ChE 204
Introduction to Transport Phenomena

Module 1

The Bernoulli's equation

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Module 1
The Bernoulli's equation

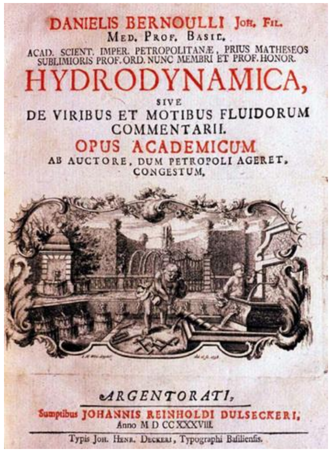
Objectives of this module:

1) Understand the use and limitations of the Bernoulli's equation
2) Apply the Bernoulli's equation to solve a variety of fluid flow problems
3) Read and Apply the Moody diagram to solve fluid flow problems including friction
4) Apply the Bernoulli's equation in the presence of closed-flow elements

1.1. Bernoulli's equation

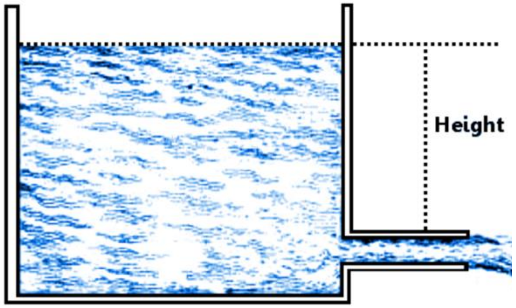
Daniel Bernoulli (1700-1782)

He was born in the city of Groningen in the Netherlands on February 8, 1700. His parents were Johann Bernoulli and Dorothea Falkner, both mathematicians and from Basel. In 1705, when Daniel was aged 5, his family relocated to Basel in Switzerland, his parents' hometown, where his father would became chair of mathematics at Basel University.



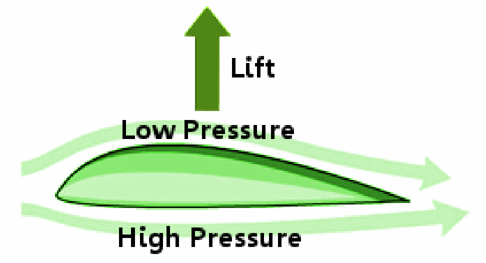
DANIELIS BERNOULLI Joh. Fil.
Med. Prof. Basil.
ACAD. SCIENT. IMPER. PETROPOLITANÆ, PRIUS MATHESEOS
SUBLIMIORIS PROF. ORD. NUNC MEMBRI ET PROF. HONOR.
HYDRODYNAMICA,
SIVE
DE VIRIBUS ET MOTIBUS FLUIDORUM
COMMENTARII.
OPUS ACADEMICUM
AB AUCTORE, DUM PETROPOLI AGERET,
CONGESTUM.

ARGENTORATI,
Sumptibus JOHANNIS REINHOLDI DULSECKERI,
Anno M D CC XXXVIII.
Typis Joh. Henr. Deckeri, Typographi Baſilienſis.



The greater the height of the water, the faster it emerges from the vessel.

The Bernoulli Effect has many real-life applications and is often cited as the reason aircraft wings provide lift.



The Bernoulli Effect on an airplane wing. The wing is shaped so that air flows faster over the upper part of the wing than the lower. This results in a pressure difference that produces lift.

1.1. Bernoulli’s equation

The Bernoulli Equation is the application of energy conservation along a streamline.

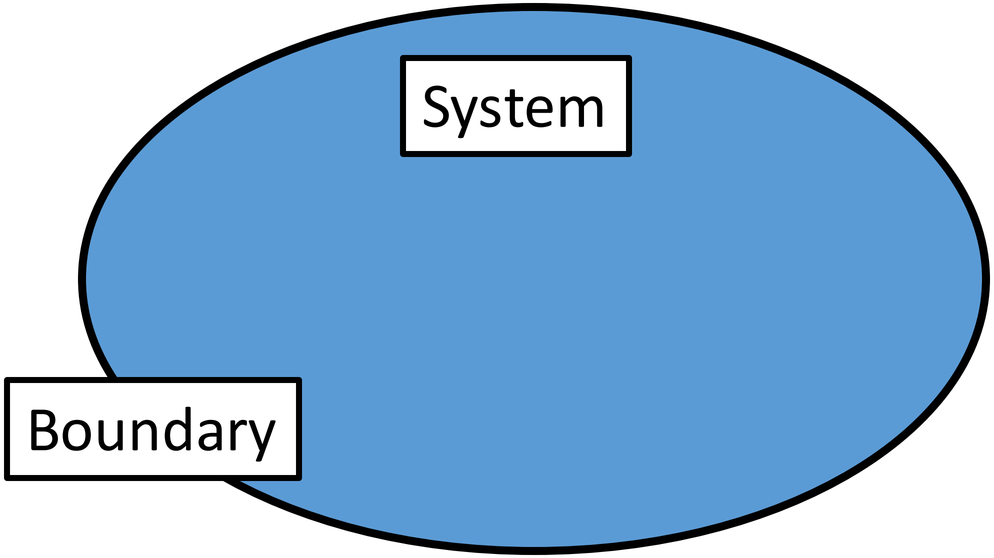
QUESTION:
How many of you are familiar with the conservation of energy?

1.1. Bernoulli’s equation

CONSERVATION OF ENERGY

The law of conservation of energy states that the total energy of an isolated system is conserved over time

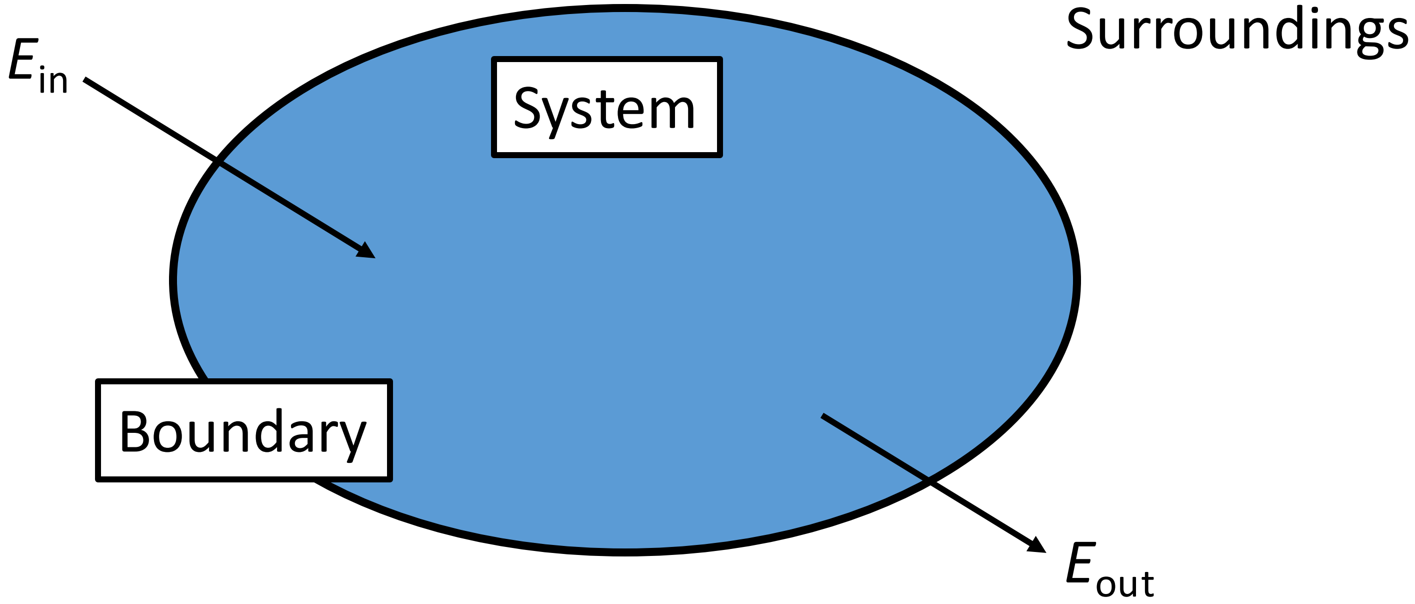
In a closed system $\Delta E = cost$



1.1. Bernoulli's equation

CONSERVATION OF ENERGY

What if a system starts to exchange energy with its surroundings?



Surroundings

E_{in}

System

Boundary

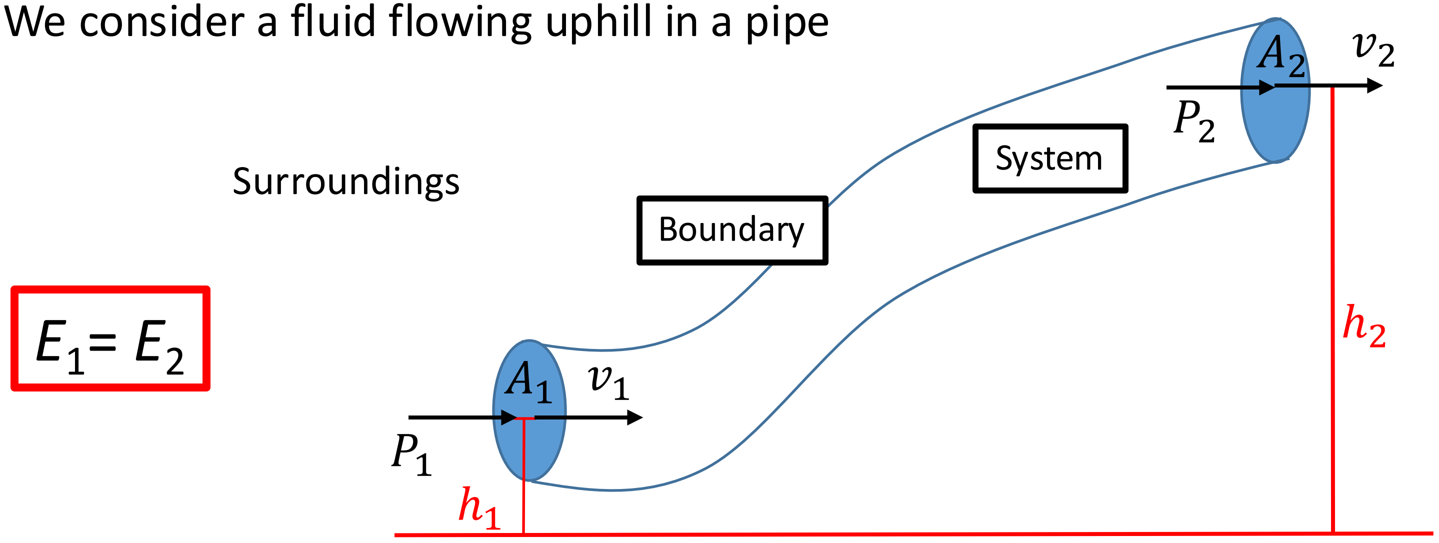
E_{out}

If the system start to interact with its surroundings and energy is transported across the boundary, the following has to be true:

$$\boxed{E_{in}= E_{out}}$$

1.1. Bernoulli's equation

We consider a fluid flowing uphill in a pipe



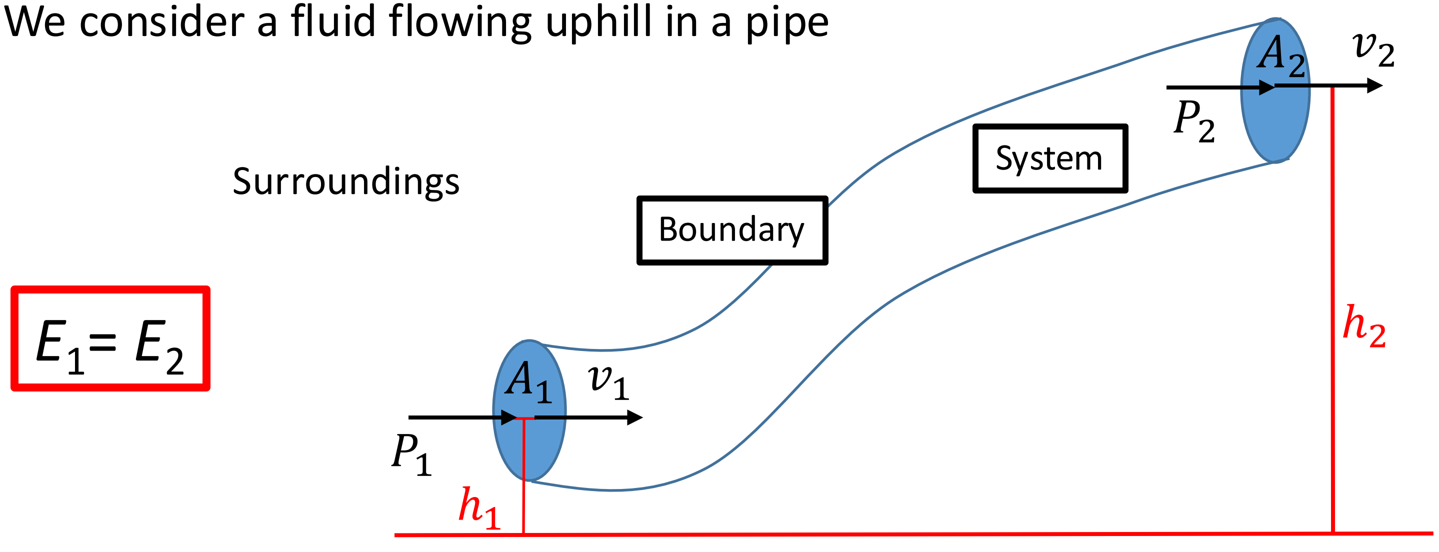
ASSUMPTIONS:

1) Steady flow ($\frac{dv}{dt}$ = 0 and $\frac{dP}{dt}$ = 0)
2) Incompressible fluid (ρ = density = const)
3) No friction

QUESTION: Which terms contribute to the energy of a fluid flowing uphill?

