} = \frac{\dot{m}\,\Delta H}{time} = 627.8\ W$

For aluminum: $k = 206\ \mathrm{W\ m^{-1}\ K^{-1}}$



$T_{top\ surface}$
↓
FOURIER'S LAW

$$\dot{Q} = \frac{k \overset{0.02\ m^2}{A} (T_{bottom} - T_{surface})}{Y}$$

Y = 7 mm

⇓

$T_{surface} = 105.7\ °C$

$\dot{Q}_{COND} = \dot{Q}_{CONV}$ Newton's Law

$\dot{Q} = h A (T_{top\ surface} - T_{bulk\ fluid})$

$h = 5507\ \frac{W}{m^2\ °C}$

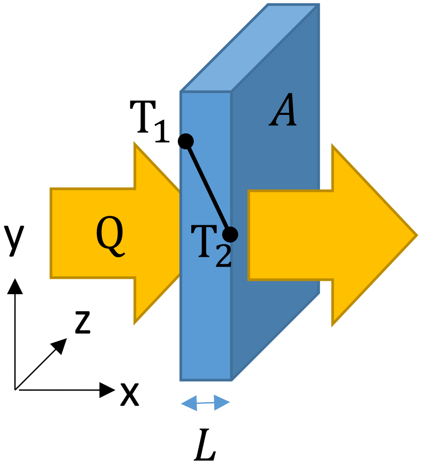
3.3. Newton’s law of cooling for convective heat transfer

DEFINITION OF THERMAL RESISTANCE

$$R_T = \frac{(T_1 - T_2)}{\dot{Q}}$$

$$\boxed{\dot{Q} = \frac{\Delta T}{\sum R_{th}}}$$

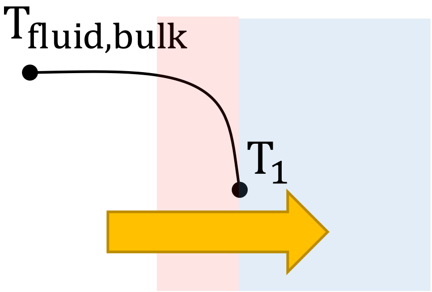
The thermal resistance of a wall (**conductive**):



$$\dot{Q} = A\,k\,\frac{(T_1 - T_2)}{L} = \frac{\Delta T}{R_{cond}}$$

$$R_{cond} = \frac{L}{kA}$$

The thermal resistance of a boundary layer (**convective**):



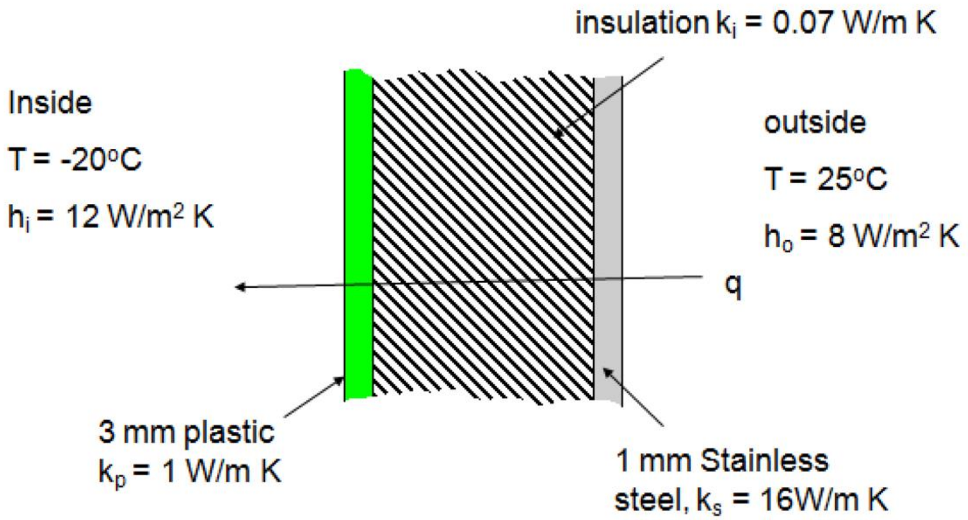
$$\dot{Q} = A\,h\,(T_{fluid,bulk} - T_{surf}) = \frac{\Delta T}{R_{conv}}$$

$$R_{conv} = \frac{1}{hA}$$

3.3. Newton's law of cooling for convective heat transfer

Example: The thermal resistance of a composite wall with convection

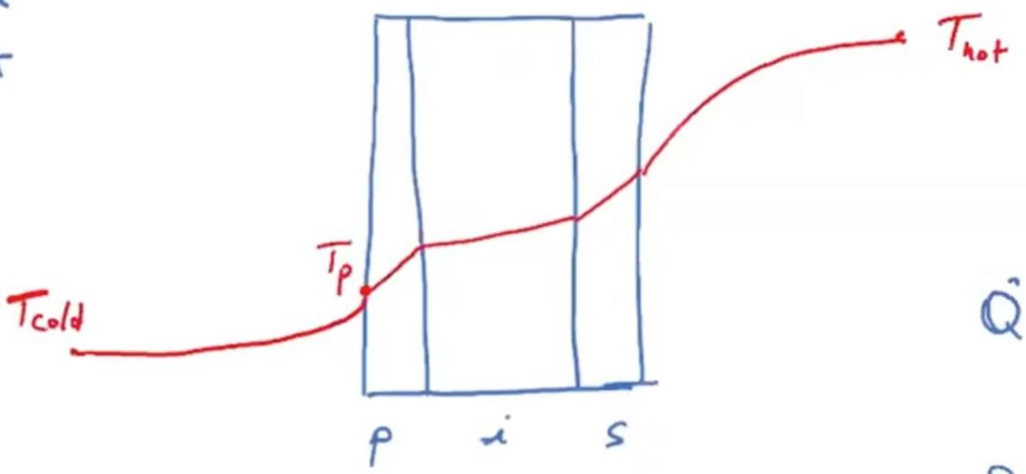
An industrial freezer is designed to operate with an internal air temperature of -20°C when the external air temperature is 25°C and the internal and external heat transfer coefficients are 12 W/m^2 K and 8 W/m^2 K, respectively. The walls of the freezer are composite construction, comprising of an inner layer of plastic (k = 1 W/m K, and thickness of 3 mm), and an outer layer of stainless steel (k = 16 W/m K, and thickness of 1 mm). Sandwiched between these two layers is a layer of insulation material with k = 0.07 W/m K. Find the width of the insulation that is required to reduce the convective heat loss to 15 W/m^2.



3.3. Newton's law of cooling for convective heat transfer

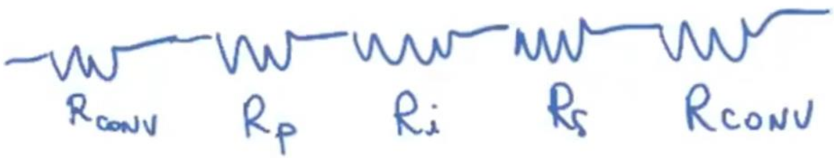
① temperature profile

② thermal circuit



$$\dot{Q} = \frac{\Delta T}{\Sigma R}$$

$$q = \frac{\dot{Q}}{A}$$



$$R_{TOT} = \frac{1}{A h_i} + \frac{L_p}{A k_p} + \frac{L_i}{A k_i} + \frac{L_s}{A k_s} + \frac{1}{A h_o}$$

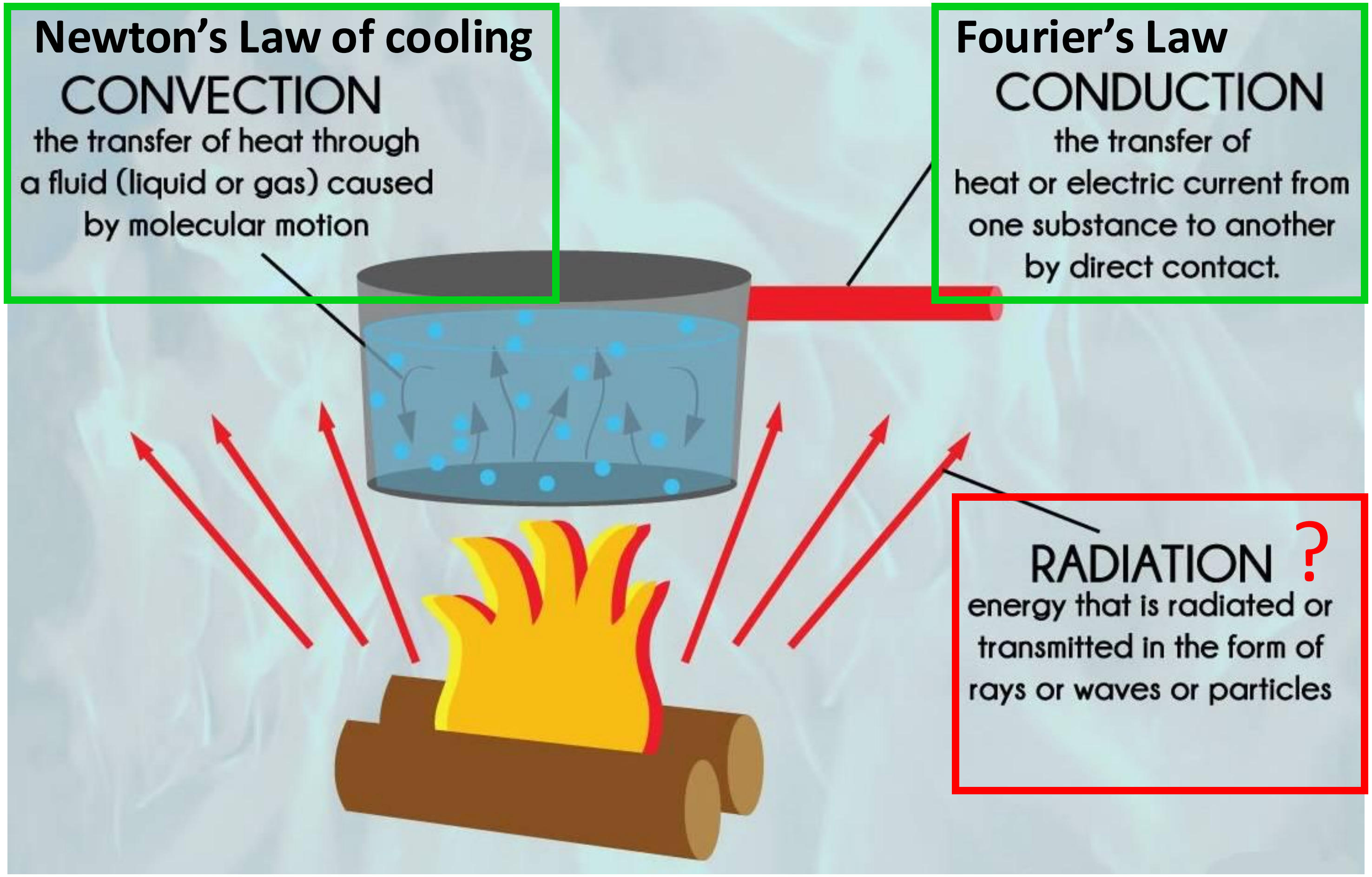
$15 \frac{W}{m^2}$ ↙

$$q = \frac{\Delta T = 25-(-20) \quad 45°C}{\left[\frac{1}{h_i} + \frac{L_p}{k_p} + \boxed{\frac{L_i}{k_i}} + \frac{L_s}{k_s} + \frac{1}{h_o}\right]} \frac{\cancel{A}}{\cancel{A}}$$

$L_i = 0.185 m$

3.0. Modes of heat transport

Heat transport is the movement of heat energy from one area of higher temperature to one area of lower temperature



Newton's Law of cooling

CONVECTION

the transfer of heat through a fluid (liquid or gas) caused by molecular motion

Fourier's Law

CONDUCTION

the transfer of heat or electric current from one substance to another by direct contact.

