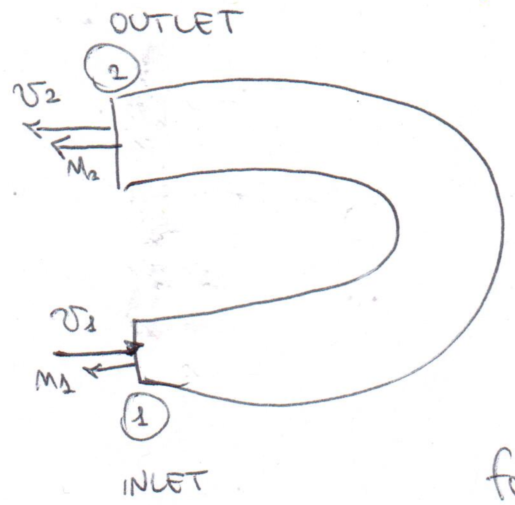
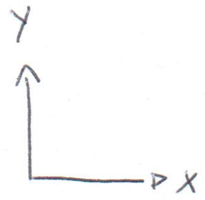
AN INTERESTING EXAMPLE : A REVERSING ELBOW

$\Downarrow$

The coordinate reference system is important





Determine the anchoring force (= reaction force) to keep the elbow in place

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$$F_{rxn} + \left(-P_1 A_1 \hat{n}_1\right) + \left(-P_2 A_2 \hat{n}_2\right) = \int_{A_1} \rho\, v_{1,avg}\left(v_{1,avg}\cdot \hat{n}_1\right) dA_1 + \int_{A_2} \rho\, v_{2\,avg}\left(v_{2\,avg}\cdot \hat{n}_2\right) dA_2$$

We will have only a component along $-x$

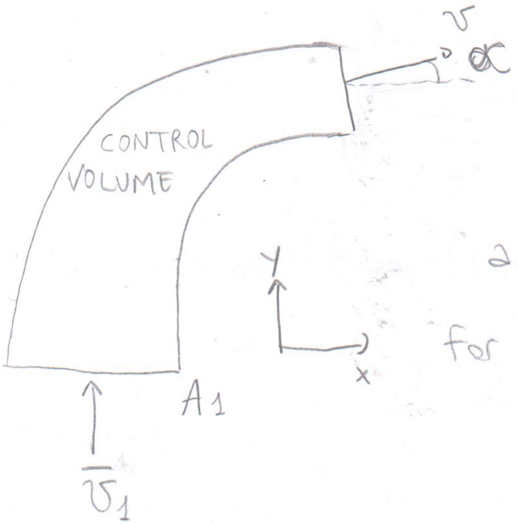
$$F_{rxn,x} + \left(-P_1 A_1(-1) + \left(-P_2 A_2(-1)\right)\right) = \overbrace{\rho\, v_{1\,avg} A_1}^{\dot{m}}\left(-v_{1,avg}\right) + \underbrace{\rho\, v_{2\,avg} A_2}_{\dot{m}}\left(-v_{2\,avg}\right)$$

$$F_{rxn,x} + P_1 A_1 + P_2 A_2 = -\dot{m}\left(v_1 + v_2\right)$$

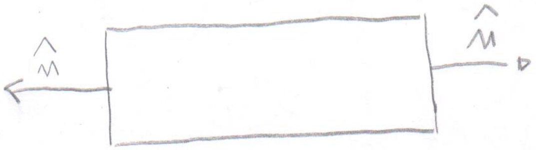
CONSERVATION OF MOMENTUM (recap)

From Module 2:

(1)



We define $\hat{n}$ as the unit vector normal to the surface and we have adopted a convention for the sign of $\hat{n}$ where $\hat{n}$ points outwards from the control surface



when writing the balance of forces we have seen that

$$\sum F_{volume} + \sum F_{surface} = \sum_{i=1}^{N} \int_{A_i} \rho v (v \cdot \hat{n})\, dA_i$$