1.1. Bernoulli's equation

We consider a fluid flowing uphill in a pipe



$$E = W + PE + KE$$

1) W is the flow energy, the work done by the static pressure on the system

$$W = (P \cdot A) \cdot l = P \cdot V = P \cdot \frac{m}{\rho}$$

2) PE is the potential energy, the work done by gravity

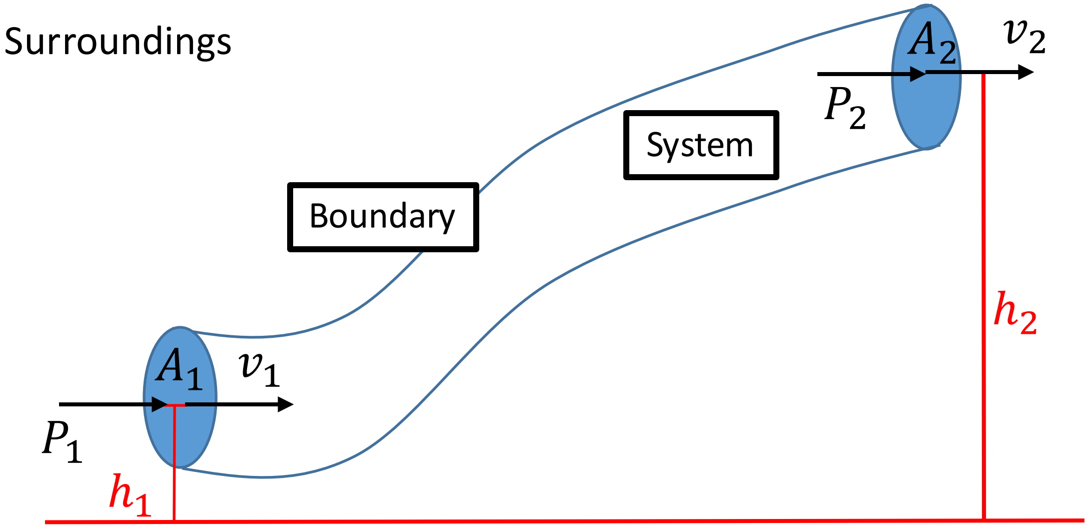
$$PE = m\, g\, h$$

3) KE is the kinetic energy

$$KE = \frac{1}{2} m v^2$$

1.1. Bernoulli's equation

$$\boxed{E_1 = E_2}$$



$$\boxed{E = W + PE + KE = \boldsymbol{P \cdot \frac{m}{\rho} + m\ g\ h + \frac{1}{2} m v^2}}$$

1) W is the flow energy, the work done by the static pressure on the system

$$W = (P \cdot A) \cdot d = P \cdot V = P \cdot \frac{m}{\rho}$$

2) PE is the potential energy, the work done by gravity

$$PE = m\ g\ h$$

3) KE is the kinetic energy

$$KE = \frac{1}{2} m v^2$$

1.1. Bernoulli's equation

$$\boxed{E_1 = E_2}$$

$$W_1 + PE_1 + KE_1 = W_2 + PE_2 + KE_2$$

$$P_1 \cdot \frac{m}{\rho} + m\,g\,h_1 + \frac{1}{2}mv_1^2 = P_2 \cdot \frac{m}{\rho} + m\,g\,h_2 + \frac{1}{2}mv_2^2$$

divide by *m* both side of the equation:

$$\frac{P_1}{\rho} + g\,h_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + g\,h_2 + \frac{1}{2}v_2^2$$

multiply by ρ both side of the equation:

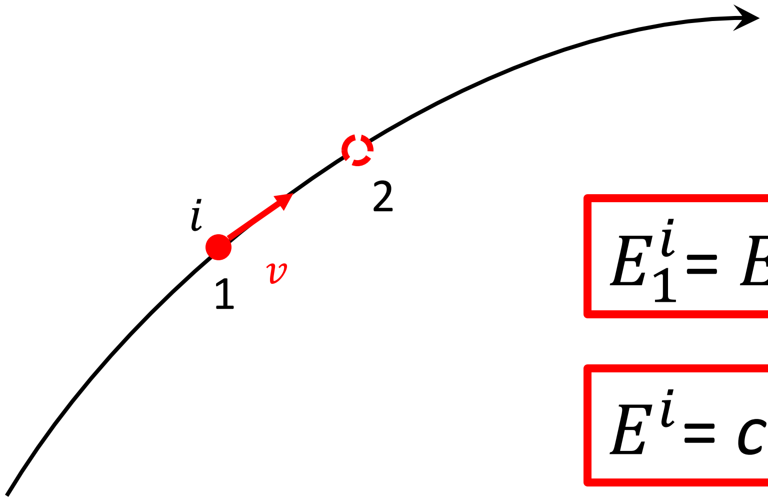
$$\boxed{P_1 + \rho g\,h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g\,h_2 + \frac{1}{2}\rho v_2^2}$$

$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = cost}$$

$$\boxed{E = cost}$$

1.1. Bernoulli’s equation (derivation from the Newton’s second law)

The Bernoulli Equation is the application of energy conservation along a streamline.



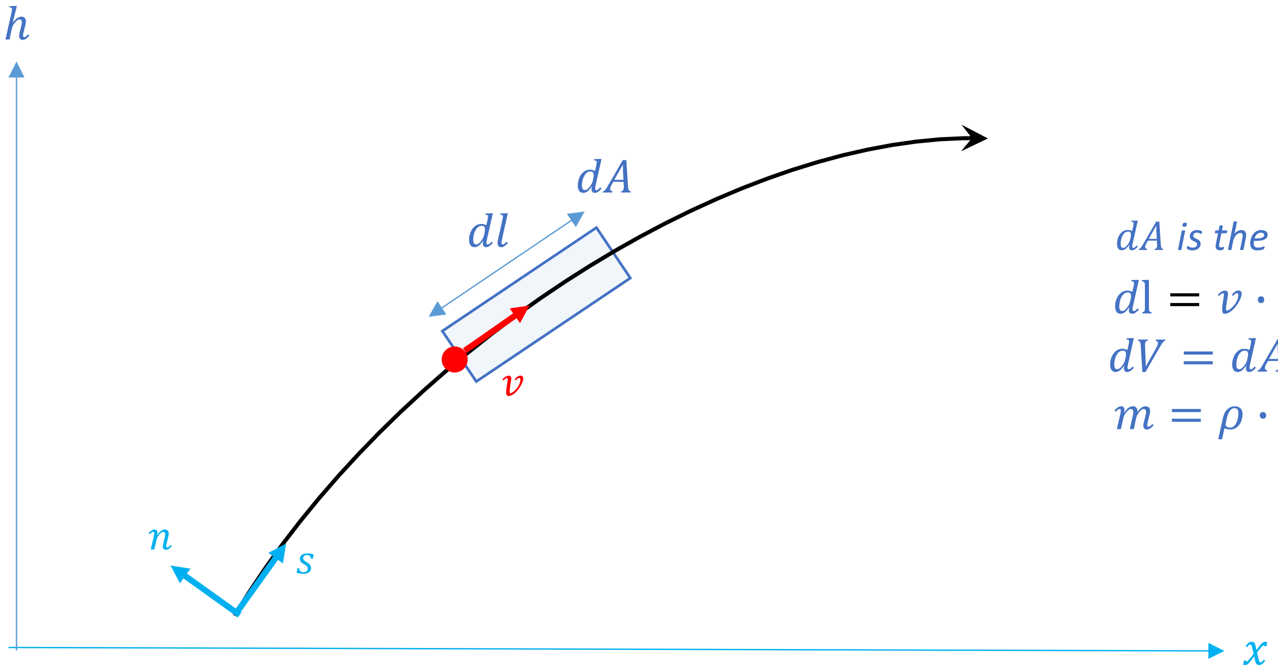
$$\boxed{E_1^i = E_2^i}$$

$$\boxed{E^i = constant}$$

The particle
i moving along a streamline (along the same path) conserves its energy

1.1. Bernoulli’s equation (derivation from the Newton’s second law)

Let’s focus on a particle moving along the streamline with velocity v and we focus on the infinitesimally small volume dV



dA is the cross sectional area

$$\mathrm{dl} = v \cdot dt$$

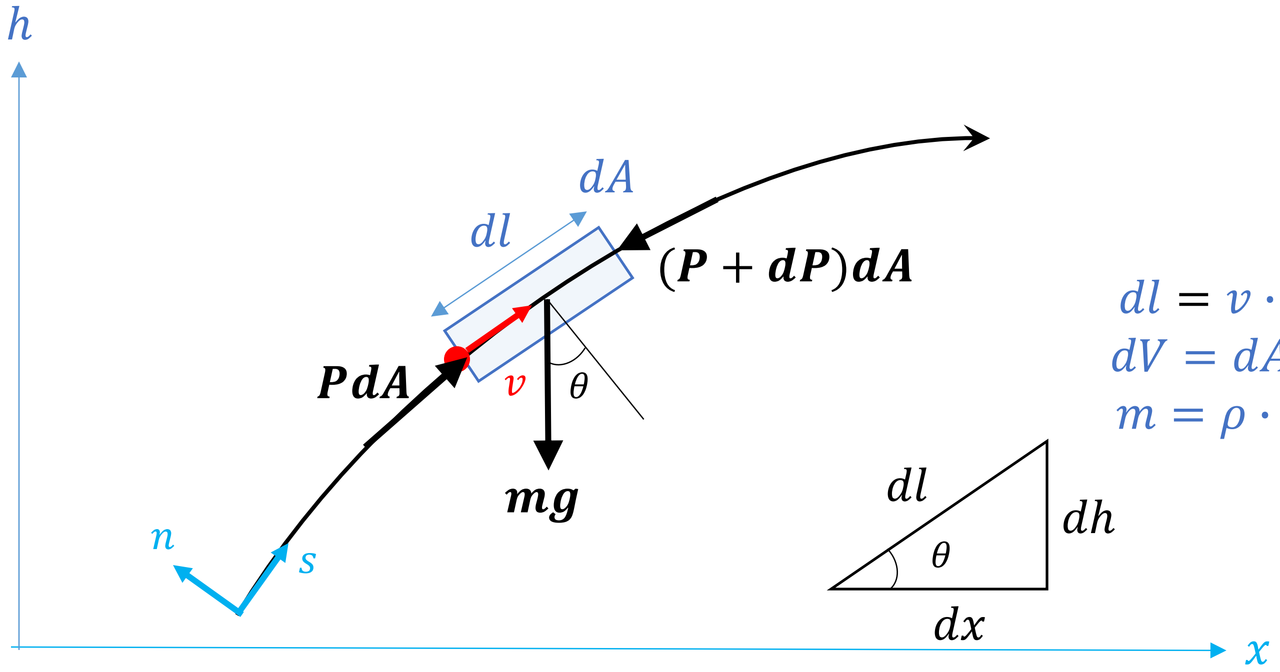
$$dV = dA \cdot dl$$

$$m = \rho \cdot dV = \rho \cdot dA\, dl$$

We can apply the Newton’s second law: $\sum F = ma$

1.1. Bernoulli's equation (derivation from the Newton's second law)

We identify the *forces acting on the particle* along the $s-direction$



$dl = v \cdot dt$
$dV = dA \cdot dl$
$m = \rho \cdot dV = \rho \cdot dA\, dl$

$$\sum F = ma$$

$$PdA\ -(P+dP)dA - mg\ sin\theta = m\frac{dv}{dt}$$

1.1. Bernoulli’s equation (derivation from the Newton’s second law)

$$PdA\ -(P+dP)dA-mg\ sin\theta=m\frac{dv}{dt}$$

Subsituting $m=\rho\ dA\ dl\ ,sin\theta=\frac{dh}{dl}\,,dt=\frac{dl}{v}$

$$-dPdA-\rho\ dA\ dl\ g\ \frac{dh}{dl}=\rho\ dA\ dl\ v\frac{dv}{dl}$$

Cancelling dA and simplifying dl

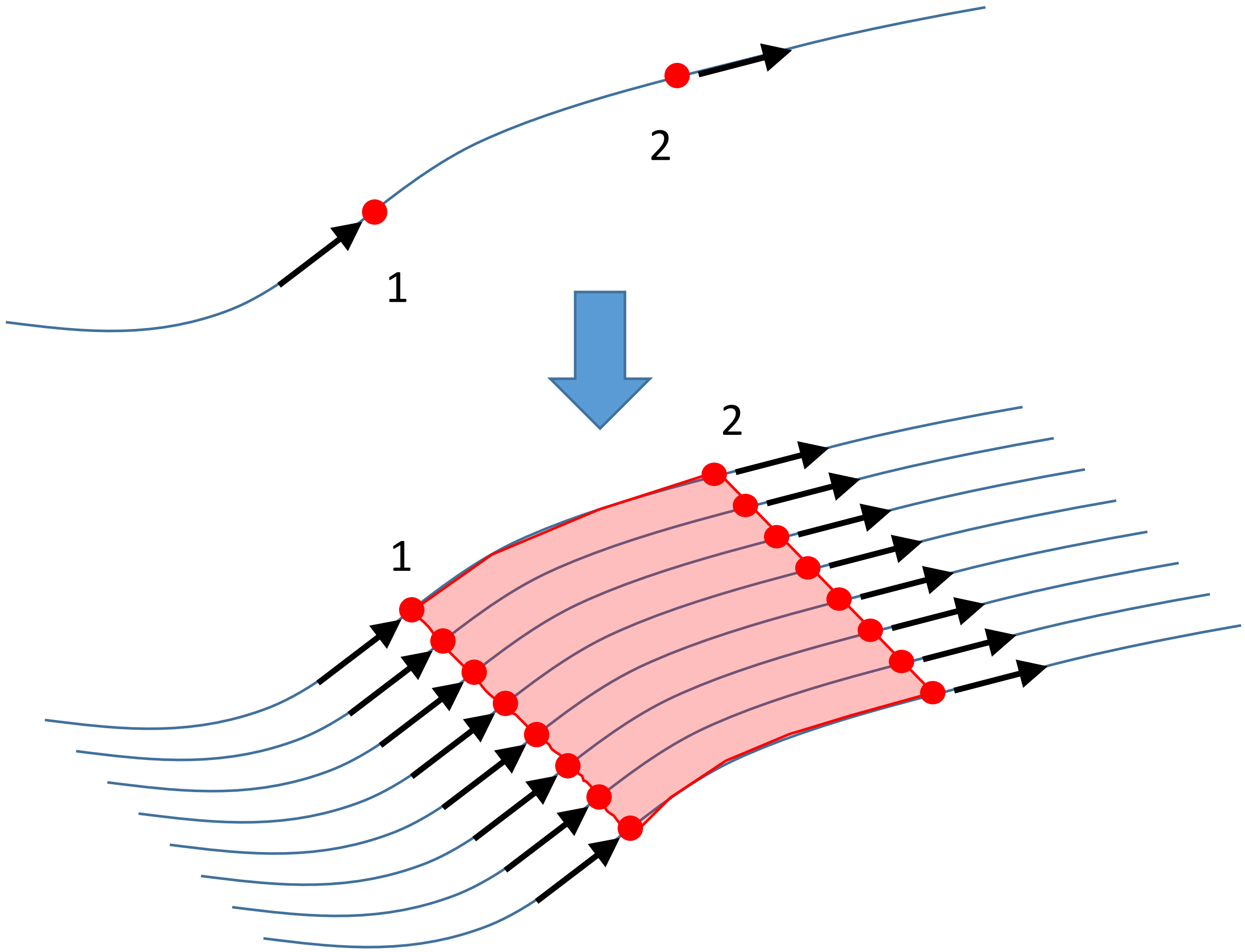
$$-dP-\rho g\ dh\ =\rho v\ dv$$

Noting that $v\ dv\ =\frac{1}{2}d(v^2)$ and diving each term by ρ

$$\frac{dP}{\rho}+g\ dh\ +\frac{1}{2}d(v^2)=0$$ **for one particle i of the fluid**

1.1. Bernoulli's equation

QUESTION:
How do we go from conservation of energy along one streamline to conservation of energy along a stream of fluid?