RADIATION ?

energy that is radiated or transmitted in the form of rays or waves or particles

Examples in real life: air conditioners, refrigerators, thermoelectrics

3.4. Radiative heat transport

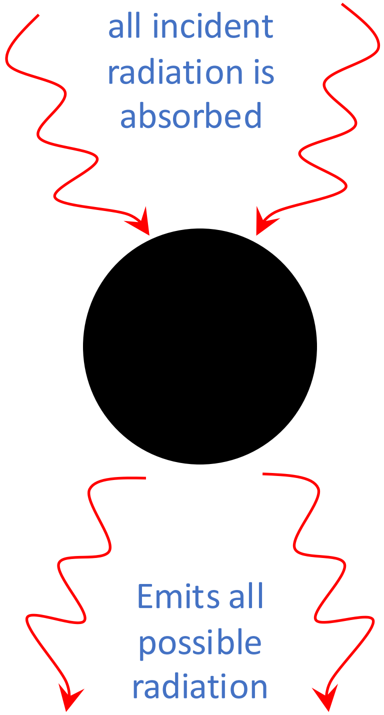
Heat can be transferred by infrared radiation. Unlike conduction and convection, which need particles, infrared radiation is a type of electromagnetic radiation that involves waves.

Radiation can even work through the vacuum of space. This is why we can still feel the heat of the Sun even though it is 150 million km away from the Earth.

$$\dot{Q} = ?$$

3.4. Radiative heat transport

A black body is a perfect absorber, it absorbs all wavelengths of thermal radiation incident on it. All black bodies heated at a given temperature emit thermal radiation



The radiation energy per unit time from a **black body** is proportional to the fourth power of the absolute temperature and can be expressed with **Stefan-Boltzmann Law** as:

$$\dot{Q} = \sigma T^4 A$$

where

$\dot{Q}$ = *heat flow per unit time (W)*

$\sigma = 5.6703\ 10^{-8}\ (W/m^2K^4)$ - *The Stefan-Boltzmann Constant*

T = *absolute temperature in Kelvin (K)*

A = *area of the emitting body* (m^2)

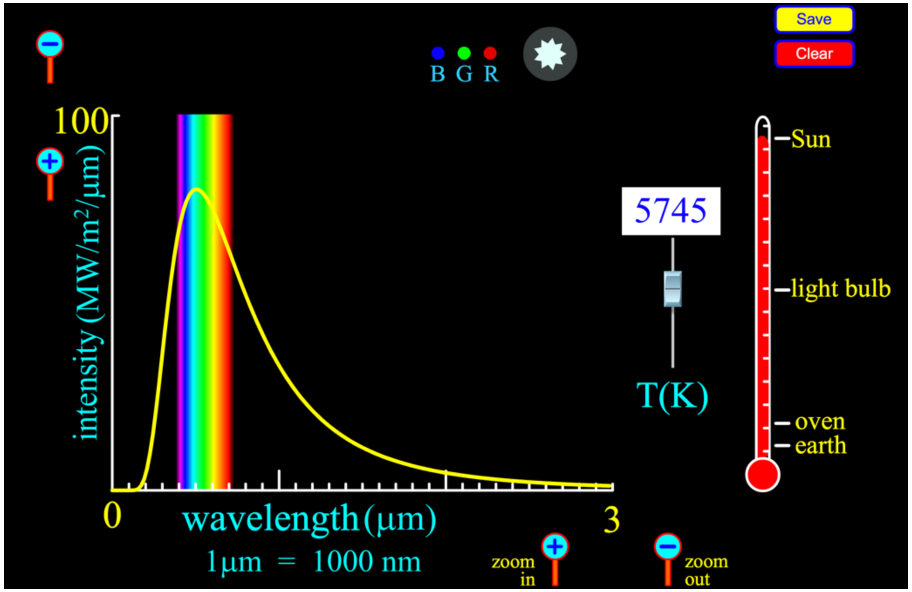
3.4. Radiative heat transport

$$\dot{Q} = \varepsilon \sigma T^4 A$$

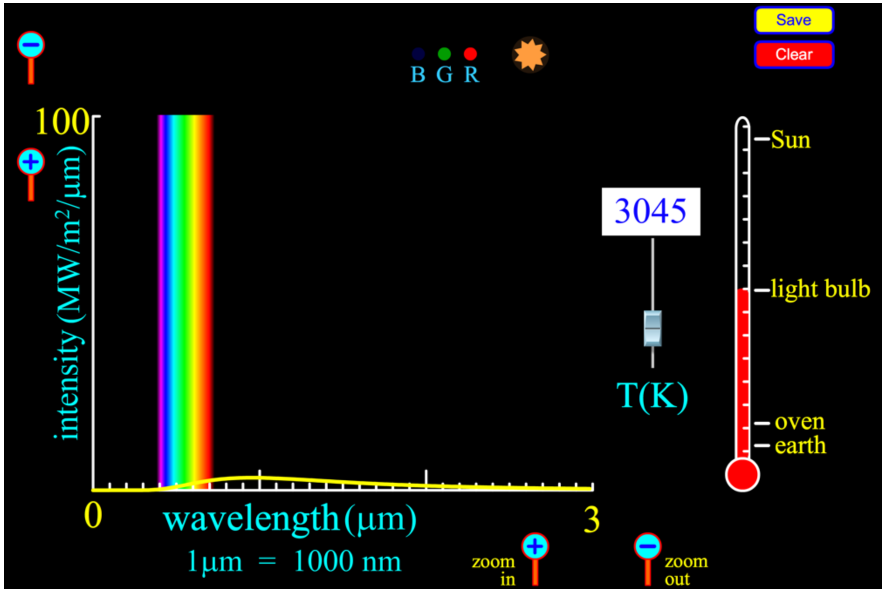
$$0 < \varepsilon < 1$$

Emissivity ($\mathcal{E}$) is the ratio of a surface's ability to emit radiant energy compared with the ability of a perfect black body of the same area at the same temperature. ***A material with high emissivity is efficient in both absorbing radiation energy as well as emitting it. Therefore, a good absorber is also a good emitter.***

Radiative emission from the Sun



Radiative emission from a light bulb



The wavelength and the intensity of the emitted radiation is proportional to the temperature

3.4. Radiative heat transport

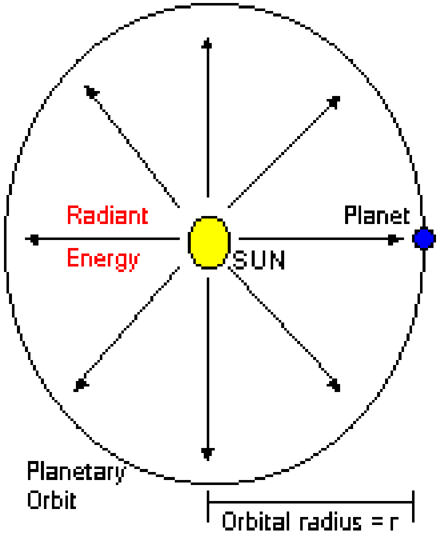
Emissivity of Some Common Materials

Materials	Emissivity Values
Carbon	0.85-0.95
Aluminum	0.11
Brass (oxidized)	0.61
Brass (unoxidized)	0.030
Copper (oxidized)	0.60
Copper (unoxidized)	0.020
Gold	0.020
Fire brick	0.75

Perfect reflectors would have = 0. Perfect absorbers would have = 1.

3.4. Radiative heat transport

Example: How to calculate the temperature on Earth's Surface



Intensity of sunlight when it reaches the Earth, called also solar constant

$$I_{sc} = q = \frac{\dot{Q}}{A}$$

$$\dot{Q}_{sun} = 3.86 \times 10^{26}\ \mathrm{W}$$

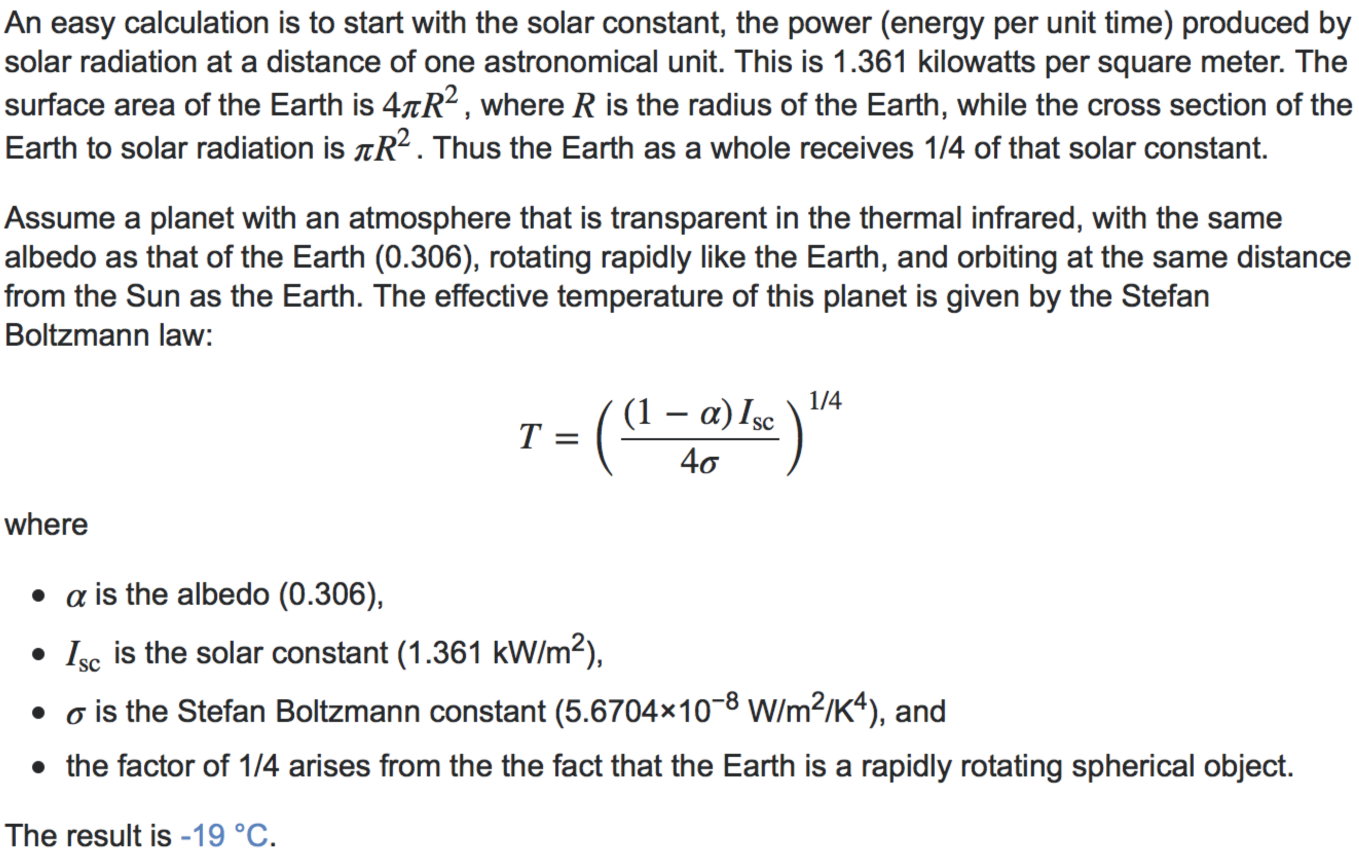
$$A = 4\pi r^2\ (r=6.96 \times 10^8\ \mathrm{m})$$

$$I_{sc} = 1.361\ \mathrm{kW/m^2}$$

Consider the hypothetical sphere surrounding the Sun with a radius r and centered on the Sun. The radius of this sphere is the same as the radius of the planetary orbit.