If we assume gravity and friction negligible we have

$$F_{reaction} + (-p_1 A_1 \hat{n}_1) + (-p_2 A_2 \hat{n}_2) = \int_{A_1} \rho v_1 (v_1 \cdot \hat{n}_1)\, dA_1$$

$$+ \int_{A_2} \rho v_2 (v_2 \cdot \hat{n})\, dA_2$$

The way in which we have written this equation $\hat{n}$ is conventional the only vector. Yet we have to consider its sign, direction compared to the reference system of coordinate and its projection along $-x$ o $-y$ when we are looking to find the component of the reaction force

In the case 1 SLIDE 20

$$F_{rxm,x} + (-p_1 A_1 (0)) + (-p_2 A_2 (+1) \cos\alpha) =$$

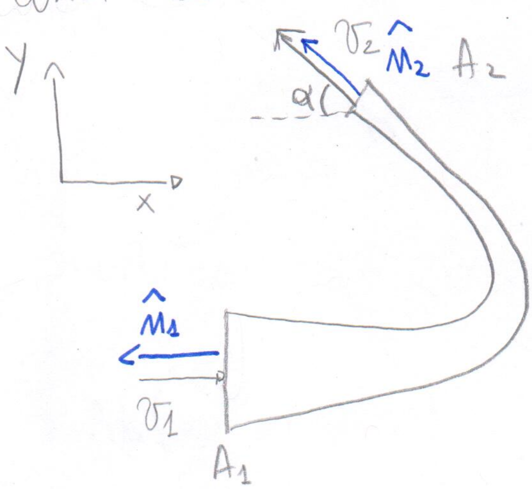
$$= \rho v_1^2 (0) A_1 + \rho v_2^2 (+1) \cos\alpha A_2$$

$$F_{rxm,y} + (-p_1 A_1 (-1)) + (-p_2 A_2 (+1) \sin\alpha) =$$

$$= \rho v_1^2 (-1) A_1 + \rho v_2^2 (+1) \sin\alpha A_2$$

IS THIS CLEAR?

What about the exam question?



$$F_{reaction} + (-p_1 A_1 \hat{n}_1) + (-p_2 A_2 \hat{n}_2) = \int_{A_1} \rho v_1 (v_1 \cdot \hat{n}_1)\, dA_1 + \int_{A_2} \rho v_2 (v_2 \cdot \hat{n}_2)\, dA_2$$

$$F_x + (-p_1 A_1 (-1)) + (-p_2 A_2 (-1) \cos\alpha) =$$

$$= \rho v_1^2 (-1) A_1 + \rho v_2^2 (-1) \cos\alpha$$

$$F_y + 0 + (-p_2 A_2 (+1) \sin\alpha) = 0 + \rho v_2^2 (+1) \sin\alpha$$

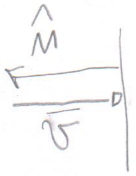
RIGOROUS TREATMENT OF MOMENTUM TERM

$$\sum \bar{F} = \sum_{i=1}^{N} \int_{A_i} \rho \bar{v} \underbrace{(\bar{v} \cdot \hat{n})}_{\text{scalar product}} dA_i$$

$$|\bar{v}||\hat{n}| \cos\theta$$

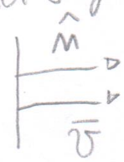
where θ is the angle in between the vectors

inlet surface



$\theta = 180°$ $\cos\theta = -1$

oulet surface



$\theta = 0°$ $\cos\theta = 1$

$\bar{v}$ is the vector which will give the sign

IF WE RE-WRITE THE SECOND TERM IN THE F_x OF THE EXAM QUESTION:

$$= \rho \bar{v}_1 \, v_1(-1) \, A_1 + \rho \bar{v}_2 \, v_2(+1) A_2$$

$$= \rho v_1^2 A_1 + \rho(-v_2 \cos\alpha) v_2 A_2$$