1.1. Bernoulli’s equation

$$\frac{dP}{\rho} + g\,dh\ + \frac{1}{2}d(v^2) = 0$$ **for one particle *i* of the fluid**

We integrate!

$$\int_i \frac{dP}{\rho} + g\,dh\ + \frac{1}{2}d(v^2) = const$$

$$\int \frac{dP}{\rho} + gh\ + \frac{v^2}{2} + \int \frac{\partial v}{\partial t}\,ds = const$$

ASSUMPTIONS:

1) Steady flow ($\frac{dv}{dt}$ = 0 and $\frac{dP}{dt}$ = 0)
2) Incompressible fluid (ρ = const)

The 3 terms are exact integrals, thus:

$$\boxed{\frac{P}{\rho} + g\ h\ + \frac{1}{2}v^2 = const\ (along\ a\ stream\ line)}$$

1.1. Bernoulli's equation

RECAP

$$\boxed{\frac{P}{\rho} + g\,h\, + \frac{1}{2}v^2 = const\ (along\ a\ stream\ line)}$$

If we multiply by the mass of the fluid m:

$$P\frac{m}{\rho} + mg\,h\, + \frac{1}{2}mv^2 = const\ (along\ a\ stream\ line)$$

Kinetic energy

Flow energy

Potential energy

$$\boxed{E = W + PE + KE = const\ (along\ a\ stream\ line)}$$

This is exactly the same expression we found by simply applying the conservation of energy

1.2. The continuity equation

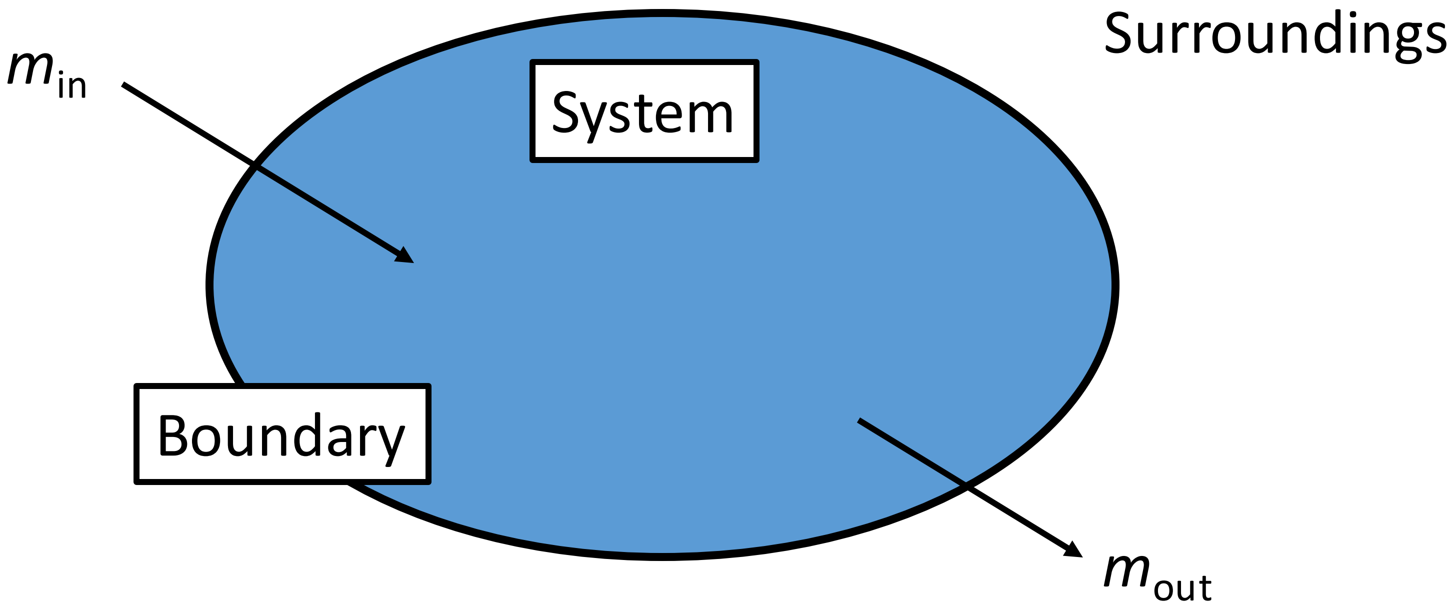
QUESTION: $a.\quad v_1 = v_2$

$b.\quad v_1 > v_2$

$c.\quad v_1 < v_2$

1.2. The continuity equation

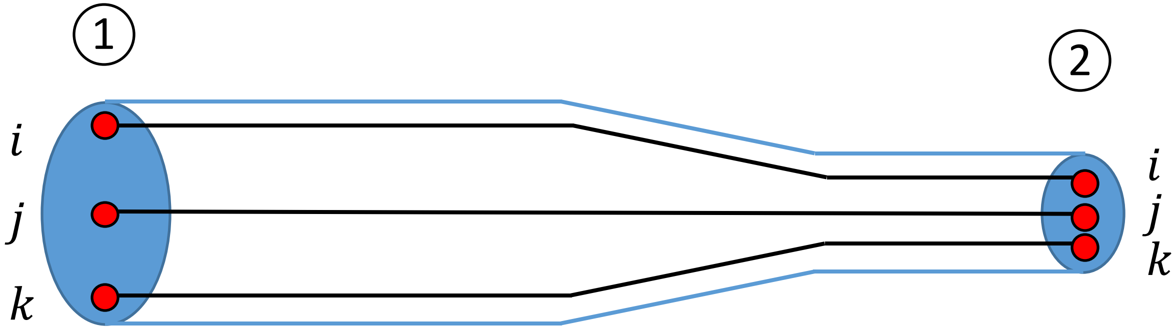
CONSERVATION OF MASS



If the system start to interact with its surroundings and mass is transported across the boundary, the following has to be true:

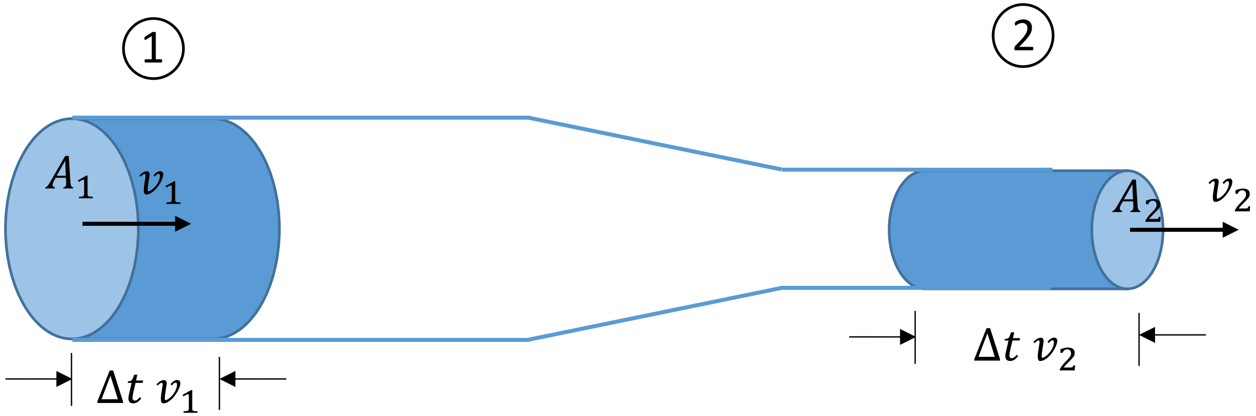
$$\boxed{m_{in} = m_{out}}$$

1.2. The continuity equation



The number of particles that enters the pipe has to exit

1.2. The continuity equation



We make always the same assumptions of steady state and uncompressible fluid.
We consider consider a time interval Δt:

$$m_1 = m_2$$

$$\rho V_1 = \rho V_2$$

$$\rho A_1 \Delta t\ v_1 = \rho A_2 \Delta t\ v_2$$

$$\boxed{A_1 v_1 = A_2 v_2}$$

https://www.youtube.com/watch?v=UJ3-Zm1wbIQ

1.3. Applications of the Bernoulli's equation

VOLUMETRIC AND MASS FLOW RATE

$$Volumetric\ flow\ rate\ \boldsymbol{Q}\ = \frac{volume\ of\ fluid}{time} = A\,v \qquad \left[\frac{L}{s}\right]\ \left[\frac{m^3}{s}\right]$$

$$Mass\ flow\ rate\ \dot{\boldsymbol{m}}\ = \frac{mass\ of\ fluid}{time} = \rho Q = \rho A\,v \qquad \left[\frac{g}{s}\right]\ \left[\frac{Kg}{min}\right]$$

1.3. Applications of the Bernoulli's equation

TWO DIFFERENT WAYS TO EXPRESS PRESSURE

Absolute pressure is zero-referenced against a perfect vacuum.

Gauge pressure is zero-referenced against ambient air pressure, so it is equal to absolute pressure minus atmospheric pressure.

Example:

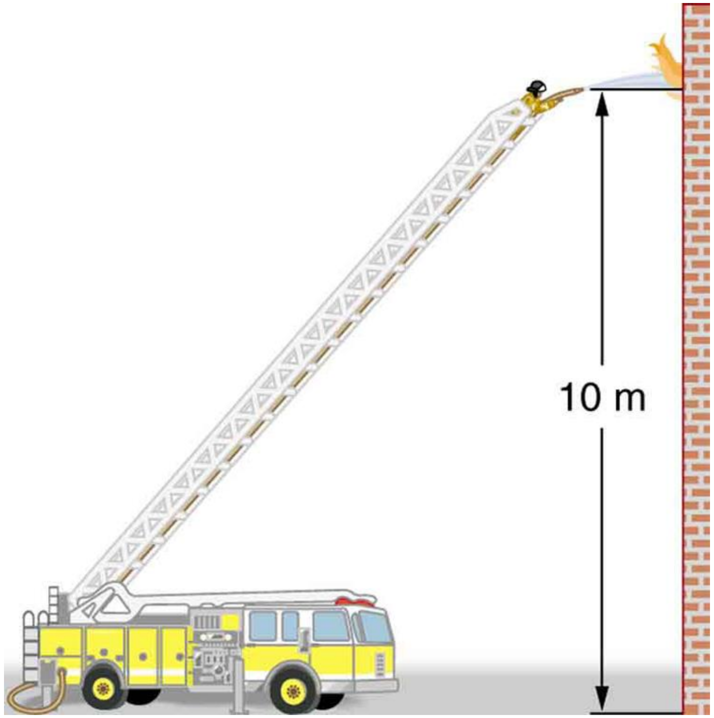
$P_{abs} = 2$ atm

$P_{gauge} = P_{abs} – P_{atm} = 2$ atm – 1 atm = 1 atm

If we express the atmospheric pressure as gauge pressure, the value is 0 atm

1.3. Applications of the Bernoulli's equation

EXAMPLE: Calculating pressure at a fire hose nozzle

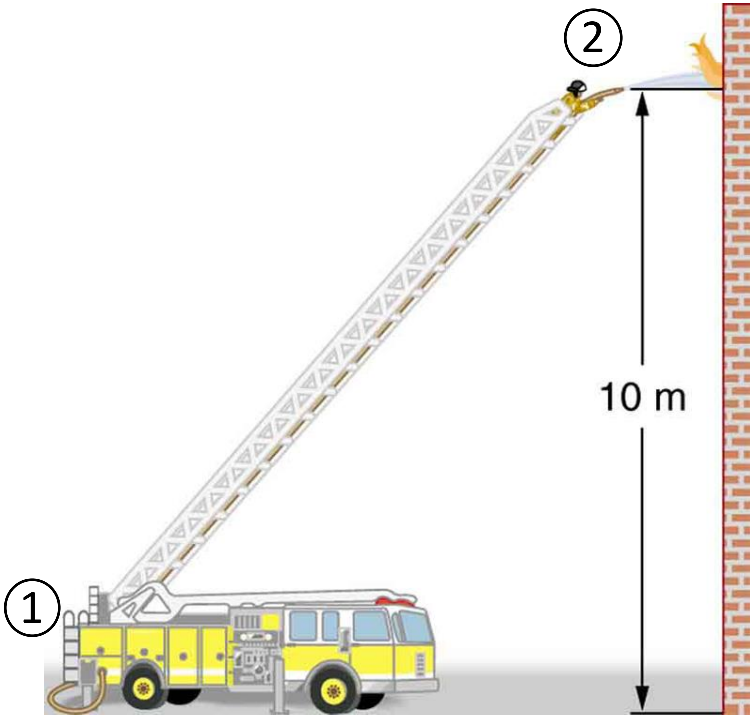


QUESTION:

Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle and speed increases in the nozzle. **In spite of its lowered pressure how can the water still exert a large force on anything it strikes? Also, what is the pressure in the water stream at the exit of the hose?**

1.3. Applications of the Bernoulli's equation

EXAMPLE: Calculating pressure at a fire hose nozzle



QUESTION:

Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle and speed increases in the nozzle. **In spite of its lowered pressure how can the water still exert a large force on anything it strikes? Also, what is the pressure in the water stream at the exit of the hose?**

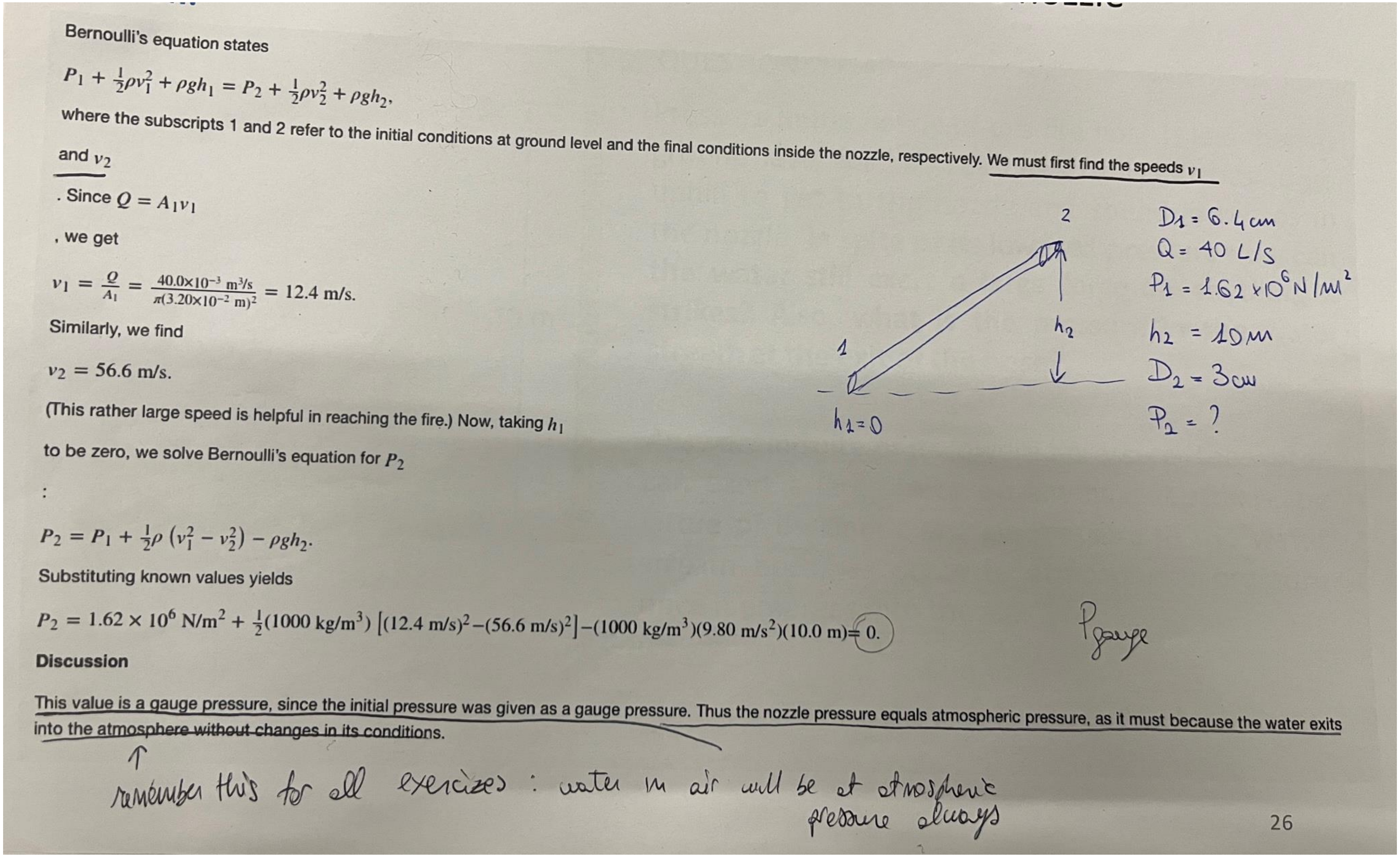
Exercize:

Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of 1.62 x 10^6 N/m^2. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the pressure at the nozzle?