3.4. Radiative heat transport

Example: How to calculate the temperature on Earth’s Surface



An easy calculation is to start with the solar constant, the power (energy per unit time) produced by solar radiation at a distance of one astronomical unit. This is 1.361 kilowatts per square meter. The surface area of the Earth is $4\pi R^2$, where R is the radius of the Earth, while the cross section of the Earth to solar radiation is πR^2. Thus the Earth as a whole receives 1/4 of that solar constant.

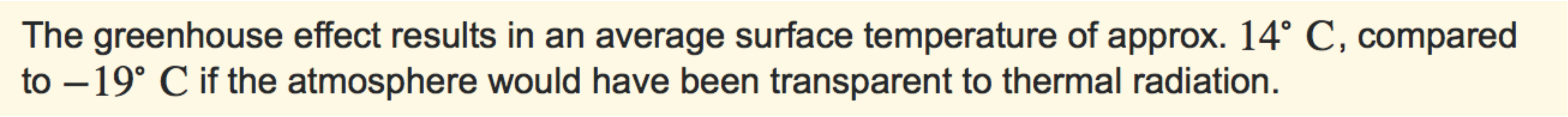
Assume a planet with an atmosphere that is transparent in the thermal infrared, with the same albedo as that of the Earth (0.306), rotating rapidly like the Earth, and orbiting at the same distance from the Sun as the Earth. The effective temperature of this planet is given by the Stefan Boltzmann law:

$$T = \left(\frac{(1-\alpha)\, I_{\text{sc}}}{4\sigma} \right)^{1/4}$$

where

- α is the albedo (0.306),
- I_{sc} is the solar constant (1.361 kW/m^2),
- σ is the Stefan Boltzmann constant (5.6704×10^{-8} W/m^2/K^4), and
- the factor of 1/4 arises from the the fact that the Earth is a rapidly rotating spherical object.

The result is -19 °C.



The greenhouse effect results in an average surface temperature of approx. 14° C, compared to -19° C if the atmosphere would have been transparent to thermal radiation.

HEAT TRANSFER RECAP: Conduction, Radiation, Convection

A cubic box with the side of 80 cm is surrounded by air at T_{air}=15ºC (with a convection coefficient of h=10W/m^{2}K). The outer surface of the box has a uniform surface temperature of T_{out}=100ºC and thermal emissivity ε=0.9. The box is made of plastic and the walls are 0.5 cm thick with thermal conductivity k = 1.3 W/m K. Calculate the temperature in the inner surface of the box.

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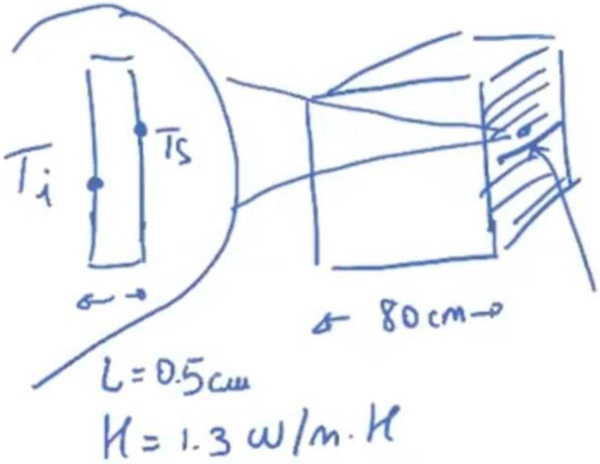


T_S = 100°C

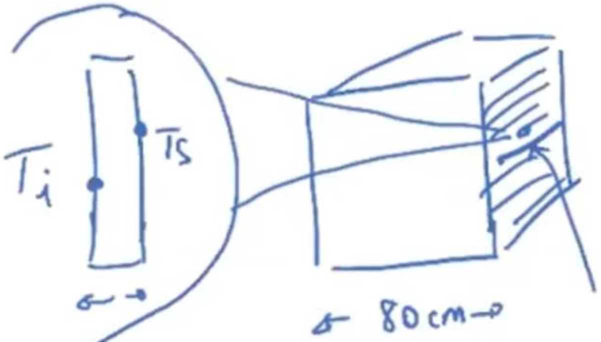
- $T_\infty = 15°C$

$h_{AIR} = 10 \frac{W}{m^2 \cdot K}$

$\varepsilon = 0.9$

HEAT TRANSFER RECAP: Conduction, Radiation, Convection

A cubic box with the side of 80 cm is surrounded by air at T_{air}=15ºC (with a convection coefficient of h=10W/m^{2}K). The outer surface of the box has a uniform surface temperature of T_{out}=100ºC and thermal emissivity ε=0.9. The box is made of plastic and the walls are 0.5 cm thick with thermal conductivity k = 1.3 W/m K. Calculate the temperature in the inner surface of the box.



$T_S = 100°C$

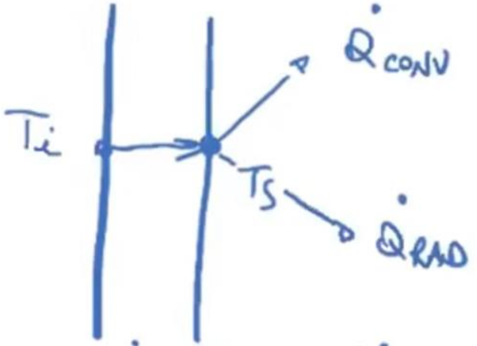
$L = 0.5 cm$

$K = 1.3 W/m \cdot K$

- $T_\infty = 15°C$

$h_{AIR} = 10 \frac{W}{m^2 \cdot K}$

$\varepsilon = 0.9$



$$\dot{Q}_{COND}\Big|_{surface} = \left|\dot{Q}_{CONV} + \dot{Q}_{RAD}\right|_{surf}$$

HEAT TRANSFER RECAP: Conduction, Radiation, Convection

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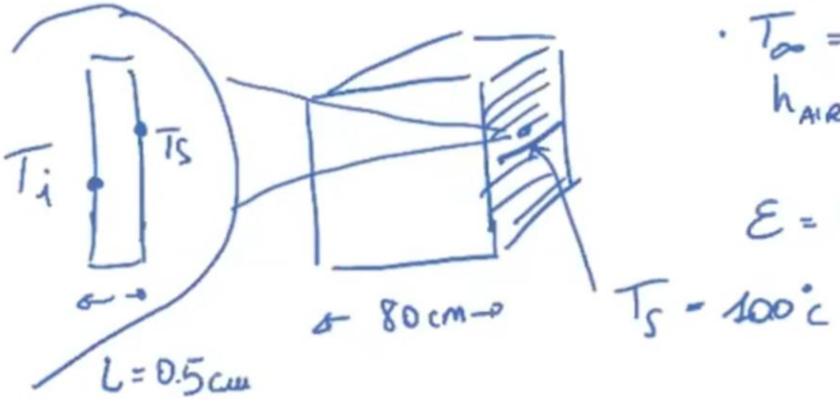


L = 0.5 cm

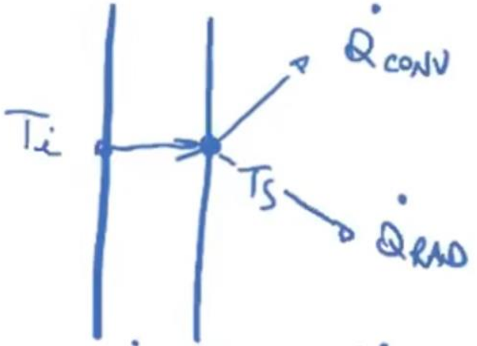
K = 1.3 W/m·K

- $T_\infty = 15°C$

$h_{AIR} = 10 \frac{W}{m^2 \cdot K}$

$\varepsilon = 0.9$

$$\frac{\dot{Q}_{COND}}{\dot{Q}_{CONV}}$$



$$\dot{Q}_{COND}\Big|_{surface} = \Big|\dot{Q}_{CONV} + \dot{Q}_{RAD}\Big|_{surf}$$

$$\dot{Q}_{CONV} = A_s\, h\,(T_s - T_\infty) = 3264\ W$$

$$A_s = 6 \times 0.8^2\ m^2 = 3.84\ m^2$$

$$\dot{Q}_{RAD} = A_s\, \varepsilon\, \sigma\, (T_s^4 - T_\infty^4) = 4.5\ W$$

$$\sigma = 5.6 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$$

HEAT TRANSFER RECAP: Conduction, Radiation, Convection

$$\dot{Q}_{COND} = \dot{Q}_{CONV} + \dot{Q}_{RAD} = \underline{\underline{3281}}\ W$$

$$\underline{\underline{\dot{Q}_{COND}}} = \frac{A_s\, k\, \left(\underset{\underset{100^\circ C}{\uparrow}}{T_s} - T_i\right)}{L}$$

$$T_i = 370^\circ C$$

RECAP FROM LECTURES: conduction and convection

CONDUCTION, Fourier's Law

$$\dot{Q}_{COND} = \frac{K\,A\,\Delta T}{L}$$

↑ thermal conductivity

temperature profile is linear

$T = f(L)$

$K(T)$ we don't consider this dependence

CONVECTION : Newton's Law

$$\dot{Q}_{CONV} = A\,h\,\Delta T$$

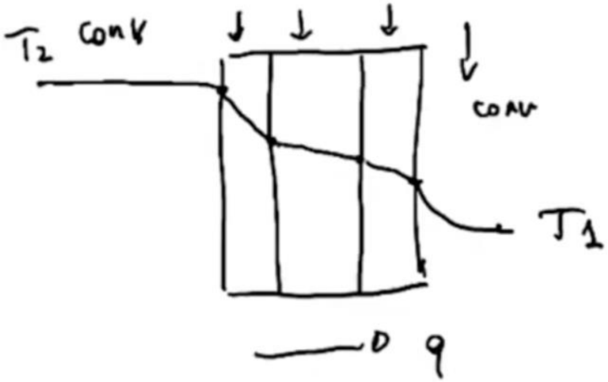
↑ heat transfer coefficient

temperature profile is parabolic within the boundary layer

$T = f(L)$

$h = f(L)$ we don't consider this dependence

CIRCUIT OF THERMAL RESISTANCES (last exercize)



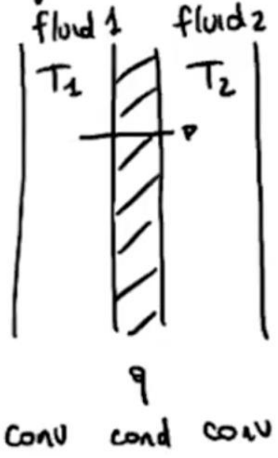
$T_2 > T_1$

$$\dot{Q} = \frac{\Delta T}{\Sigma R_{TOT}} = \frac{T_2 - T_1}{R_{TOT}}$$

$R_{CONV1} + R_{COND1} + R_{COND2} + R_{COND3} + R_{CONV2}$

HEAT EXCHANGERS:

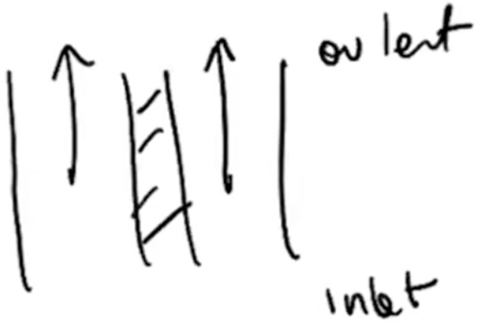
are devices that transfer heat between two fluids at different temperature without the fluids mixing!



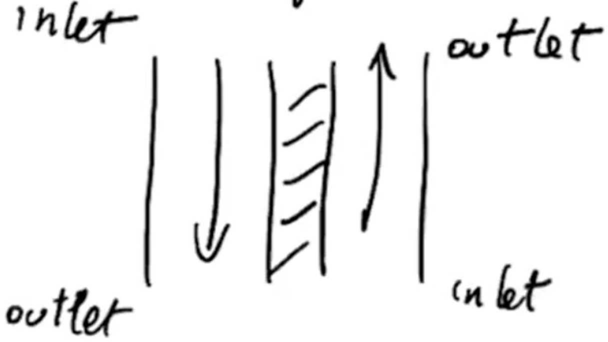
$T_1 > T_2$

ΔT_m

parallel flow



counter flow



3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1

The general function of a heat exchanger is to transfer heat from one fluid to another. The basic component of a heat exchanger can be viewed as a tube with one fluid running through it and another fluid flowing by on the outside. To note that the fluids never mix.

U-tube heat exchanger

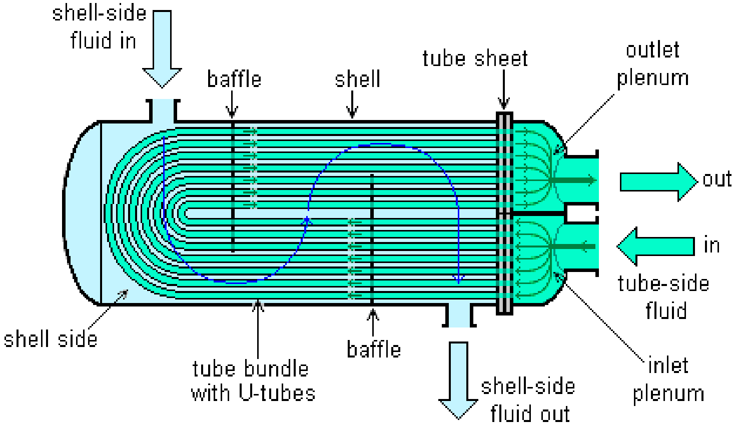
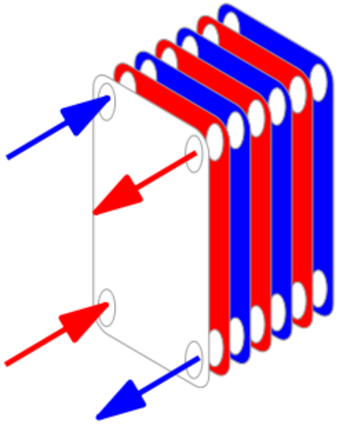
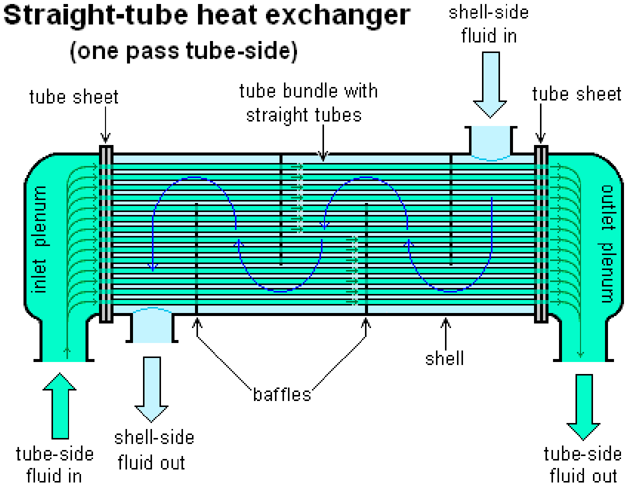


Plate heat exchanger



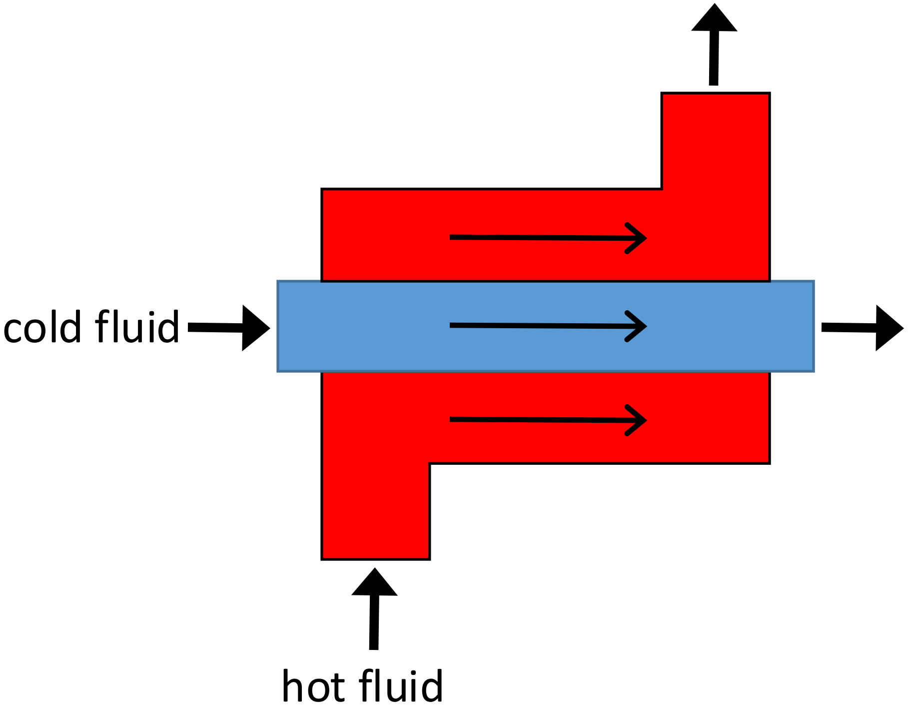
Straight-tube heat exchanger

(one pass tube-side)