1.3. Applications of the Bernoulli's equation

EXAMPLE: Calculating pressure at a fire hose nozzle

Solution: (This exercize will be solved during the lecture)



Bernoulli's equation states

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2,$$

where the subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0\times10^{-3}\ \text{m}^3/\text{s}}{\pi(3.20\times10^{-2}\ \text{m})^2} = 12.4\ \text{m/s}.$$

Similarly, we find

$$v_2 = 56.6\ \text{m/s}.$$

(This rather large speed is helpful in reaching the fire.) Now, taking h_1 to be zero, we solve Bernoulli's equation for P_2:

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g h_2.$$

Substituting known values yields

$$P_2 = 1.62\times10^6\ \text{N/m}^2 + \frac{1}{2}(1000\ \text{kg/m}^3)\left[(12.4\ \text{m/s})^2 - (56.6\ \text{m/s})^2\right] - (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10.0\ \text{m}) = \boxed{0}.$$

P_{gauge}

$D_1 = 6.4\ cm$

$Q = 40\ L/s$

$P_1 = 1.62\times10^6\ N/m^2$

$h_2 = 10\ m$

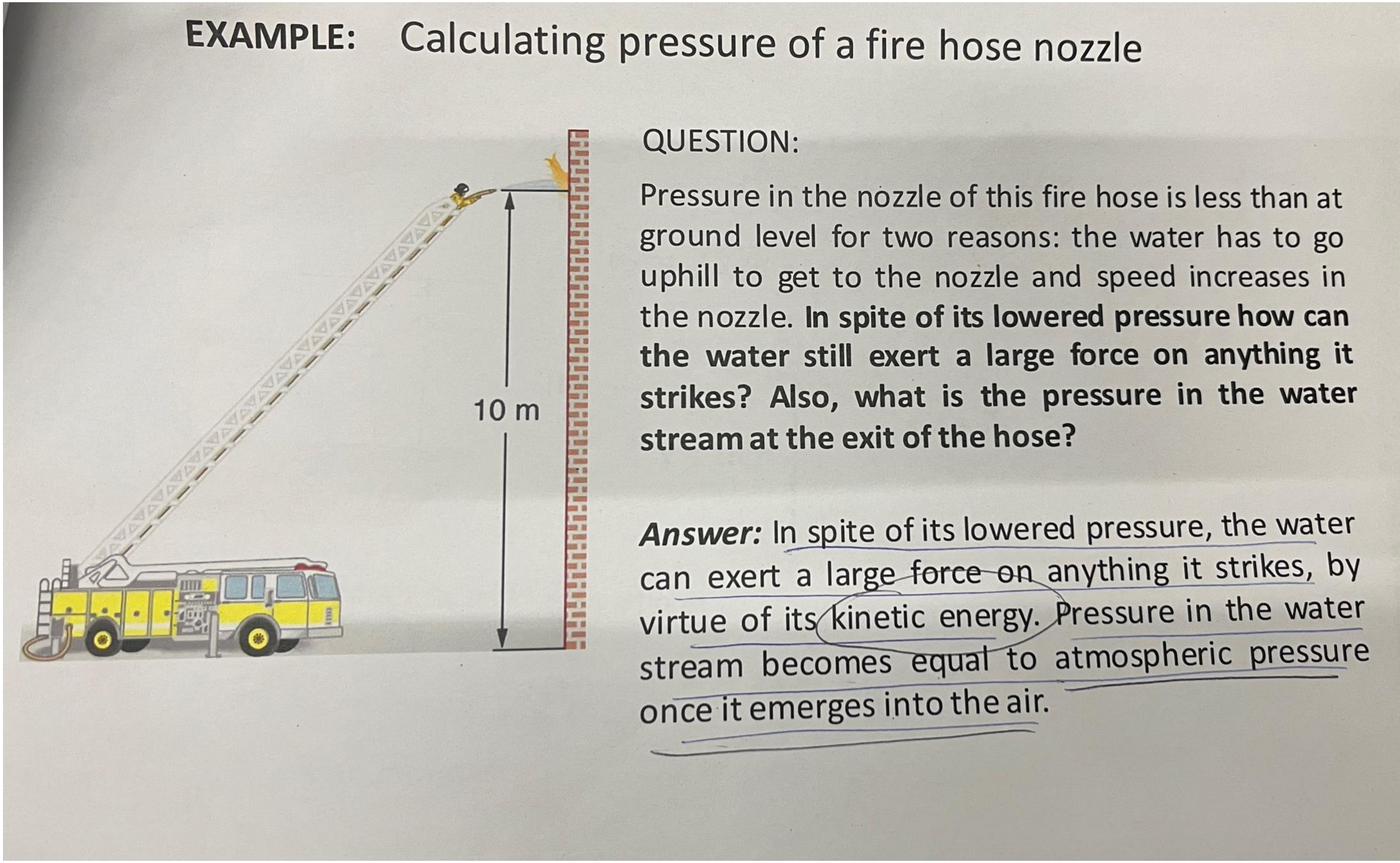
$D_2 = 3\ cm$

$P_2 = ?$

Discussion

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus the nozzle pressure equals atmospheric pressure, as it must because the water exits into the atmosphere without changes in its conditions.

↑
remember this for all exercizes : water in air will be at atmospheric pressure always



EXAMPLE: Calculating pressure of a fire hose nozzle

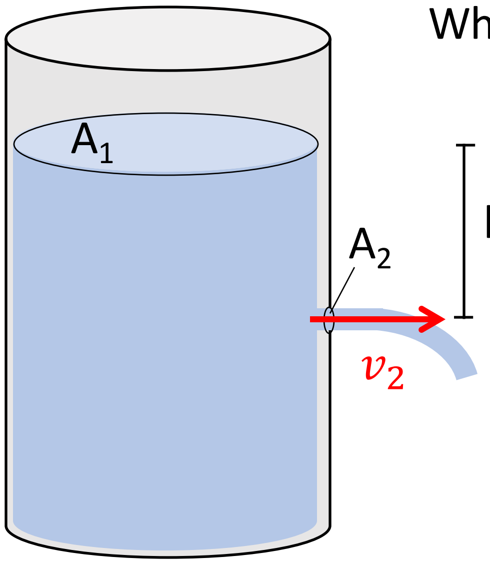
QUESTION:

Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle and speed increases in the nozzle. **In spite of its lowered pressure how can the water still exert a large force on anything it strikes? Also, what is the pressure in the water stream at the exit of the hose?**

Answer: In spite of its lowered pressure, the water can exert a large force on anything it strikes, by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

1.3. Applications of the Bernoulli's equation

THE TORRICELLI'S THEOREM



What is the output velocity of the fluid as a function of this height?

$$\boxed{P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2}$$

If we express them as gauge pressure we can write

$$\boldsymbol{P_{gauge\,1} = P_{gauge\,2} = P_{atm} - P_{atm} = 0}$$

$$\rho g(\cancel{0}) + \frac{1}{2}\rho(\cancel{0})^2 = \rho g\,(-h) + \frac{1}{2}\rho v_2^2$$

$$\cancel{\rho} gh = \frac{1}{2}\cancel{\rho} v_2^2$$

$$\boxed{v_2 = \sqrt{2gh}}$$

$$A_2 \ll A_1$$

$$\boldsymbol{P_1 = P_2 = P_{atm}}$$

$$h_1 = 0$$

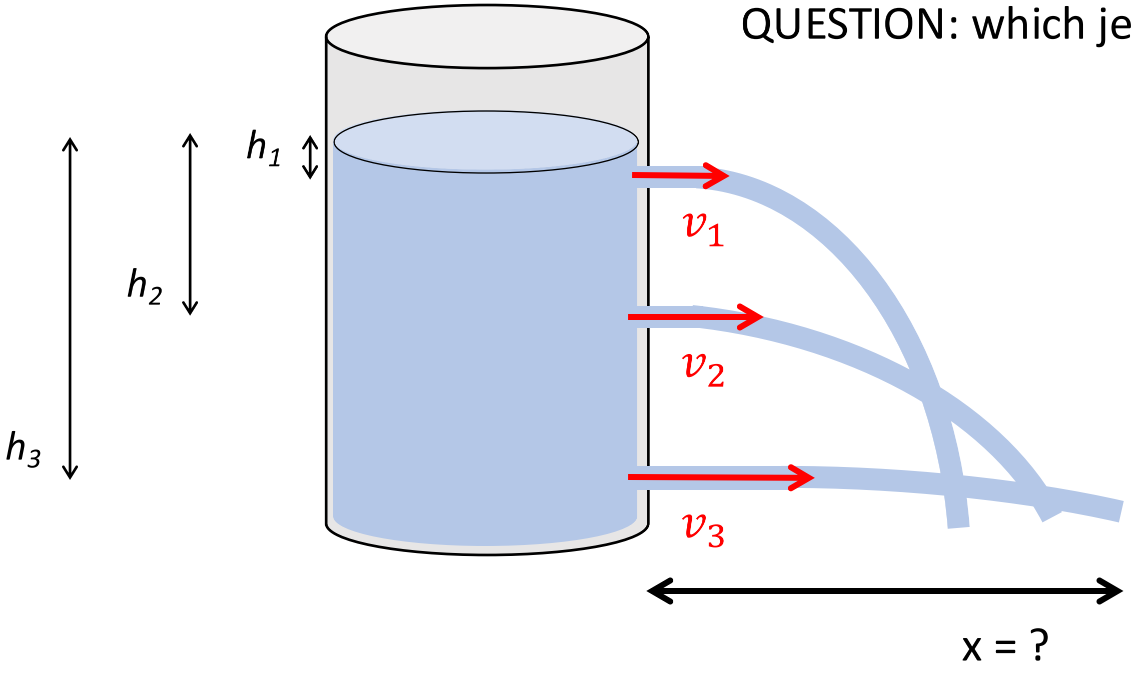
$v_1 A_1 = v_2 A_2$ (continuity equation)

$$v_1 \ll v_2$$

1.3. Applications of the Bernoulli's equation

THE TORRICELLI'S THEOREM

QUESTION: which jet of water is the fastest?



$v_3 > v_2 > v_1$

$$\boxed{v_2 = \sqrt{2gh}}$$

1.3. Applications of the Bernoulli's equation

THE TORRICELLI'S THEOREM

Exercize:

A large tank is filled with water to a depth of 15 meters. A spout is located 10m above the bottom of the tank. (a) With what speed v_2 will water emerge from the spout? (b) What horizontal distance from the base of the large tank does it land away?

Solution: (This exercize will be solved during the lecture)

Video on parabolic motion equations:

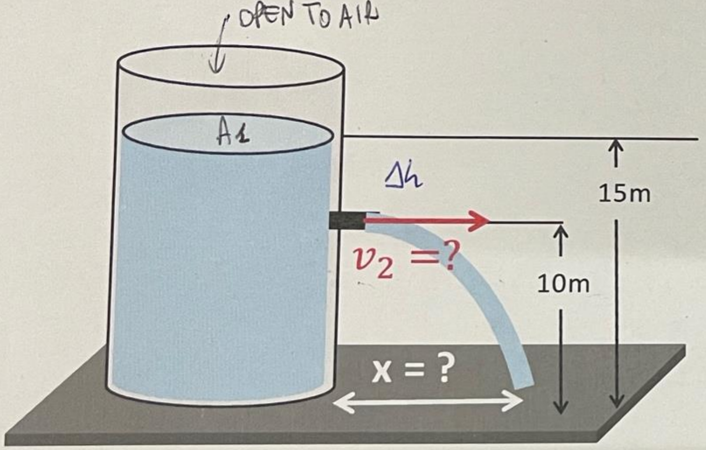
https://www.youtube.com/watch?v=X71H56Fdk6I

1.3. Applications of the Bernoulli's equation

THE TORRICELLI'S THEOREM

Exercize:

A large tank is filled with water to a depth of 15 meters. A spout is located 10m above the bottom of the tank is, then opened as shown in the drawing. (a) With what speed v_2 will water emerge from the spout? (b) What horizontal distance from the base of the large tank does it land away?

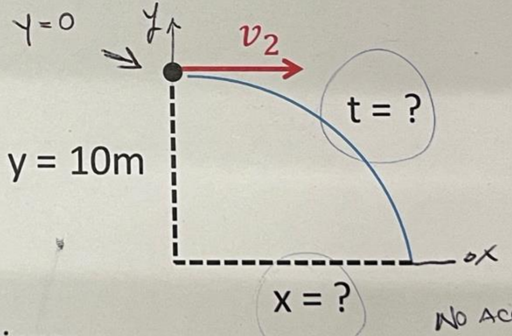


(a)

only thing to notice

$$v_2 = \sqrt{2gh} = \sqrt{2\left(9.8\frac{m}{s^2}\right)(5m)} = 9.9\ \text{m/s}$$

(b)



$v_{0y}t$

starting point

parabolic motion equations:

$$y = \cancelto{0}{y_0} + \cancelto{0}{v_{0y}}\, t - \frac{1}{2}gt^2$$

v_2t the projection of v_2 along x is equal to 0

$$10m = \frac{1}{2}\left(\frac{9.8\text{m}}{\text{s}^2}\right)t^2$$

just the module

$t = 1.43$ s

NO ACCELERATION IN OUR SYSTEM

$$x = \cancelto{0}{x_0} + v_{0x}\, t + \frac{1}{2}\cancelto{0}{a_x}\, t^2$$

v_2

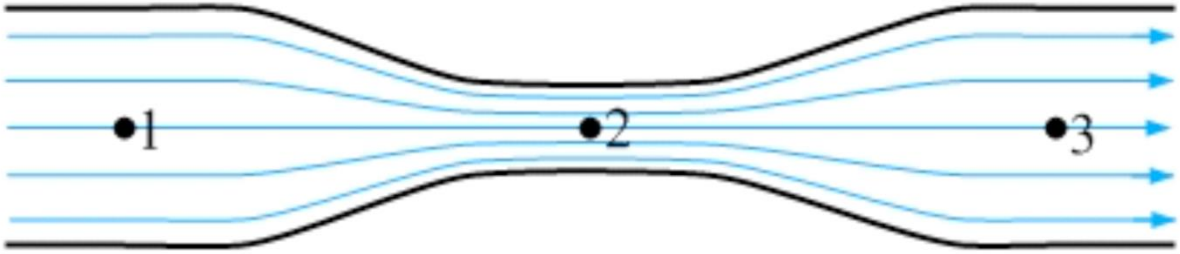
the projection of v_2 along x is v_2

$$x = \left(9.9\frac{m}{s}\right)(1.43\text{s}) = 14.1\ m$$

1.3. Applications of the Bernoulli's equation

THE VENTURI EFFECT

Consider a fluid flowing through a pipe with a constriction. Make a prediction regarding the pressure. Is the pressure higher at point 1 or at point 2?