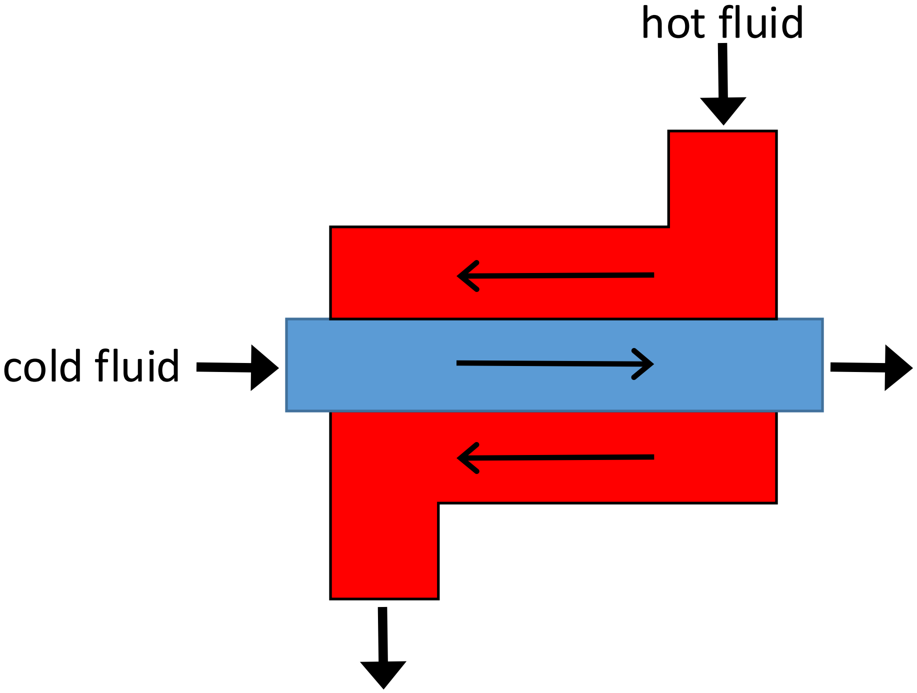


3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1



parallel flow



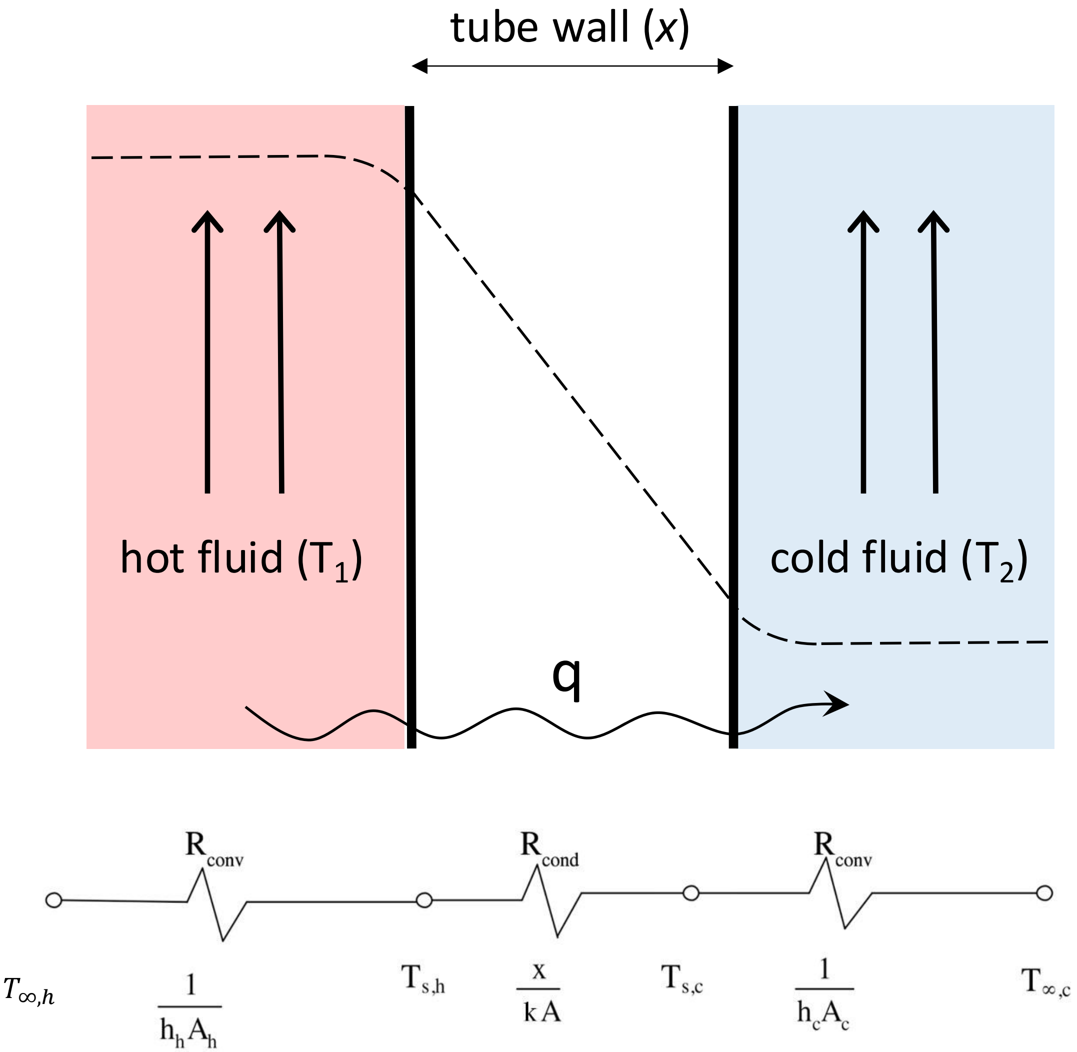
counter flow

There are three heat transfer operations that need to be described:

1. Convective heat transfer from the fluid to the inner wall of the tube
2. Conductive heat transfer through the tube wall
3. Convective heat transfer from the outer tube wall to the outside fluid.

3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1



$$\boxed{q = \frac{\dot{Q}}{A} = \frac{\Delta T}{A \sum R_T}}$$

Reminder

Fourier's First Law (conduction):

$$q_{cond} = -k\nabla T \quad \dot{Q}_{cond} = -kA\,\frac{(T_2 - T_1)}{x}$$

$$R_{T,cond} = \frac{x}{kA}$$

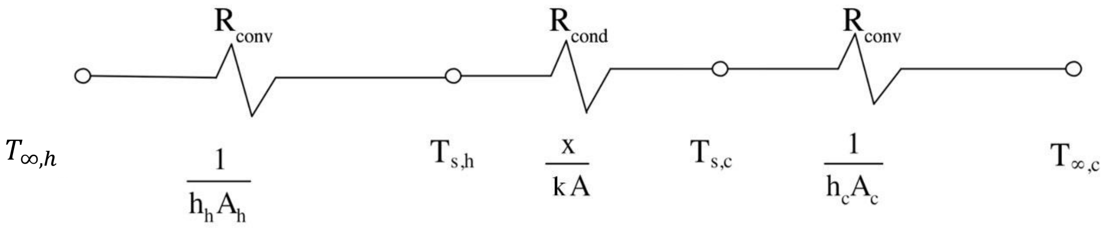
Newton's law of cooling (convection)

$$\dot{Q}_{conv} = A\,h\,\left(T_{surface} - T_{fluid}\right)$$

$$R_{T,conv} = \frac{1}{hA}$$

3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1



$$q = \frac{\dot{Q}}{A} = \frac{\Delta T}{A\sum R_T}$$

$$q = \frac{(T_{\infty,h} - T_{\infty,c})}{A\sum R_T} = \frac{(T_{\infty,h} - T_{\infty,c})}{\frac{1}{h_h}+\frac{x}{k}+\frac{1}{h_c}}$$

$$\frac{1}{UA} = \left[\sum R_T\right]^{-1} = \left[R_{conv,hotfluid} + R_{cond,wall} + R_{conv,cold\,fluid})\right]^{-1}$$

$$= \left[\frac{1}{h_h}+\frac{x}{k}+\frac{1}{h_c}\right]^{-1}$$

$$Q = AU\Delta T_m$$

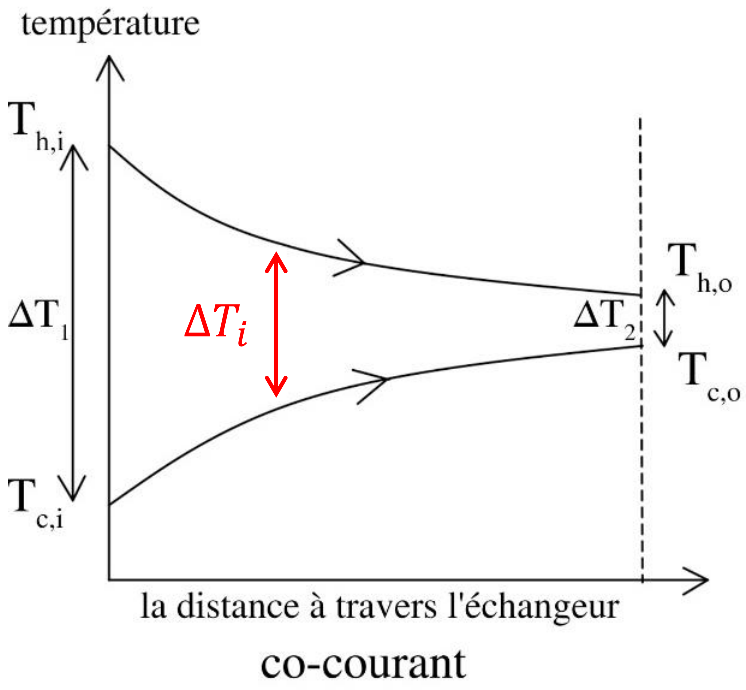
$$\boxed{q = U(T_{\infty,h} - T_{\infty,c})}$$

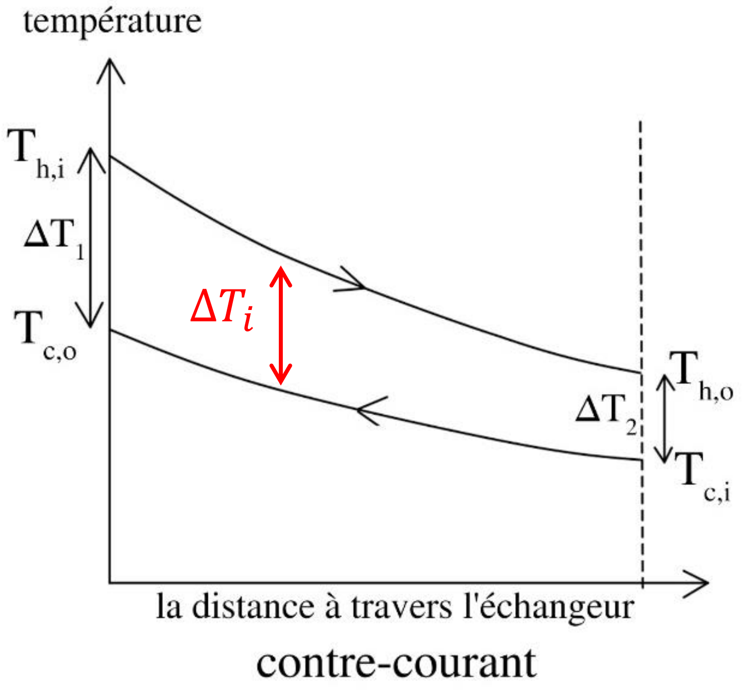
***U* = overall heat transfer coefficient per unit area** [$W \cdot m^{-2} \cdot K^{-1}$]

3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1

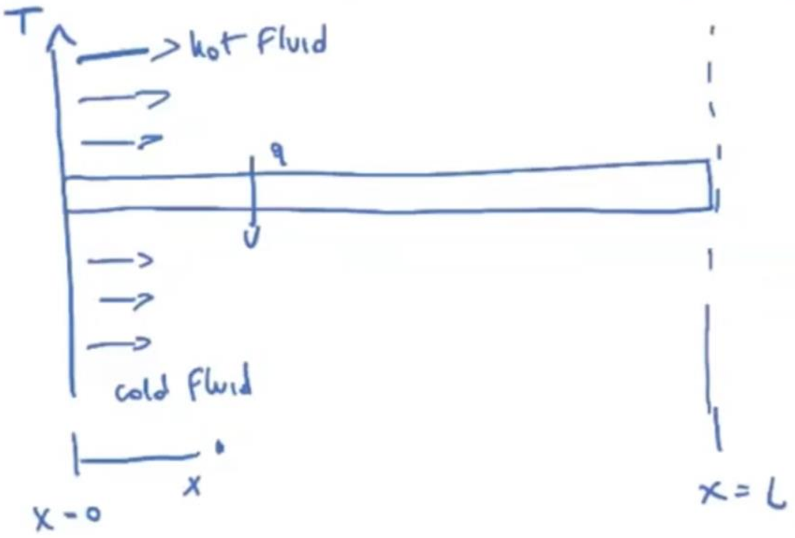
Temperature Profile in the heat exchangers



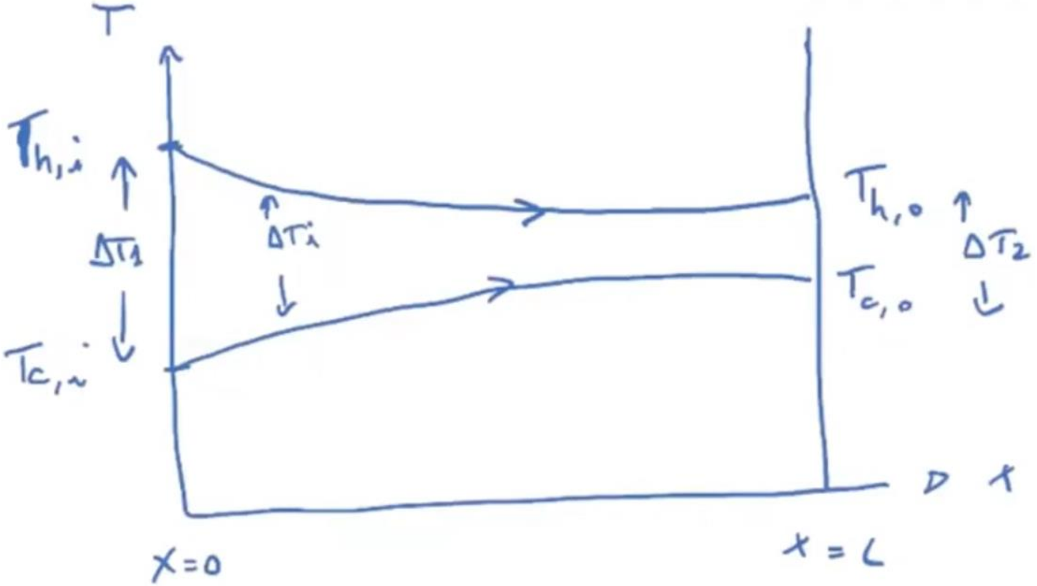


$\mathbf{T_{h,i}}$ is the temperature of the hot fluid at the entrance of the heat exchanger
$\mathbf{T_{h,o}}$ is the temperature of the hot fluid at the exit of the heat exchanger
$\mathbf{T_{c,i}}$ is the temperature of the cold fluid at the entrance of the heat exchanger
$\mathbf{T_{c,o}}$ is the temperature of the cold fluid at the exit of the heat exchanger

Temperature profile in parallel flow heat exchangers



$$\Delta T_{lm} = f(\Delta T_1, \Delta T_2)$$

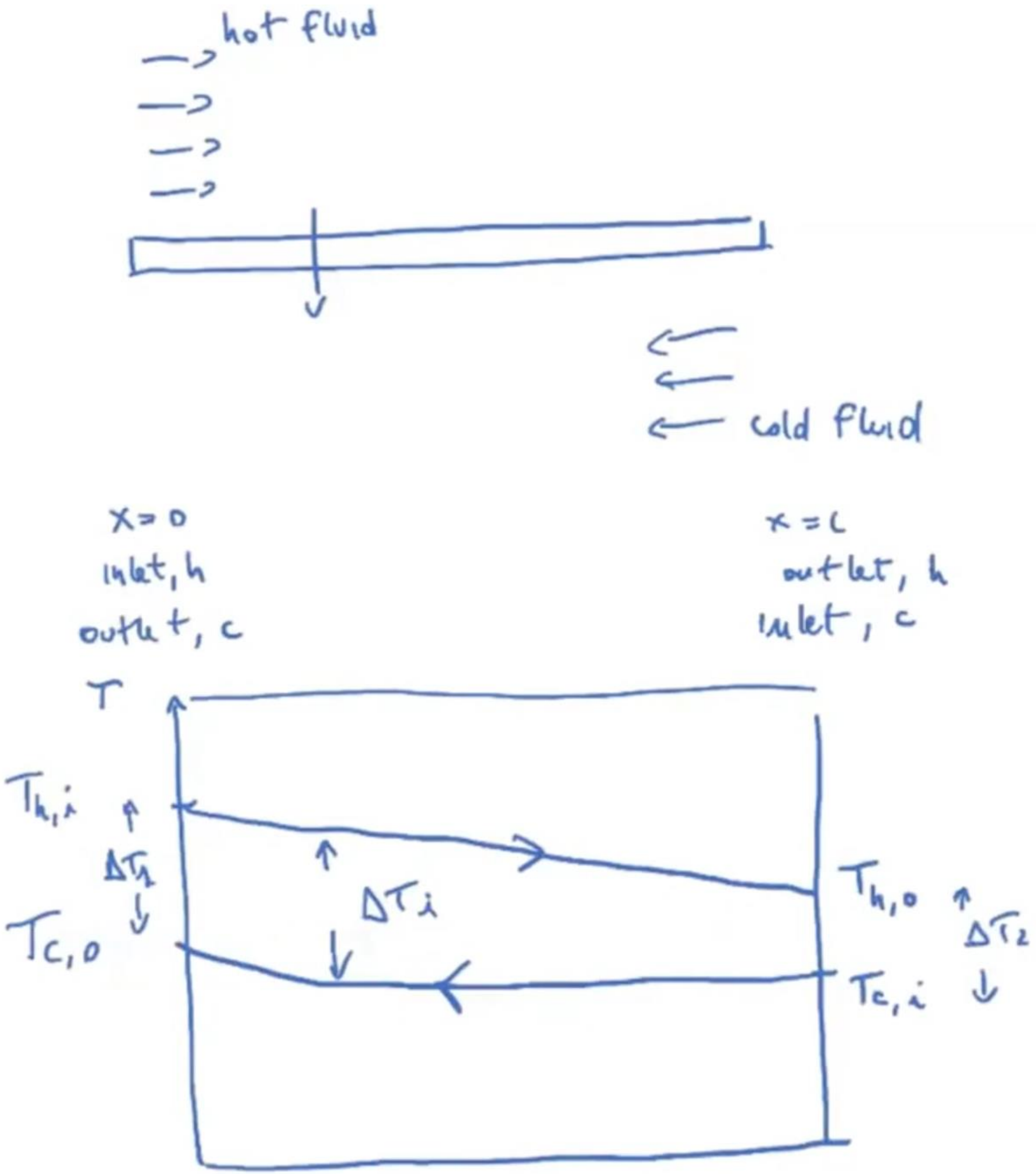


$$\Delta T_1 > \Delta T_2$$

$$\Delta T_1 = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,o}$$

Temperature profile in counter-flow heat exchangers



$$\Delta T_1 > \Delta T_2$$

$$\Delta T_1 = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1

$$\dot{Q} = UA\Delta \mathrm{T}_m$$

Because T is changing, which ΔT are we going to consider?

$$^{*}\ \boxed{\Delta \mathrm{T}_{lm} = \frac{\Delta \mathrm{T}_1 - \Delta \mathrm{T}_2}{\ln\left(\frac{\Delta \mathrm{T}_1}{\Delta \mathrm{T}_2}\right)}}$$

$\Delta T_1 = T_{h,i} - T_{c,i}$ et $\Delta T_2 = T_{h,o} - T_{c,o}$ parallel flow

$\Delta T_1 = T_{h,i} - T_{c,o}$ et $\Delta T_2 = T_{h,o} - T_{c,i}$ counter flow

assuming U, $T_{h,i}$ and $T_{c,i}$ are the same

$$\Delta \mathrm{T}_{lm}, \mathrm{counter} > \Delta \mathrm{T}_{lm}, \mathrm{parallel}$$

which means

$$A\ \mathrm{counter} < A\ \mathrm{parallel}$$

Therefore, counter-flow heat exchangers are more efficient than parallel-flow heat exchangers!

* This expression will be derived in class

3.5. Heat Exchangers

RELEVANT FOR ChE-203 TP-1

How can we estimate **U**?

$$\dot{Q} = UA\Delta \mathrm{T}_m$$

Heat loss

$$\dot{Q}_h = \dot{m}_h C_{p,h}(\mathrm{T}_{h,o} - \mathrm{T}_{h,i})$$

$$\dot{Q}_c = \dot{m}_c C_{p,c}(\mathrm{T}_{c,i} - \mathrm{T}_{c,o})$$

Heat gain

or

$$\dot{Q}_h = \dot{m}_h C_{p,h}(\mathrm{T}_{h,i} - \mathrm{T}_{h,o})$$

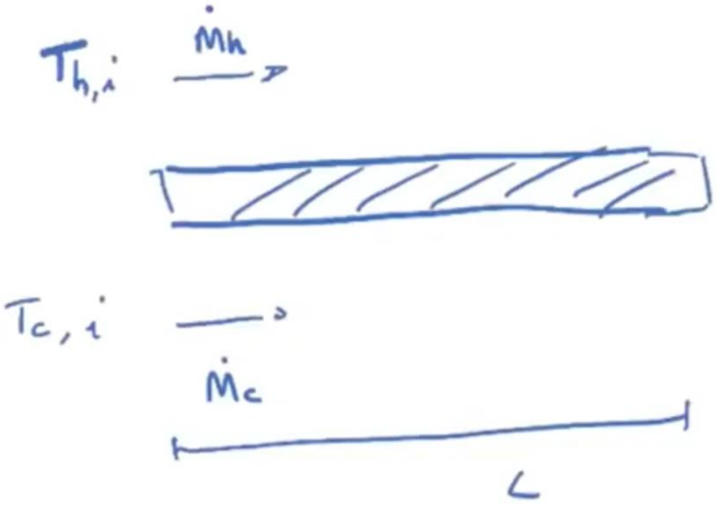
$$\dot{Q}_c = \dot{m}_c C_{p,c}(\mathrm{T}_{c,o} - \mathrm{T}_{c,i})$$

$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rate [kg s^{-1}]
$C_{p,h}$ and $C_{p,c}$ are specific heat capacity [J kg^{-1}K^{-1}]

$$\boxed{\mathrm{U\,A} = \frac{\dot{Q}_h}{\Delta \mathrm{T}_{lm}} = \frac{\dot{Q}_h}{\dfrac{\Delta \mathrm{T}_1 - \Delta \mathrm{T}_2}{\ln\left(\dfrac{\Delta \mathrm{T}_1}{\Delta \mathrm{T}_2}\right)}}}$$

where Q can be derived from one of the two expression above because for the energy balance **the heat lost by the hot fluid must be equal to the heat gained by the cold fluid.**

Derivation of log mean temperature ΔTm



$$A = L \cdot \underbrace{2\pi R}_{\text{circumference} \rightarrow B}$$

$$dA = B\,dx$$

$$d\dot{Q} = U\,\Delta T\,dA = \underbrace{-\dot{m}_h\,C_{p,h} \cdot dT_h}_{\text{enthalphy change on the hot side}} = \underbrace{\dot{m}_c\,C_{p,c} \cdot dT_c}_{\text{enthalpy change on the cold side}}$$

$$Q_h = Q_c$$

The “-” sign is there because the dT of the hot fluid is negative

Derivation of log mean temperature ΔTm

$$A = L \cdot \underbrace{2\pi R}_{\text{circumference} \rightarrow B}$$

$$dA = B\,dx$$

$$d(\Delta T) = dT_h - dT_c = -\frac{dQ}{\dot{m}_h C_{ph}} - \frac{dQ}{\dot{m}_c C_{pc}} =$$

$$= -U\,\Delta T\,dA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_p}\right)$$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_p}\right) B \int_0^L dx$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

Based on the fact that

$$\dot{Q} = \dot{m}_h C_{p,h}(T_{h,i} - T_{h,o})$$

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,o} - T_{c,i})$$

Derivation of log mean temperature ΔTm

$$= -\frac{UA}{\dot{Q}}\left(T_{h,i} - T_{h,o} + T_{c,o} - T_{c,i}\right) = P$$

$$\dot{Q} = UA \boxed{\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}}$$

$$\dot{Q} = UA \quad \Delta T_m$$

3.5. Heat Exchangers

Exercize: Heat transfer area and log mean temperature difference

A heavy hydrocarbon oil which has Cp=2.30 kJ · Kg^{-1} · K^{-1} is being cooled in a heat exchanger from 371.9K to 349.7K and flows inside the tube at a rate of 3630 Kg/h. A flow of 1450Kg water/h enters at 288.6K for cooling and flows outside the tube.

a) Calculate the water outlet temperature and heat-transfer area if the overall Ui=340W/m^2 · K and the streams are countercurrent.