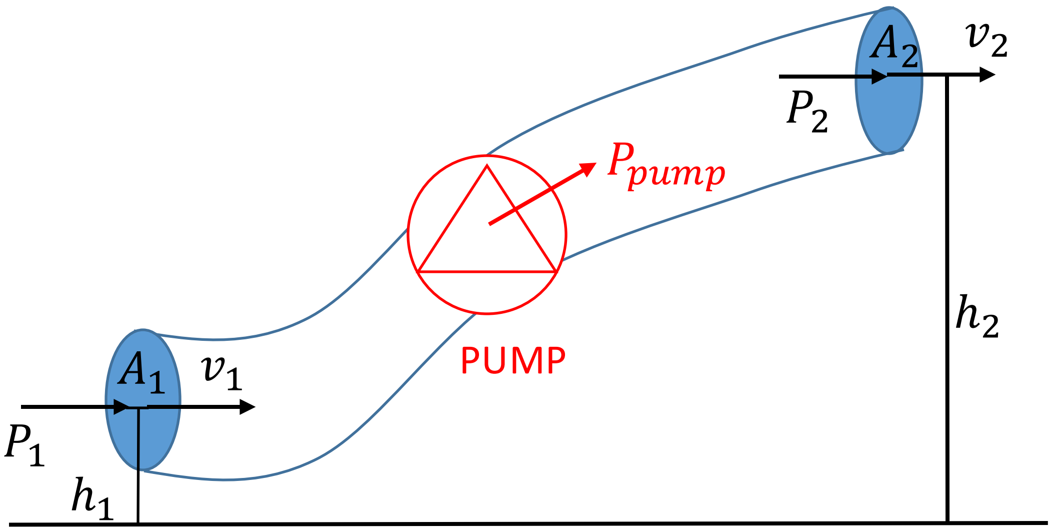


$$P_1 + \rho g \cancel{h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \rho g \cancel{h_2} + \frac{1}{2}\rho v_2^2$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$A_1 v_1 = A_2 v_2 = A_3 v_3 ;\ \ A_1 = A_3 > A_2 \Rightarrow v_2 > v_1 = v_3 \Rightarrow \boxed{P_1 > P_2}$$

1.4. Extension of Bernoulli's Equation to include pumps



In the energy balance we need to include the mechanical energy (or power) transmitted by the pump to the fluid

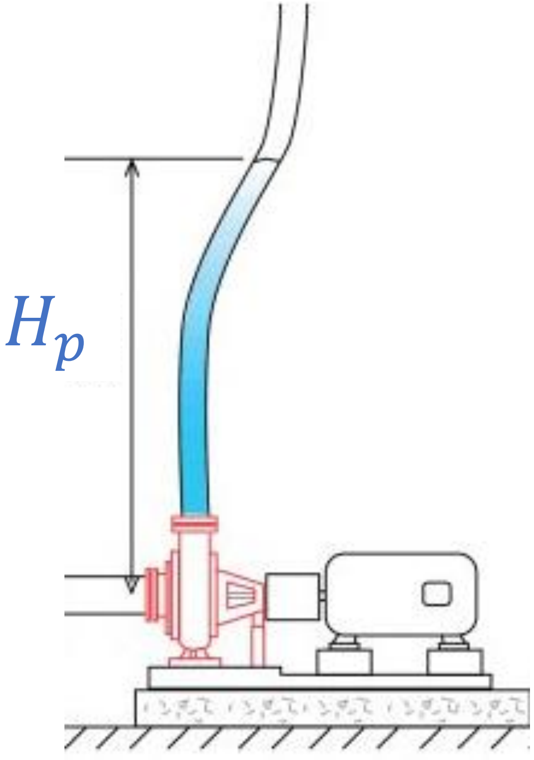
$$\boxed{E_1 = E_2}$$

$$W_1 + PE_1 + KE_1 + W_{pump} = W_2 + PE_2 + KE_2$$

$$P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

1.4. Extension of Bernoulli's Equation to include pumps

Definition of POWER OF A PUMP and PUMP HEAD



$$\boxed{\text{Power} = Q\rho\, g\boldsymbol{H_p}} \quad \text{[W]}$$

Q = volumetric flow [m^3/s]
ρ = density of fluid [kg/m^3]
g = gravity (9.81 m/s^2)
H_p =pump head (m)

The **pump "pressure-head"** is the vertical lift in height - usually measured meters of water - at which a pump can no longer exert enough pressure to move water. At this point, the pump may be said to have reached its "shut-off" head pressure.

1.4. Extension of Bernoulli's Equation to include pumps

Definition of EFFICIENCY OF A PUMP

A pump will have losses, thus we define the efficiency as:

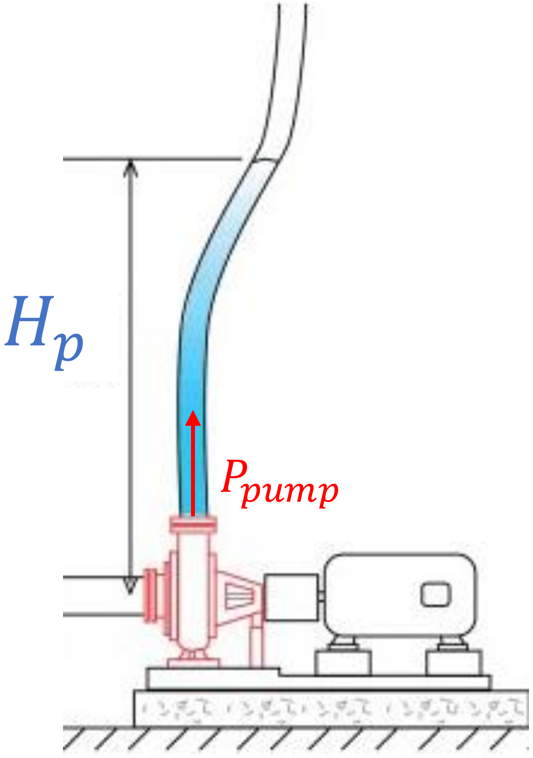
$$\eta = \frac{power\ output}{power\ input} \times 100 = \frac{Q\rho\ g\boldsymbol{H_p}}{power\ input} \times 100$$

Let's say that a pump is 80% efficient. This means that we have to supply

$$Power\ input\ = \frac{Q\rho\ g\boldsymbol{H_p}}{80} \times 100$$

1.4. Extension of Bernoulli's Equation to include pumps

CONVERSION FROM PUMP HEAD TO PRESSURE



The below equations may be used to convert between head and pressure when those measures are in the metric units kPa and m. Gravity is measured in m/s^2 and density in kg/m^3

$$H_P = \frac{P_{pump}}{\rho g} \qquad P_{pump} = \rho g H_P$$

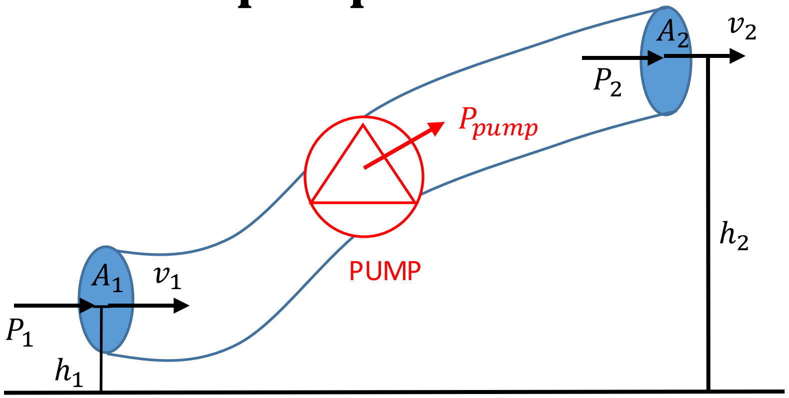
Example: H_p = 0.61 m P_{pump} = ?

$$P_{pump} = 1000 \text{ kg m}^{-3} \text{ x } 9.81 \text{ m s}^{-2} \text{ x } 0.61\text{m} = 5.98\ kPa$$

Note: During the course, Ppump will refer to the pump pressure. If we want to refer to the power, we will write "Power"

1.4. Extension of Bernoulli’s Equation to include pumps

The relation between pressure head and pressure allows us to write the Bernoulli’s equation in head terms



$$P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

divide everything by ρg

$$\boxed{\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + \frac{P_{pump}}{\rho g} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g}}$$

All the terms have length as dimension!

pump head $\boldsymbol{H_p}$

velocity head

static head

pressure head

$$\boxed{H_1 + \boldsymbol{H_p} = H_2}$$

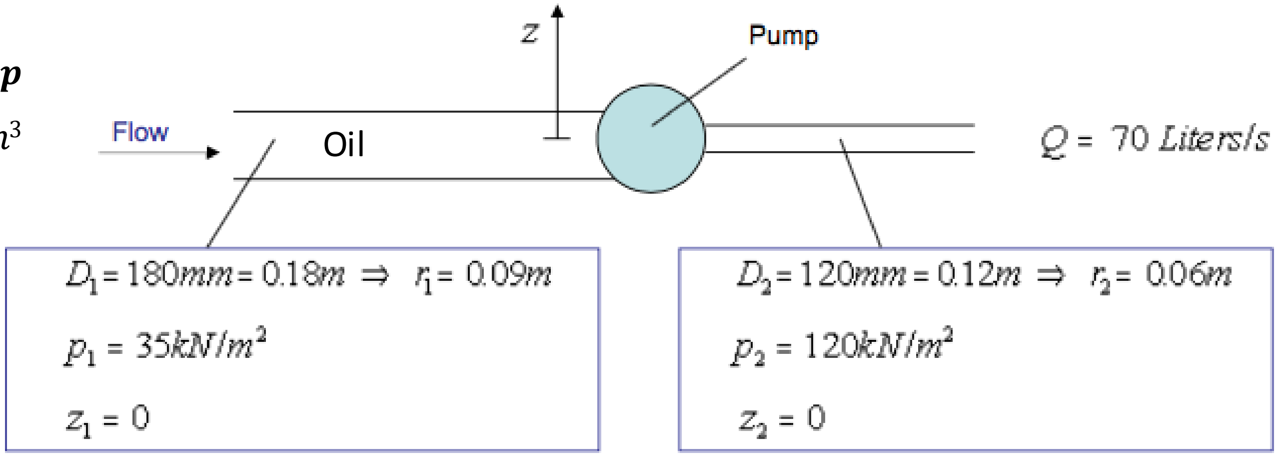
1.4. Head of a pump and Bernoulli's equation in head terms

Exercize:

Determine the Power of the pump

$$\text{Power} = Q\rho g \boldsymbol{H_p}$$

$$\rho(oil) = 0.82\ g/cm^3$$



$Q = 70\ Liters/s$

$D_1 = 180mm = 0.18m \Rightarrow r_1 = 0.09m$

$p_1 = 35kN/m^2$

$z_1 = 0$

$D_2 = 120mm = 0.12m \Rightarrow r_2 = 0.06m$

$p_2 = 120kN/m^2$

$z_2 = 0$

$$\frac{P_1}{\rho g} + \cancelto{0}{h_1} + \frac{v_1^2}{2g} + \boldsymbol{H_p} = \frac{P_2}{\rho g} + \cancelto{0}{h_2} + \frac{v_2^2}{2g}$$

$$v_1 = \frac{Q}{A_1} = \frac{0.07m^3/s}{\pi(0.09m)^2} = 2.7508\, m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.07m^3/s}{\pi(0.06m)^2} = 6.18936\, m/s$$

$$\boldsymbol{H_p} = \frac{P_2 - P_1}{\rho g} + \frac{v_2^2 - v_1^2}{2g} = \frac{(120-35)kN/m2}{0.82\,(9.81)kN/m^3} + \frac{\left(6.19\frac{m}{s}\right)^2 - \left(2.75\frac{m}{s}\right)^2}{2\,(9.81)kN/m^3} = 12.13\text{m}$$

1.4. Head of a pump and Bernoulli's equation in head terms

Exercize:

Determine the power of the pump

$$\text{Power} = Q\rho g\boldsymbol{H_p} = (0.07\,\frac{m^3}{s})(0.82 \text{ x } 9.81\,\frac{kN}{m^3})(12.13m) = 6.83\,\frac{m\,kN}{s} = 6.83 \text{ kW}$$

Let's say in the exercize we just solved, the pump is 90% efficient and we require 6.83kW output:

$$Power\ input = \frac{6.83\ kW}{0.9} = 7.59\ kW$$

1.4. Extension of Bernoulli's Equation to include pumps

RECAP

Conservation of energy along a stream line

$$\boxed{E = W + PE + KE = const\ (along\ a\ stream\ line)}$$

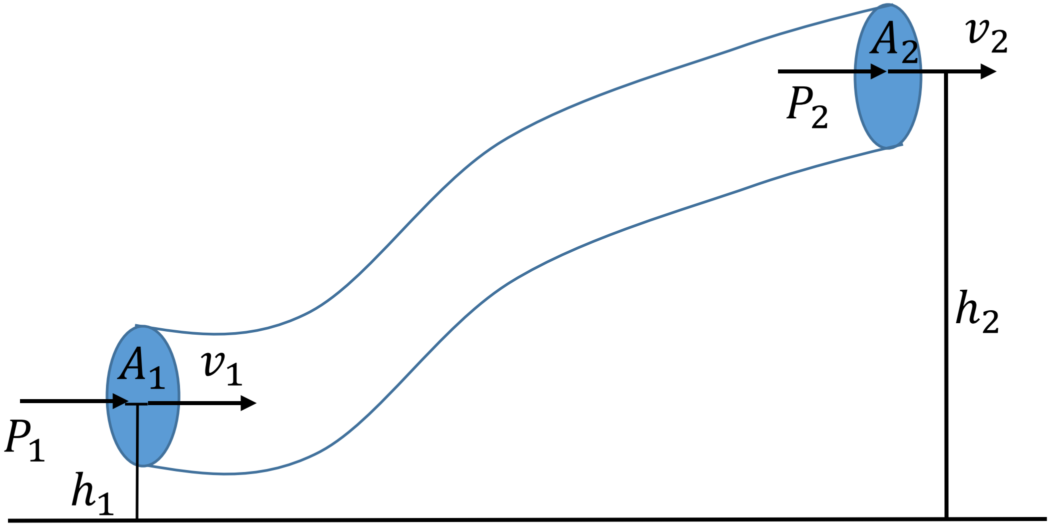
$$P_1 + \rho g\ h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g\ h_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + \boldsymbol{H_p} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g}$$

$$H_P = \frac{P_{pump}}{\rho g}$$

1.5. Friction factors due to viscosity

In real pipe line there are energy losses due to friction and these must be taken into account as they can be pretty significant.



$$P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 \neq P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2$$

frictional pressure drop

$$\boxed{P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2 + \boldsymbol{\Delta P_f}}$$

1.5. Friction factors due to viscosity

What is friction?

Reaction Force Resisting to Motion

FRICTION FORCE (F_f)

What is viscosity?

The viscosity of a fluid is a measure to its resistance to flow (i.e. honey is more viscous than water)

$\boldsymbol{\mu} = dynamic\ viscosity$ [Kg · m^{-1} · s^{-1} or Pa · s]

Resistance to flow when an external force is applied.

$\mathbf{v} = kinematic\ viscosity = \dfrac{\mu}{\rho}$ [m^{2} · s^{-1}]

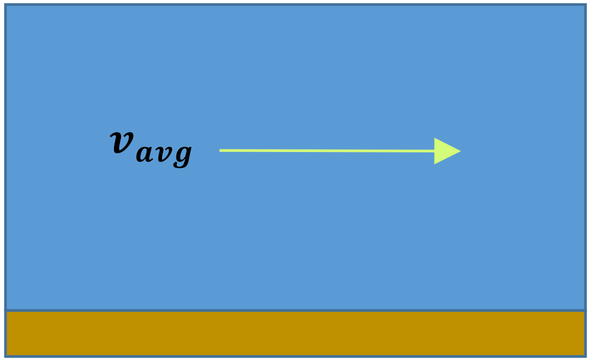
Intrinsic resistance to flow when no external force is applied, except gravity.