b) Repeat for parallel flow

SOLUTION

a)

$$\dot{\boldsymbol{Q}}_{\boldsymbol{oil}} = \dot{\boldsymbol{m}}_{\boldsymbol{oil}}\boldsymbol{C}_{\boldsymbol{p,oil}}\left(\mathbf{T}_{\boldsymbol{oil,i}} - \mathbf{T}_{\boldsymbol{oil,o}}\right) = \left(3630\ \frac{Kg}{h}\right)\left(2.30\ \frac{kJ}{Kg\ K}\right)(371.9 - 349.7\ K) = 185400\ \frac{KJ}{h} = 51490\ W$$

Heat balance $\dot{\boldsymbol{Q}}_{\boldsymbol{oil}} = \dot{\boldsymbol{Q}}_{\boldsymbol{water}}$

$$\dot{\boldsymbol{Q}}_{\boldsymbol{water}} = \dot{\boldsymbol{m}}_{\boldsymbol{water}}\boldsymbol{C}_{\boldsymbol{p,water}}\left(\mathbf{T}_{\boldsymbol{water,o}} - \mathbf{T}_{\boldsymbol{water,i}}\right) = \left(1450\ \frac{Kg}{h}\right)\left(4.187\ \frac{kJ}{Kg\ K}\right)\left(\mathrm{T}_{water,o} - 288.6\ K\right) = 185400\ \frac{KJ}{h}$$

⇩

$$\mathbf{T}_{\boldsymbol{water,o}} = 319.1K$$

$$Q = UA\Delta \mathrm{T}_{lm} \quad \Rightarrow \quad \boldsymbol{A} = \frac{\boldsymbol{Q}}{\boldsymbol{U\Delta \mathrm{T}_{lm}}}$$

$$\Delta \mathrm{T}_{lm} = \frac{\Delta \mathrm{T}_1 - \Delta \mathrm{T}_2}{\ln\left(\frac{\Delta \mathrm{T}_1}{\Delta \mathrm{T}_2}\right)} = \frac{\left(\mathrm{T}_{oil,i} - \mathrm{T}_{water,o}\right) - \left(\mathrm{T}_{oil,o} - \mathrm{T}_{water,i}\right)}{\ln\left(\frac{\mathrm{T}_{oil,i} - \mathrm{T}_{water,o}}{\mathrm{T}_{oil,o} - \mathrm{T}_{water,i}}\right)} == \frac{52.8 - 61.1}{\ln\frac{52.8}{61.1}} = 56.9K$$

$$\boldsymbol{\Delta \mathrm{T}_1} = \boldsymbol{\mathrm{T}_{oil,i}} - \boldsymbol{\mathrm{T}_{water,o}} = 52.8$$

$$\boldsymbol{\Delta \mathrm{T}_2} = \boldsymbol{\mathrm{T}_{oil,o}} - \boldsymbol{\mathrm{T}_{water,i}} = 61.1$$

$$A = \frac{Q}{U_i \Delta \mathrm{T}_{lm}} = \frac{51490}{340 \times 56.9} = 2.66\, m^2$$

b) In **parallel flow** we have that:

$$\boldsymbol{\Delta \mathrm{T}_1} = \boldsymbol{\mathrm{T}_{oil,i}} - \boldsymbol{\mathrm{T}_{water,i}} = 371.9 - 288.6 = 83.3K$$

$$\boldsymbol{\Delta \mathrm{T}_2} = \boldsymbol{\mathrm{T}_{oil,o}} - \boldsymbol{\mathrm{T}_{water,o}} = 349.7 - 319.1 = 30.6K$$

$$\Delta \mathrm{T}_{lm} = 52.7K$$

$$\boldsymbol{A = 2.87 m^2}$$

(note that the exchange area needed in parallel flow is bigger than the one needed in counter flow)

3.6. Fouling

- The performance of a heat exchanger depends upon surfaces being clean but deposits form over time

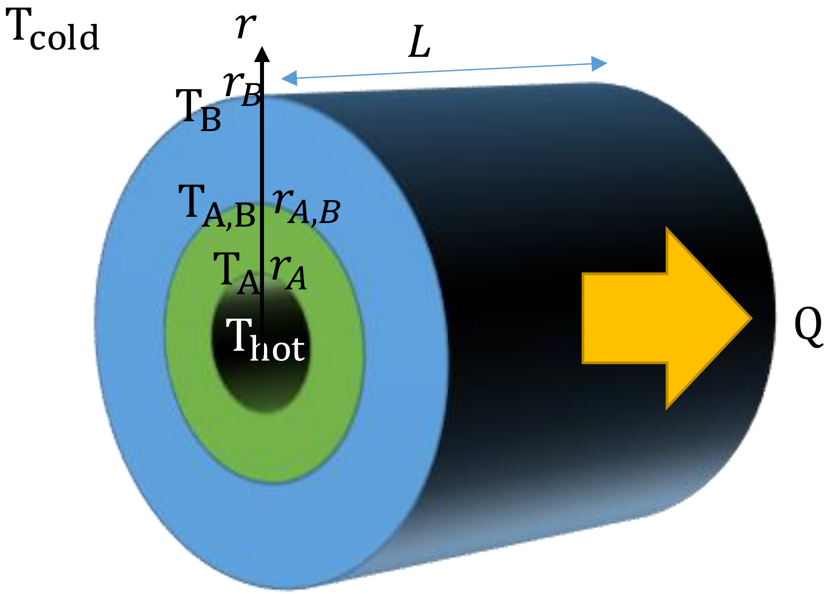
- The layer of deposits presents additional resistance to heat transfer and must be accounted for by a fouling factor, R_f

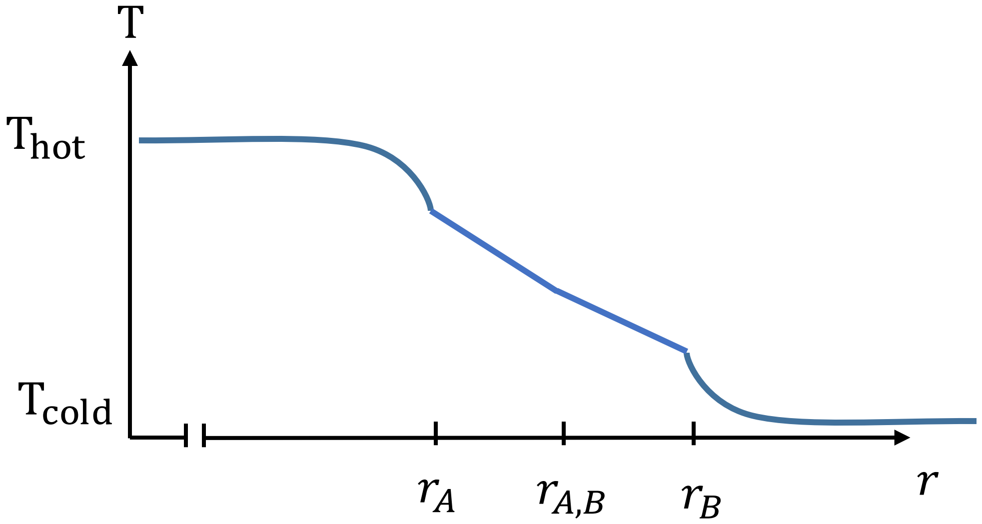
$$\frac{1}{U} = R_{conv,hotfluid} + R_{cond,wall} + \boxed{R_f} + R_{conv,cold\ fluid}$$

- Deposits can occur by the precipitation of solid deposits (i.e. calcium in a kettle), corrosion or chemical fouling due to chemical reactions, and the growth of algae, biological fouling

- Periodic cleaning of exchangers and the resulting downtime are additional penalties associated with fouling.

3.7. Heat transfer in composite systems: the thermal resistance of a composite wall with convection in a cylindrical geometry





For material “A”(green):

$$\mathrm{q} = -k\boldsymbol{\nabla}T \qquad \text{(Fourier's)}$$

$$\mathrm{q_r} = -k\frac{\partial T}{\partial r}$$

$$\mathrm{Q_r} = -kA\frac{\partial T}{\partial r} \qquad A = 2\pi rL$$

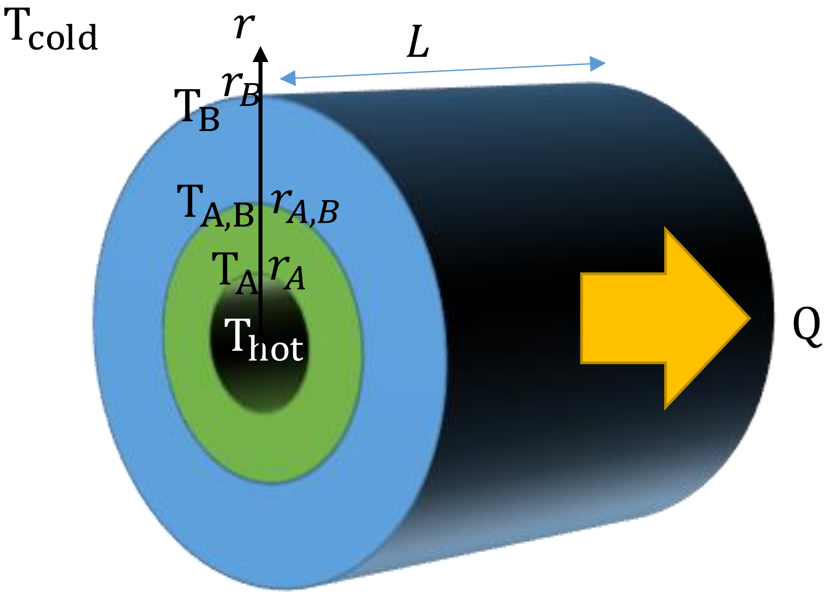
$$\mathrm{Q_r} = -2\pi krL\frac{\partial T}{\partial r}$$

$$\mathrm{Q_r}\int_{r_A}^{r_{A,B}}\frac{dr}{r} = -2\pi k_A L\int_{\mathrm{T_A}}^{\mathrm{T}_{\mathrm{A},B}} dT$$

$$\mathrm{Q_r}\ln\left(\frac{r_{A,B}}{r_A}\right) = 2\pi k_A L\left(\mathrm{T_A} - \mathrm{T}_{\mathrm{A},B}\right)$$

$$\mathrm{Q_r} = \frac{2\pi k_A L\,\Delta T}{\ln\left(\frac{r_{A,B}}{r_A}\right)} \qquad R_{con,\,cyl} = \frac{\ln\left(\frac{r_{A,B}}{r_A}\right)}{2\pi k_A L}$$

3.7. Heat transfer in composite systems: the thermal resistance of a composite wall with convection in a cylindrical geometry