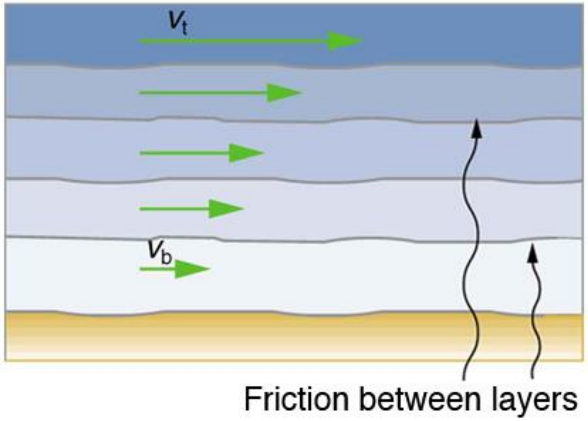
1.5. Friction factors due to viscosity

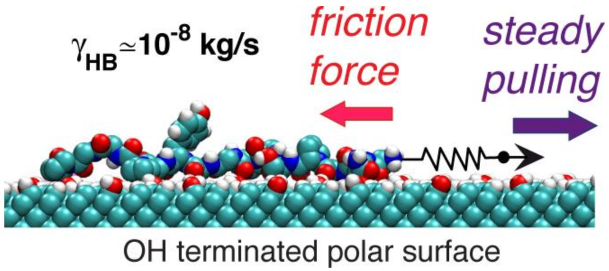
Macro

Micro

Molecular

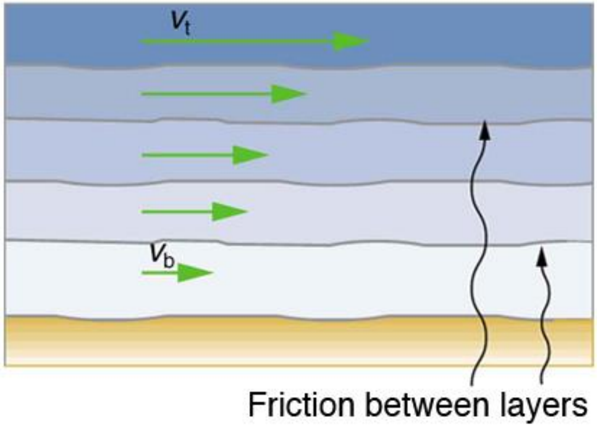






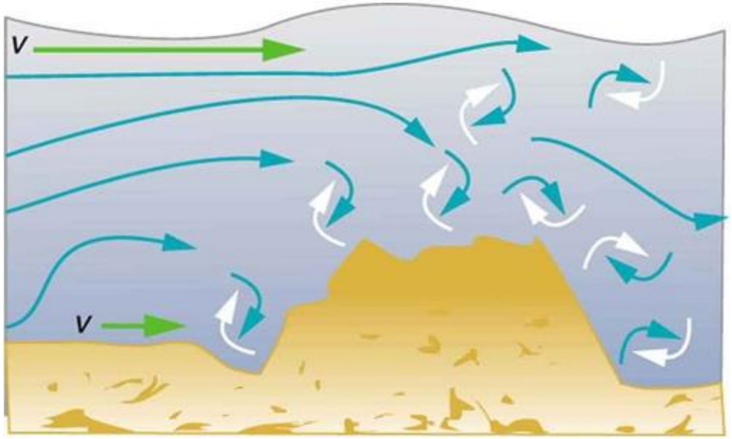
1.5. Friction factors due to viscosity

LAMINAR FLOW



The fluid flows in parallel layers, with no disruption or mixing between layers

TURBOLENT FLOW



The fluid flows in a chaotic manner with strong currents, irregular velocity and intermixing between layers

Nice demonstration of laminar flow

1.5. Friction factors due to viscosity

density of the fluid [Kg/m^3]

average velocity [m/s]

$$Re \;=\; Reynolds\ number \;=\; \frac{\rho v_{avg} D}{\mu}$$

tube diameter [m]

dynamic viscosity
[Kg · m^{-1} · s^{-1} or Pa · s]

$$\text{with v} = kinematic\ viscosity = \frac{\mu}{\rho}$$

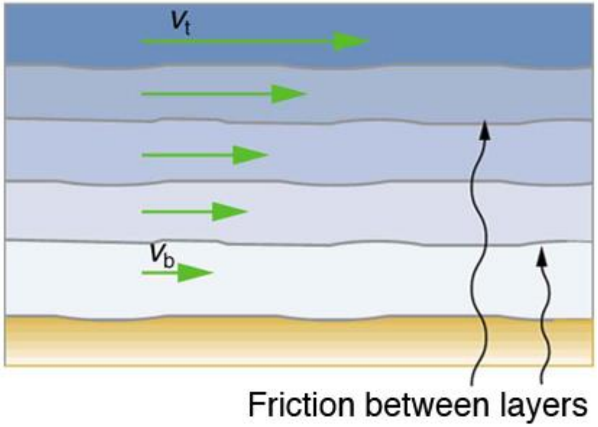
$$\boxed{Re \;=\; Reynolds\ number \;=\; \frac{v_{avg} D}{\mathrm{v}}}$$

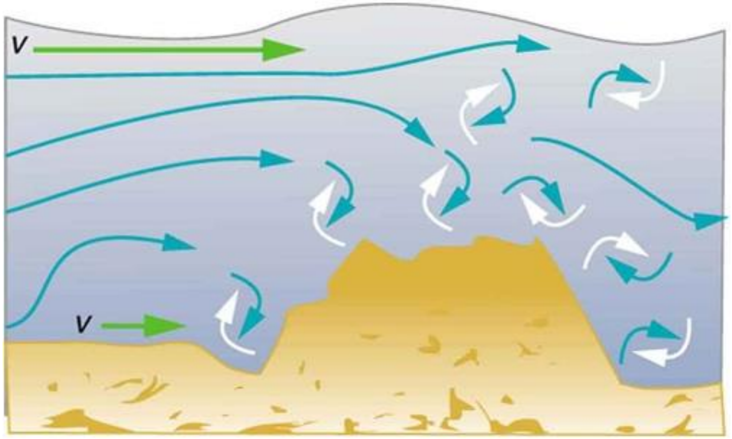
It represents the ratio between the inertial and the friction forces

1.5. Friction factors due to viscosity

LAMINAR FLOW

TURBOLENT FLOW



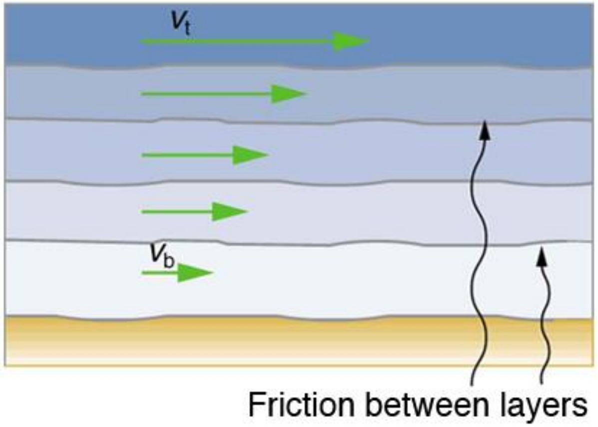


Re < 2000

Re > 4000

1.5. Friction factors due to viscosity

LAMINAR FLOW



Re < 2000

Parabolic velocity profile

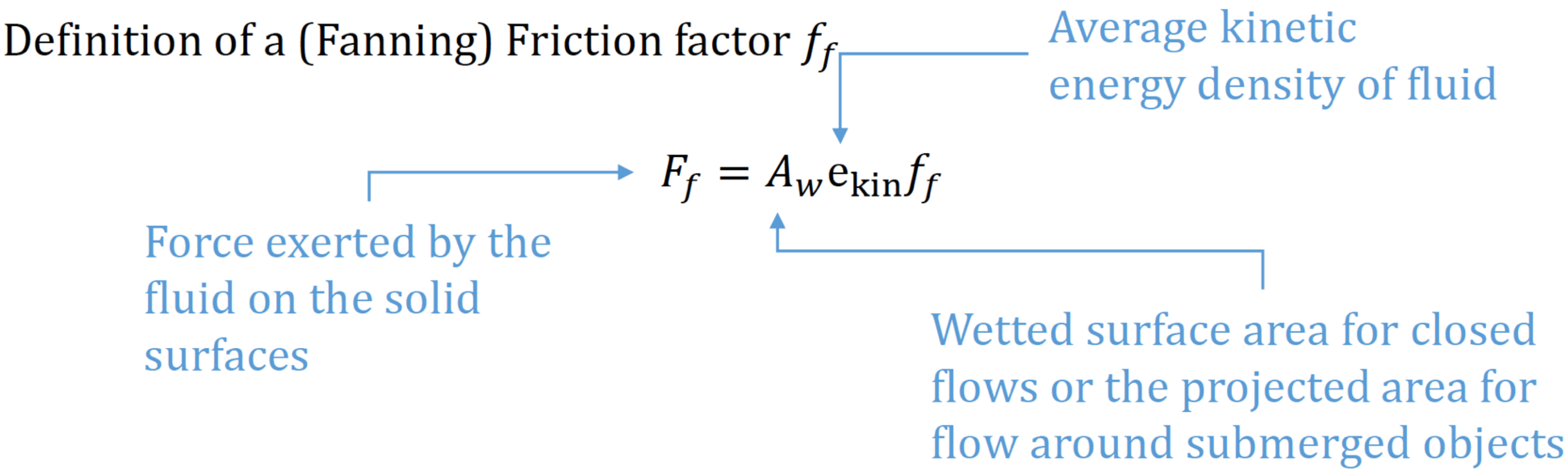
$$v(r) = v_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$$

To remember: the velocity at the wall is zero

1.5. Friction factors due to viscosity

Definition of the friction factor f_f

We consider a fluid flowing in a straight pipe in the **laminar flow regime.** Ultimately we need the friction force to insert the friction term in the Bernoulli's equation.



Definition of a (Fanning) Friction factor f_f

Average kinetic energy density of fluid

$$F_f = A_w e_{\mathrm{kin}} f_f$$

Force exerted by the fluid on the solid surfaces

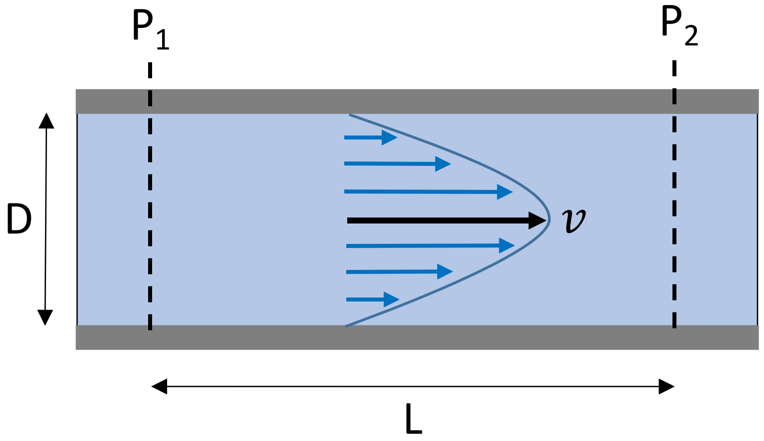
Wetted surface area for closed flows or the projected area for flow around submerged objects

Now the goal is to derive an expression for the Friction Factor f_f!

Note: Darcy Friction Factor $(f_D) = 4f_f$

1.5. Friction factors due to viscosity

Definition of the friction factor f_f



$$F_f = A_w \mathrm{e}_{\mathrm{kin}} f_f$$

$$F_f = (2\pi R L)\left(\tfrac{1}{2}\rho v_{avg}^2\right) f_f$$

$$(P_1 - P_2)\pi R^2 = (2\pi R L)\left(\tfrac{1}{2}\rho v_{avg}^2\right) f_f$$

$$F_f = (\Delta P \cdot A) = (P_1 - P_2)\pi R^2$$

written in terms of a viscous resistance force to the flow of the fluid

$$\boxed{f_f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{p_1 - p_2}{\frac{1}{2}\rho v_{avg}^2}\right)}$$

We need v_{avg}!

1.5. Friction factors due to viscosity

Definition of the friction factor f_f

The velocity profile for viscous fluid in laminar flow regime is a parabolic one:

$$\boxed{v(r) = v_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)}$$

It can be shown that: $$v_{max} = \frac{R^2}{4\mu}\frac{(P_1 - P_2)}{L}$$

Thus: $$v(r) = \frac{(P_1 - P_2)}{4\mu L}(R^2 - r^2)$$

$$v_{z,avg} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_z(r) r\, dr\, d\theta \Rightarrow v_{z,avg} = \frac{v_{max}}{2} = \frac{R^2}{8\mu}\frac{(P_1 - P_2)}{L}$$

Keep this in mind!

1.5. Friction factors due to viscosity

Definition of the friction factor f_f

$$f_f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{p_1 - p_2}{\frac{1}{2}\rho v_{avg}^2}\right)$$

$$v_{z,avg} = \frac{v_{max}}{2} = \frac{R^2}{8\mu}\,\frac{(P_1 - P_2)}{L}$$

$$f_f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{p_1 - p_2}{\frac{1}{2}\rho v_{avg}}\right)\left(\frac{1}{v_{avg}}\right) \Rightarrow$$

$$f_f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{p_1 - p_2}{\frac{1}{2}\rho v_{avg}}\right)\left(\frac{8}{D^2}\right)\left(\frac{4\mu L}{p_1 - p_2}\right) = 16\underbrace{\boxed{\frac{\mu}{\rho v_{avg} D}}}_{\frac{1}{Reynolds\ number}}$$

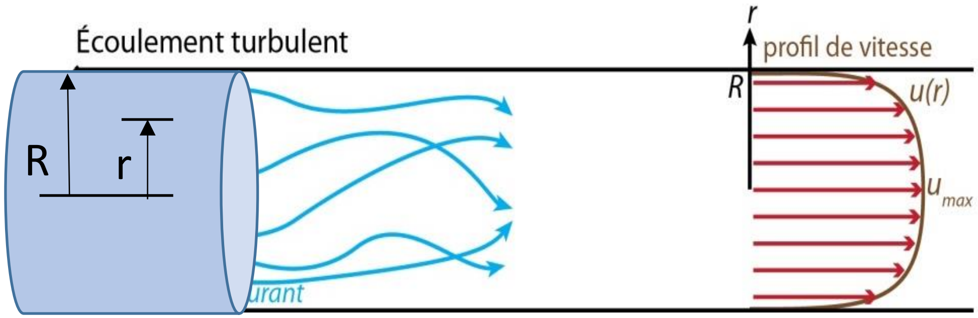
In laminar flow regime $\boldsymbol{f_f = \frac{16}{Re}}$

Note: $f_D = \frac{64}{Re}$

1.5. Friction factors due to viscosity

Definition of the friction factor f_f

TURBOLENT FLOW   Re > 4000



$$v(r) = v_{max}\left(1 - \frac{r}{R}\right)^{\frac{1}{n}}$$

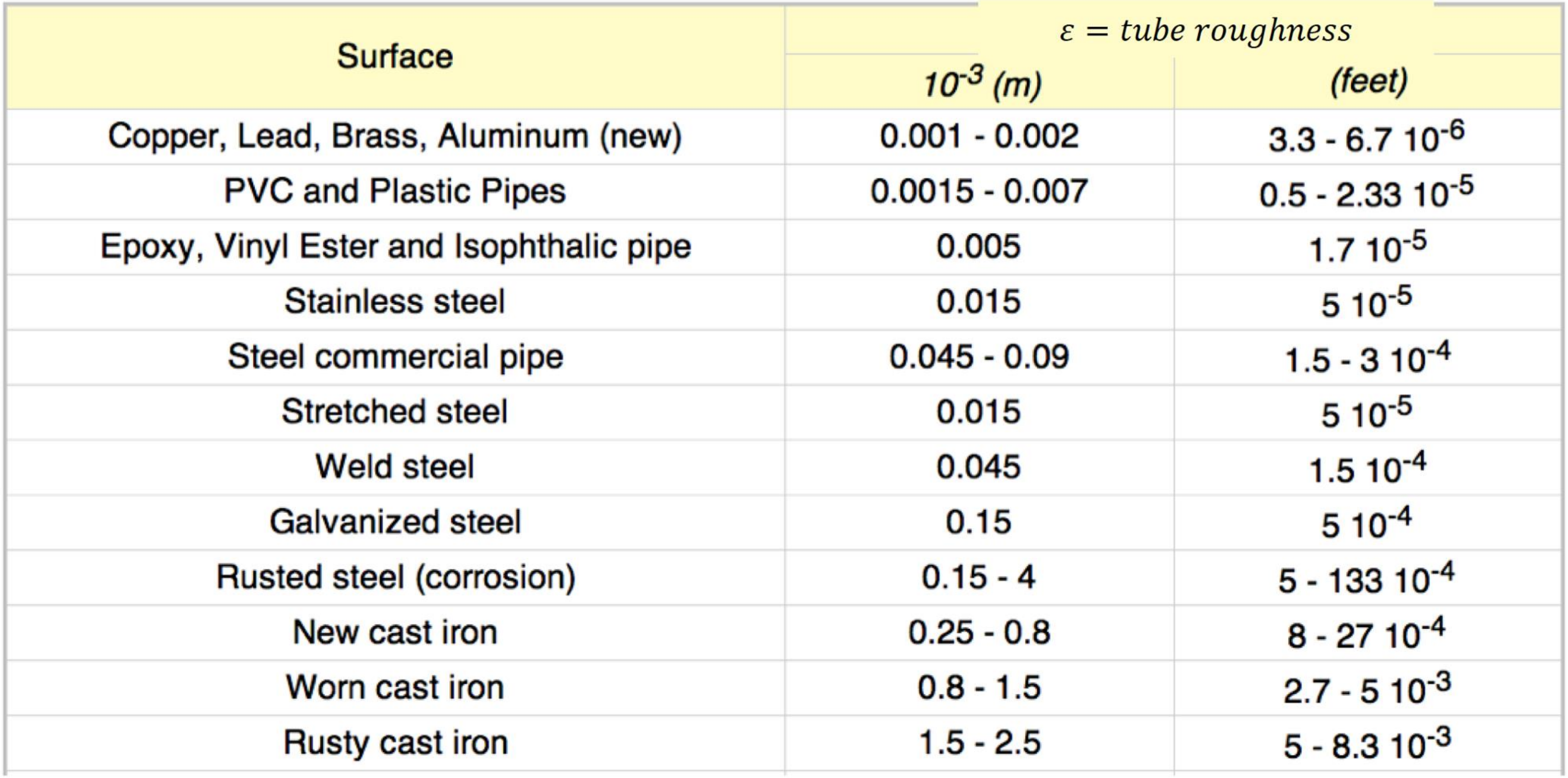
with $5 < n < 7\ and\ n(v)$

Colebrook Equation

$$\frac{\mathbf{1}}{\sqrt{\boldsymbol{f_f}}} = -\boldsymbol{4log_{10}}\left(\frac{\boldsymbol{\varepsilon}}{\mathbf{3.7}\boldsymbol{D}} + \frac{\mathbf{2.51}}{\boldsymbol{Re}\sqrt{\boldsymbol{f_f}}}\right)$$

Where $\boldsymbol{\varepsilon}$ is the tube roughness and D is the diameter of the tube.

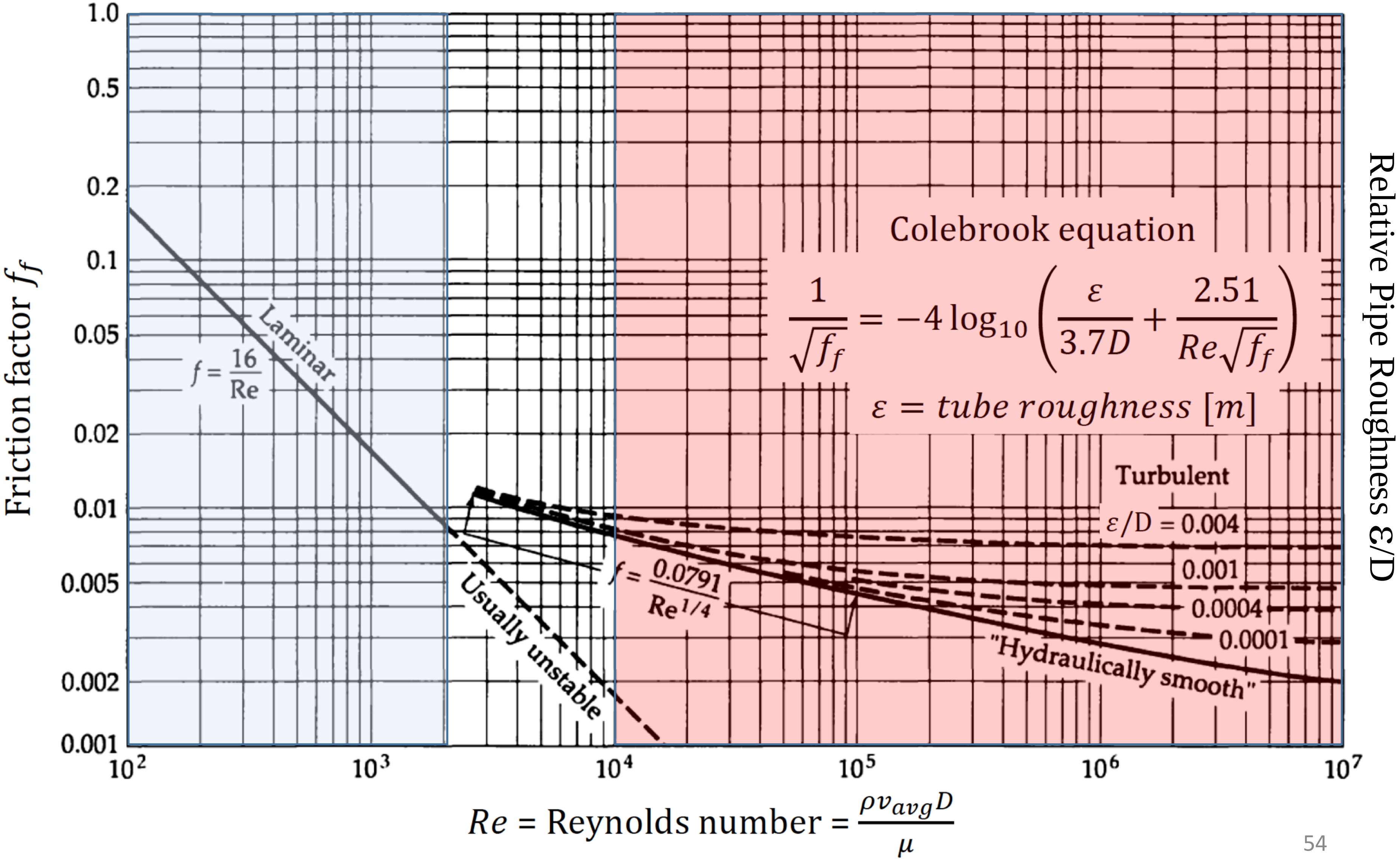
1.5. Friction factors due to viscosity



Surface	$\varepsilon = tube\ roughness$	
	10^{-3} (m)	*(feet)*
Copper, Lead, Brass, Aluminum (new)	0.001 - 0.002	3.3 - 6.7 10^{-6}
PVC and Plastic Pipes	0.0015 - 0.007	0.5 - 2.33 10^{-5}
Epoxy, Vinyl Ester and Isophthalic pipe	0.005	1.7 10^{-5}
Stainless steel	0.015	5 10^{-5}
Steel commercial pipe	0.045 - 0.09	1.5 - 3 10^{-4}
Stretched steel	0.015	5 10^{-5}
Weld steel	0.045	1.5 10^{-4}
Galvanized steel	0.15	5 10^{-4}
Rusted steel (corrosion)	0.15 - 4	5 - 133 10^{-4}
New cast iron	0.25 - 0.8	8 - 27 10^{-4}
Worn cast iron	0.8 - 1.5	2.7 - 5 10^{-3}
Rusty cast iron	1.5 - 2.5	5 - 8.3 10^{-3}

1.5. Friction factors due to viscosity

The Moody diagram: for flow of a Newtonian fluid in a straight circular pipe



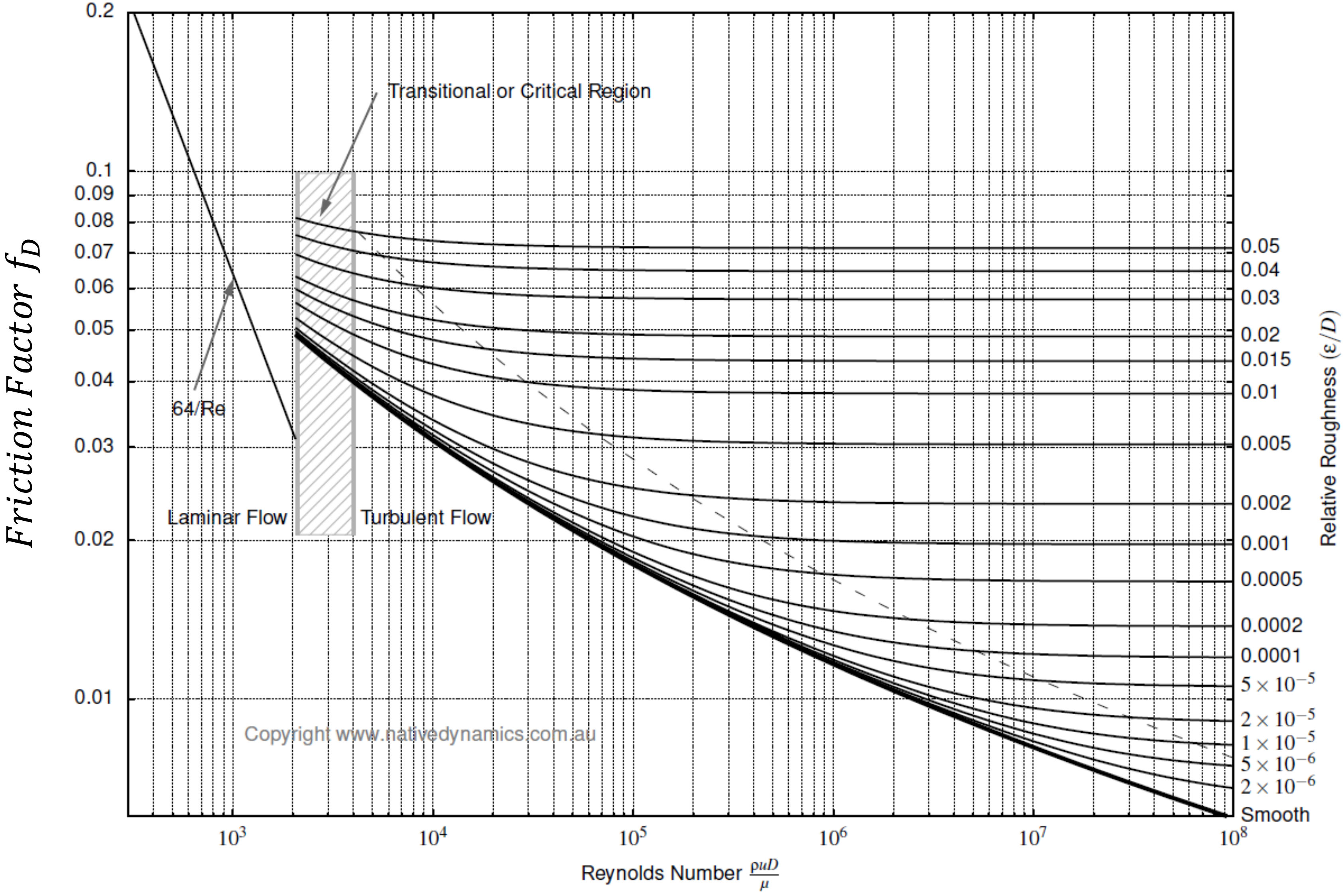
Colebrook equation

$$\frac{1}{\sqrt{f_f}} = -4\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f_f}}\right)$$

$$\varepsilon = tube\ roughness\ [m]$$

1.5. Friction factors due to viscosity

The Moody diagram: for flow of a Newtonian fluid in a straight circular pipe



1.5. Friction factors due to viscosity

The Moody diagram: for flow of a Newtonian fluid in a straight circular pipe

How to use the Moody diagram

1.5. Friction factors due to viscosity

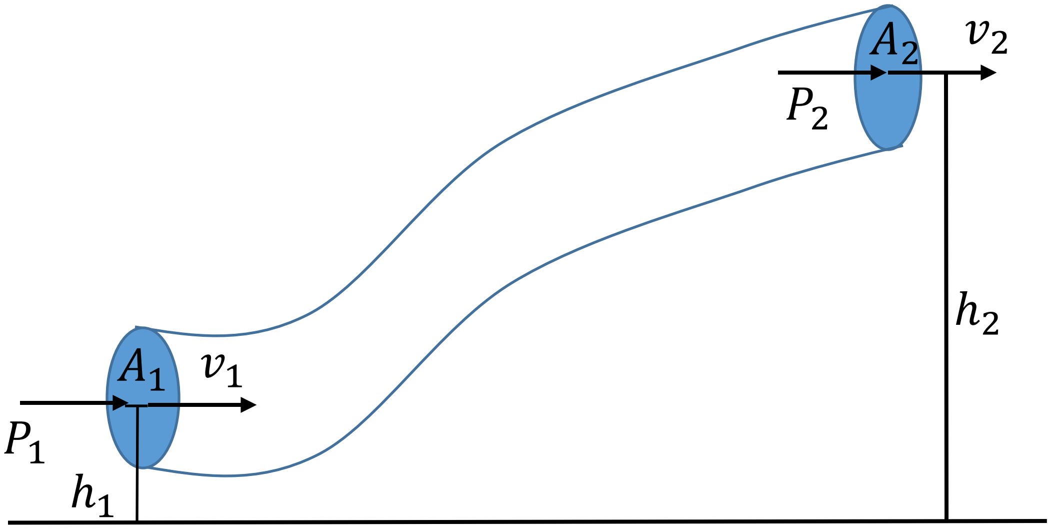
NEWTONIAN vs NON-NEWTONIAN FLUIDS

Newtonian fluids exhibit constant viscosity under applied shear stress for a constant temperature. They have no elasticity (memory effect) Examples are: water, mineral oil, gasoline.

In non-Newtonian fluids the viscosity changes under applied shear stress They have elasticity (memory effect) Examples are: glue, silly putty, paint.

1.5. Friction factors due to viscosity

In real pipe line there are energy losses due to friction and these must be taken into account as they can be pretty significant.