Subscript *i* : internal
Subscript *e* : external

$$A_{lm,A} = 2\pi L \frac{r_{A,B} - r_A}{\ln\left(\frac{r_{A,B}}{r_A}\right)}$$

$$\mathrm{Q_r} = \frac{k_A A_{lm,A}\, \Delta T}{r_{A,B} - r_A}$$

$$\mathrm{Q} = \frac{\Delta T}{\sum R_{th}}$$

$$R_{cond,cyl,A} = \frac{r_{A,B} - r_A}{k_A A_{lm}} = \frac{\ln\left(\frac{r_{A,B}}{r_A}\right)}{2\pi k_A L}$$

$$R_{conv,i} = \frac{1}{A_A h_i} = \frac{1}{2\pi r_A L h_i}$$

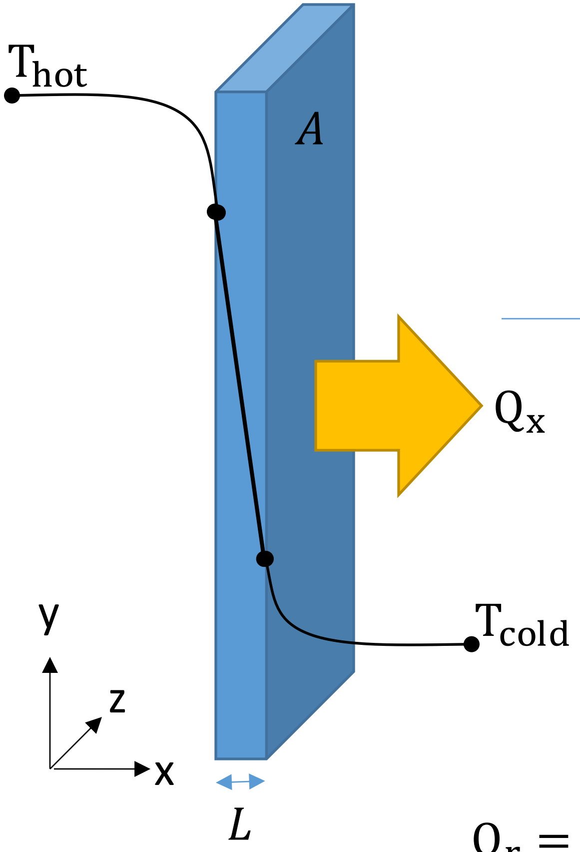
$$R_{cond,cyl,B} = \frac{r_B - r_{A,B}}{k_B A_{lm,B}} = \frac{\ln\left(\frac{r_B}{r_{A,B}}\right)}{2\pi k_B L}$$

$$R_{conv,e} = \frac{1}{A_B h_e} = \frac{1}{2\pi r_B L h_e}$$

$$\mathrm{Q_r} = \frac{\Delta T}{R_{conv,i} + R_{cond,cyl,A} + R_{cond,cyl,B} + R_{conv,e}}$$

$$\mathrm{Q_r} = \frac{\Delta T}{\frac{1}{2\pi r_A L h_i} + \frac{\ln\left(\frac{r_{A,B}}{r_A}\right)}{2\pi k_A L} + \frac{\ln\left(\frac{r_B}{r_{A,B}}\right)}{2\pi k_B L} + \frac{1}{2\pi r_B L h_e}}$$

3.7. Heat transfer in composite systems: relations to the overall heat transfer coefficient U



$$Q_x = \frac{(T_c - T_f)}{\frac{1}{h_c A} + \frac{L}{kA} + \frac{1}{h_f A}} = \frac{A\Delta T}{\frac{1}{h_c} + \frac{L}{k} + \frac{1}{h_f}}$$

$$Q_x = UA\Delta T \qquad U = \frac{1}{\frac{1}{h_c} + \frac{L}{k} + \frac{1}{h_f}}$$

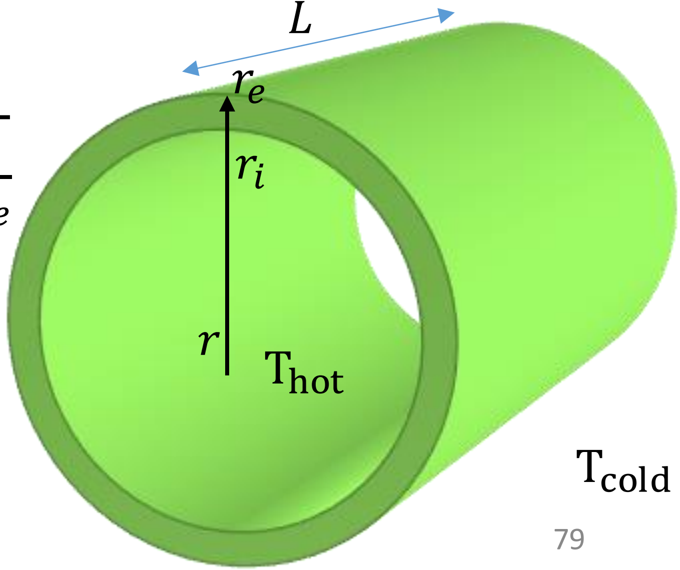
overall heat transfer coefficient per unit area

$$Q_r = \frac{\Delta T}{\frac{1}{2\pi r_i L h_i} + \frac{\ln(r_e/r_i)}{2\pi k L} + \frac{1}{2\pi r_e L h_e}}$$

$$Q_r = \frac{2\pi L \Delta T}{\frac{1}{r_i h_i} + \frac{\ln(r_e/r_i)}{k} + \frac{1}{r_e h_e}}$$

$$Q_r = U_e A_e \Delta T$$

$$\frac{1}{r_e U_e} = \frac{1}{r_i h_i} + \frac{\ln(r_e/r_i)}{k} + \frac{1}{r_e h_e}$$



3.8. Prandtl and Nusselt numbers

The properties of the fluids and the different forms of heat transfer can be described by some dimensionless numbers:

Prandtl is the ratio between the fluid ability to store heat and to transfer heat through conduction, independent of the system geometry:

$$Pr = \frac{viscous\ diffusion\ rate}{thermal\ diffusion\ rate} = \frac{\mu/\rho}{k/\rho C_p} = \frac{\mu C_p}{k}$$

dynamic (or absolute) viscosity [Kg · m^{-1} · s^{-1} or Pa · s]

specific heat capacity [J · Kg^{-1} · K^{-1}]

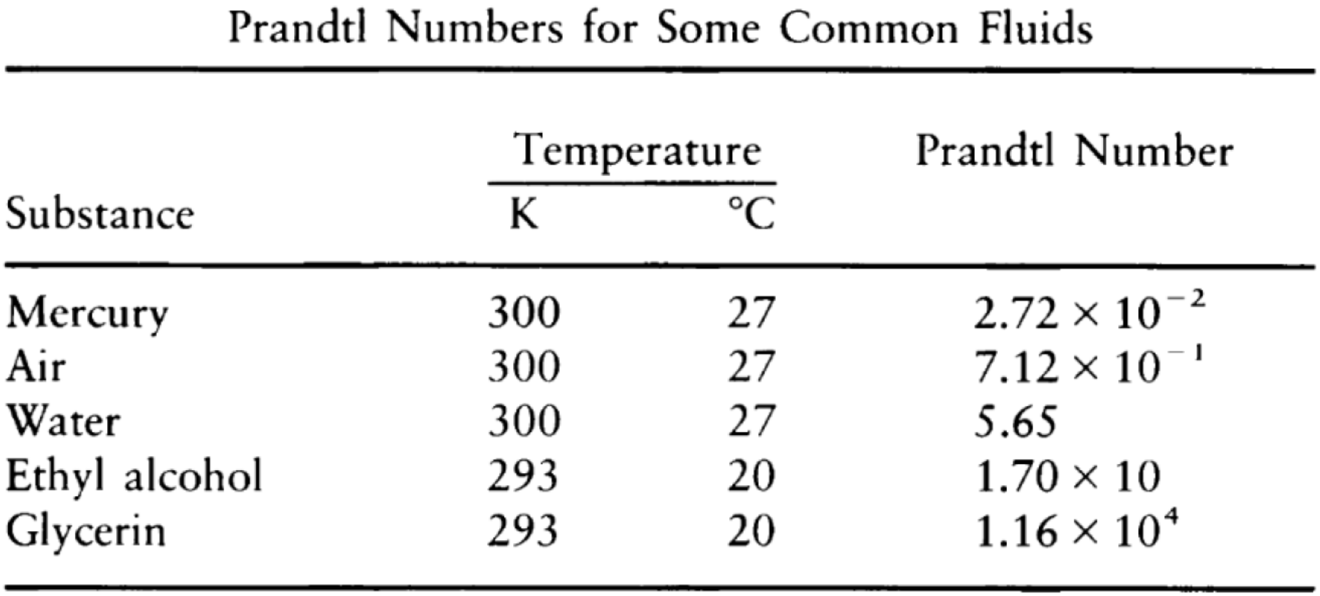
thermal conductivity [W · m^{-1} · K^{-1} or J · s^{-1} · m^{-1} · K^{-1}]

$Pr \ll 1$ Thermal diffusivity dominates

$Pr \gg 1$ Momentum diffusivity dominates

Substances with high *Pr* will store heat rather than transferring it! (ex. Water and glycerin in the next slide)

3.8. Prandtl and Nusselt numbers



Prandtl Numbers for Some Common Fluids

	Temperature		Prandtl Number
Substance	K	°C	
Mercury	300	27	2.72×10^{-2}
Air	300	27	7.12×10^{-1}
Water	300	27	5.65
Ethyl alcohol	293	20	1.70×10
Glycerin	293	20	1.16×10^{4}

For example, the listed value for liquid mercury indicates that the heat conduction is more significant compared to convection, so thermal diffusivity is dominant.

3.8. Prandtl and Nusselt numbers

Nusselt describes the ratio of the **thermal energy convected** to the fluid to the **thermal energy conducted** within the fluid

heat transfer coefficient
[$W \cdot m^{-2} \cdot K^{-1}$ or $J \cdot s^{-1} \cdot m^{-2} \cdot K^{-1}$]

$$Nu = \frac{hL}{k}$$

Thickness of the fluid layer
[m]

thermal conductivity
[$W \cdot m^{-1} \cdot K^{-1}$ or $J \cdot s^{-1} \cdot m^{-1} \cdot K^{-1}$]

For illustration, consider a fluid layer of thickness **L** and temperature difference **ΔT**. Heat transfer through the fluid layer will be by convection when the fluid involves some motion and by conduction when the fluid layer is motionless.

In case of conduction, the heat flux can be calculated using Fourier's law of conduction. In case of convection, the heat flux can be calculated using Newton's law of cooling. Taking their ratio gives:

$$convection: \quad q_{conv} = h\Delta T$$

$$conduction: \quad q_{cond} = k\frac{\Delta T}{L}$$

$$\frac{q_{conv}}{q_{cond}} = \frac{h\Delta T}{h\frac{\Delta T}{L}} = \frac{hL}{k} = Nu_L$$

The preceding equation defines the **Nusselt number**. Therefore, the **Nusselt number** represents the enhancement of heat transfer through a fluid layer as a result of **convection relative to conduction** across the same fluid layer. A **Nusselt number** of **Nu=1** for a fluid layer represents heat transfer across the layer by **pure conduction**. The larger the **Nusselt number**, the more effective the convection. A larger Nusselt number corresponds to more effective convection, with turbulent flow typically in the 100–1000 range. For turbulent flow, the **Nusselt number** is usually a function of the Reynolds number and the Prandtl number.