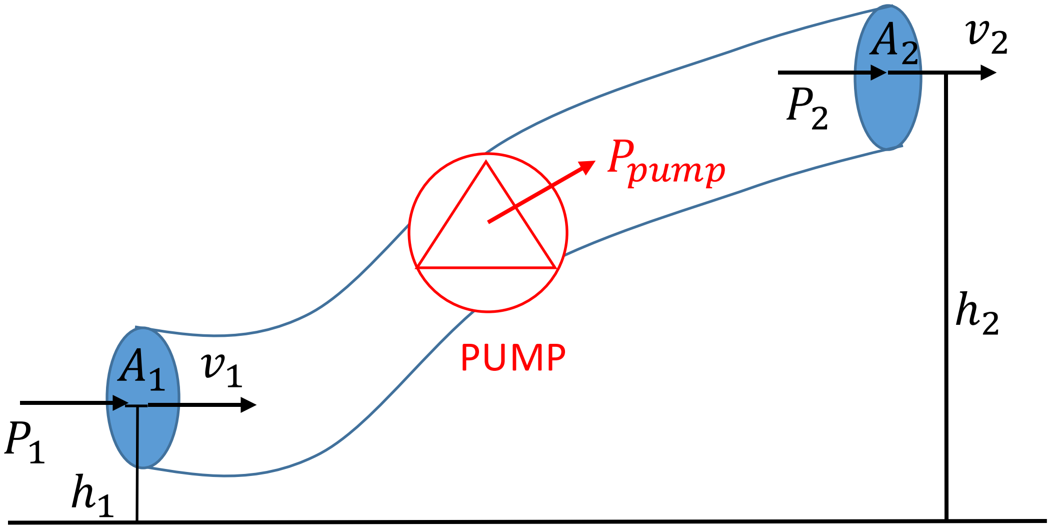


frictional pressure drop

$$\boxed{P_1 + \rho g\, h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g\, h_2 + \frac{1}{2}\rho v_2^2 + \boldsymbol{\Delta P_f}}$$

$$\boldsymbol{\Delta P_f = f_f \frac{4L}{D}\frac{\rho v_{avg}^2}{2}}$$

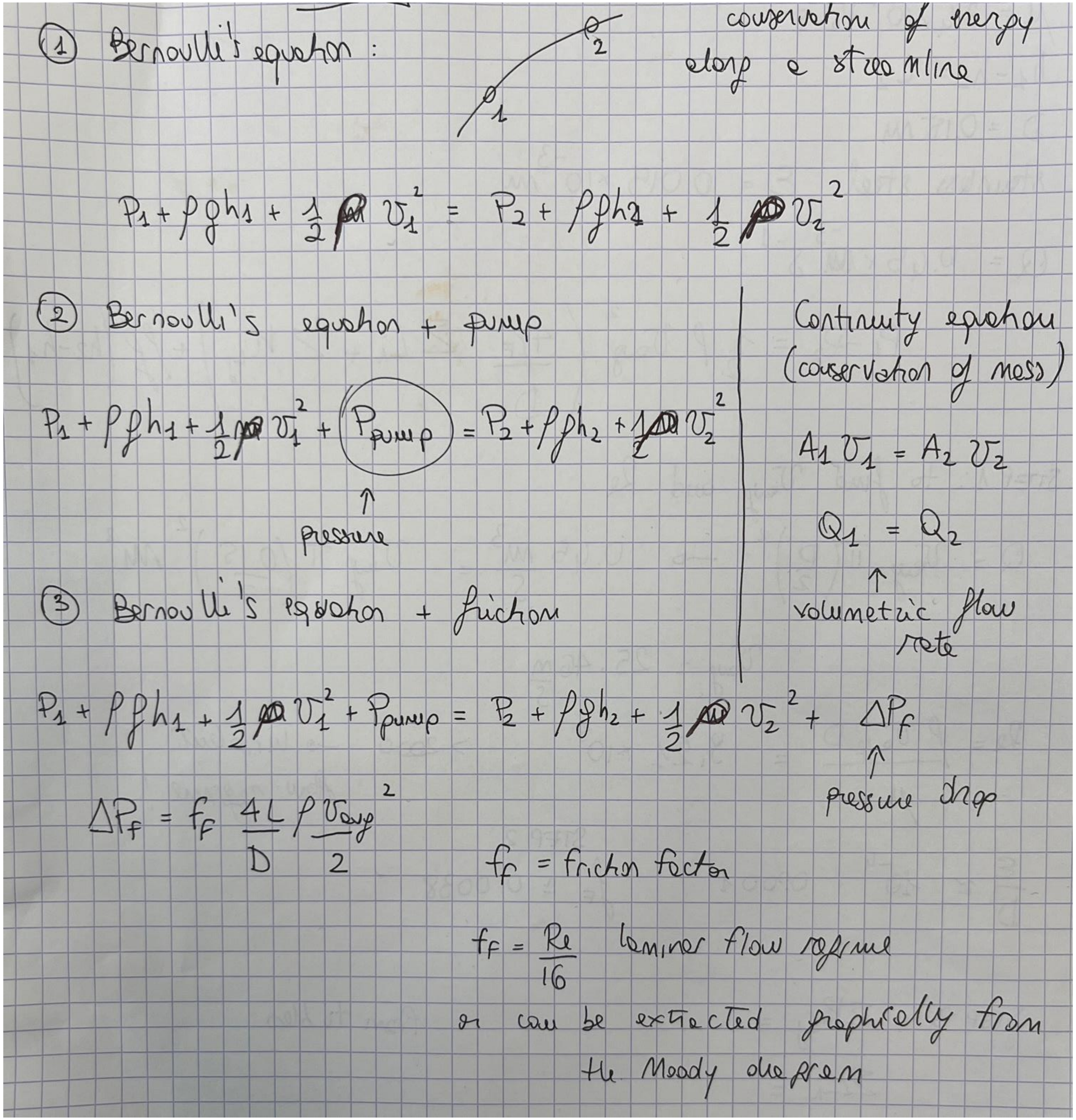
1.5. Friction factors due to viscosity



$$\frac{P_1}{\rho g} + h_1 + \frac{v_1^2}{2g} + \boldsymbol{H_p} = \frac{P_2}{\rho g} + h_2 + \frac{v_2^2}{2g} + \boldsymbol{H_f}$$

Extension of the Bernoulli's equation to include the energy gain from a pump and the energy loss from friction

RECAP



① Bernoulli's equation : conservation of energy along a streamline

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

② Bernoulli's equation + pump

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 + \boxed{P_{pump}} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

↑ pressure

Continuity equation (conservation of mass)

$$A_1 v_1 = A_2 v_2$$

$$Q_1 = Q_2$$

↑ volumetric flow rate

③ Bernoulli's equation + friction

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 + P_{pump} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 + \Delta P_f$$

↑ pressure drop

$$\Delta P_f = f_F \frac{4L}{D} \rho \frac{v_{avg}^2}{2}$$

f_F = friction factor

$f_F = \frac{Re}{16}$ laminar flow regime

or can be extracted graphically from the Moody diagram

IN REAL LIFE NOTHING IS SIMPLE

1.6. Pressure drop in various closed-flow "elements"

Each of these "minor losses" can be calculated by:

Pressure drop over an element (e.g. 90° elbow, gate valve, Y-connector) due to friction

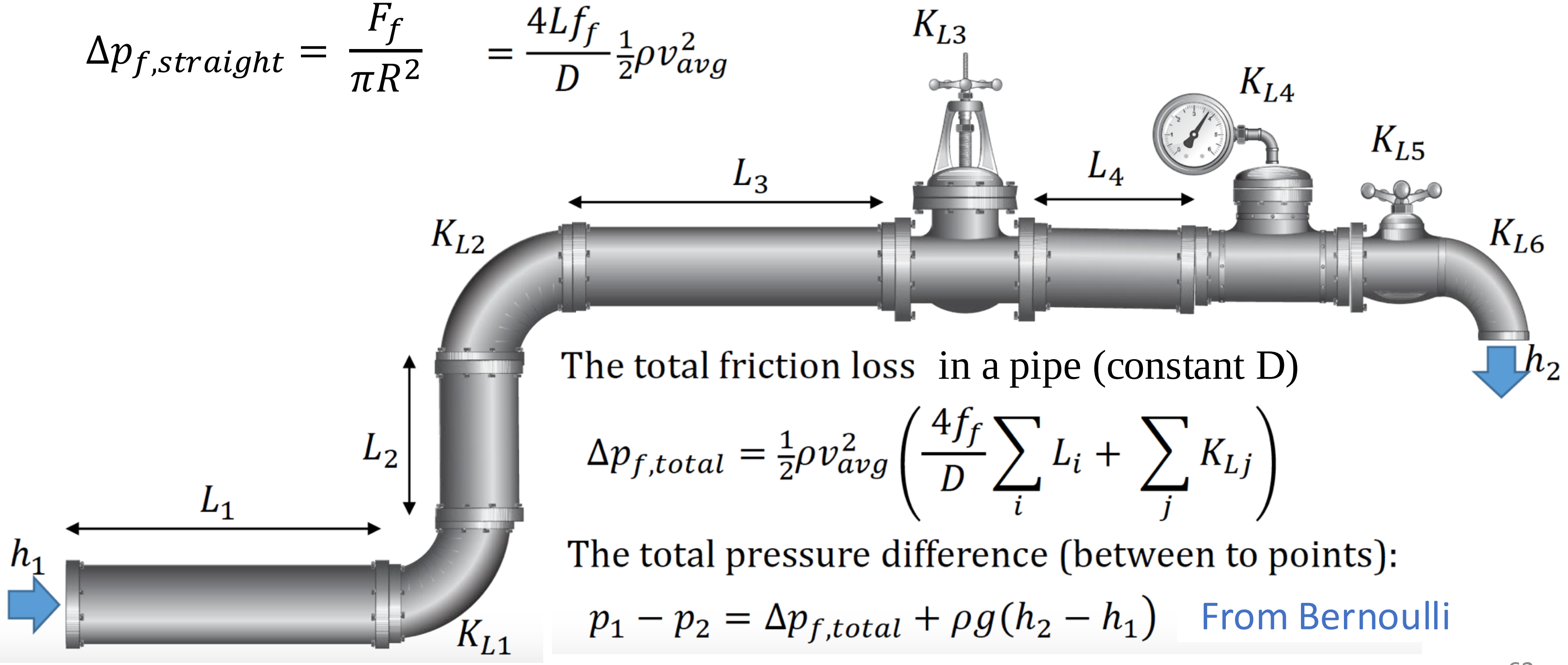
$$\Delta p_{f,element} = K_L \underbrace{\tfrac{1}{2}\rho v_{avg}^2}$$

Average kinetic energy density of fluid

Empirical "loss" coefficient (dimensionless)

The total friction loss in a **straight** section of the tube:

$$\Delta p_{f,straight} = \frac{F_f}{\pi R^2} \quad = \frac{4Lf_f}{D}\tfrac{1}{2}\rho v_{avg}^2$$



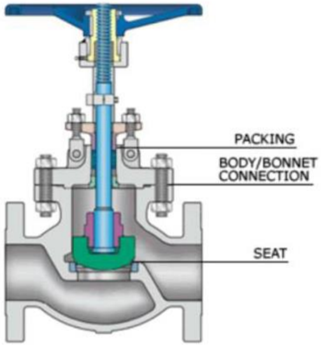
The total friction loss in a pipe (constant D)

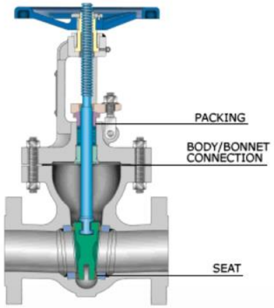
$$\Delta p_{f,total} = \tfrac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D} \sum_i L_i + \sum_j K_{Lj} \right)$$

The total pressure difference (between to points):

$$p_1 - p_2 = \Delta p_{f,total} + \rho g (h_2 - h_1)$$

From Bernoulli



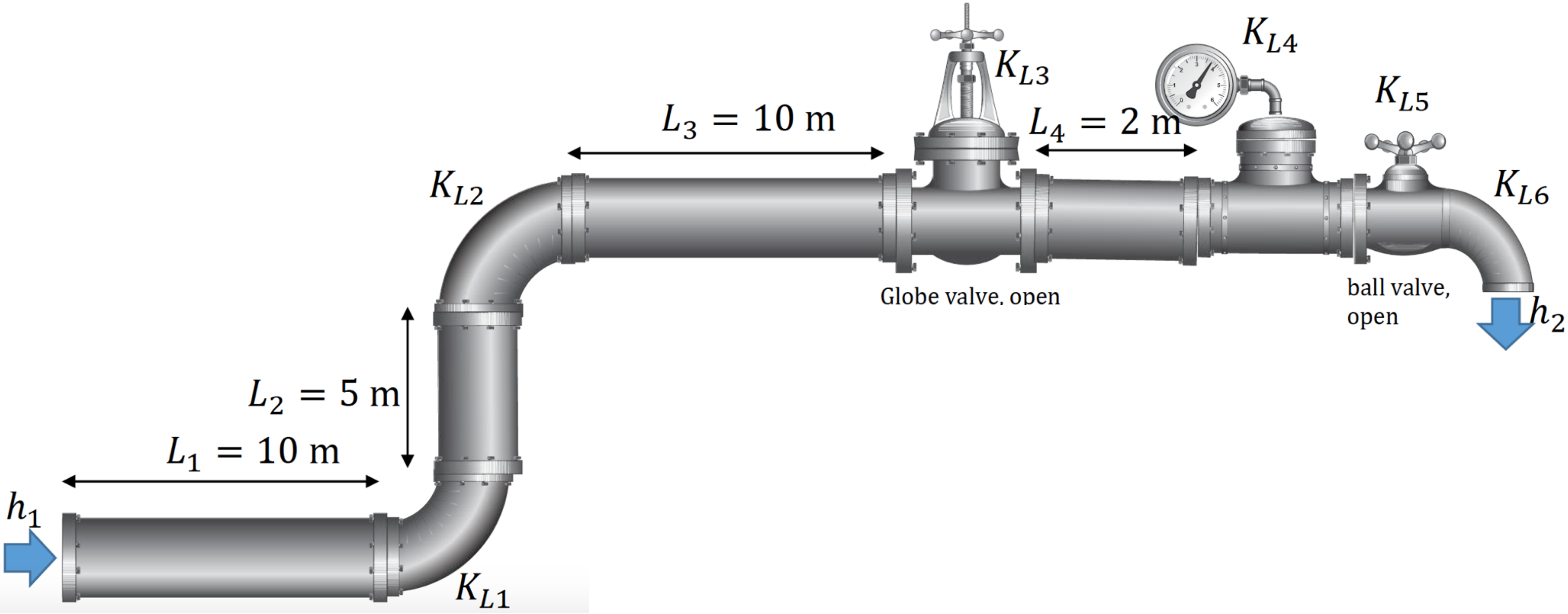


Type of Component or Fitting	Minor Loss K_L
Tee, Flanged, Dividing Line Flow	0.2
Tee, Threaded, Dividing Line Flow	0.9
Tee, Flanged, Dividing Branched Flow	1.0
Tee, Threaded , Dividing Branch Flow	2.0
Union, Threaded	0.08
Elbow, Flanged Regular 90°	0.3
Elbow, Threaded Regular 90°	1.5
Elbow, Threaded Regular 45°	0.4
Elbow, Flanged Long Radius 90°	0.2
Elbow, Threaded Long Radius 90°	0.7
Elbow, Flanged Long Radius 45°	0.2
Return Bend, Flanged 180°	0.2
Return Bend, Threaded 180°	1.5
Globe Valve, Fully Open	10
Angle Valve, Fully Open	2
Gate Valve, Fully Open	0.15
Gate Valve, 1/4 Closed	0.26
Gate Valve, 1/2 Closed	2.1
Gate Valve, 3/4 Closed	17
Swing Check Valve, Forward Flow	2
Ball Valve, Fully Open	0.05
Ball Valve, 1/3 Closed	5.5
Ball Valve, 2/3 Closed	200
Diaphragm Valve, Open	2.3
Diaphragm Valve, Half Open	4.3
Diaphragm Valve, 1/4 Open	21
Water meter	7

1.6. Pressure drop in various closed-flow ”elements”

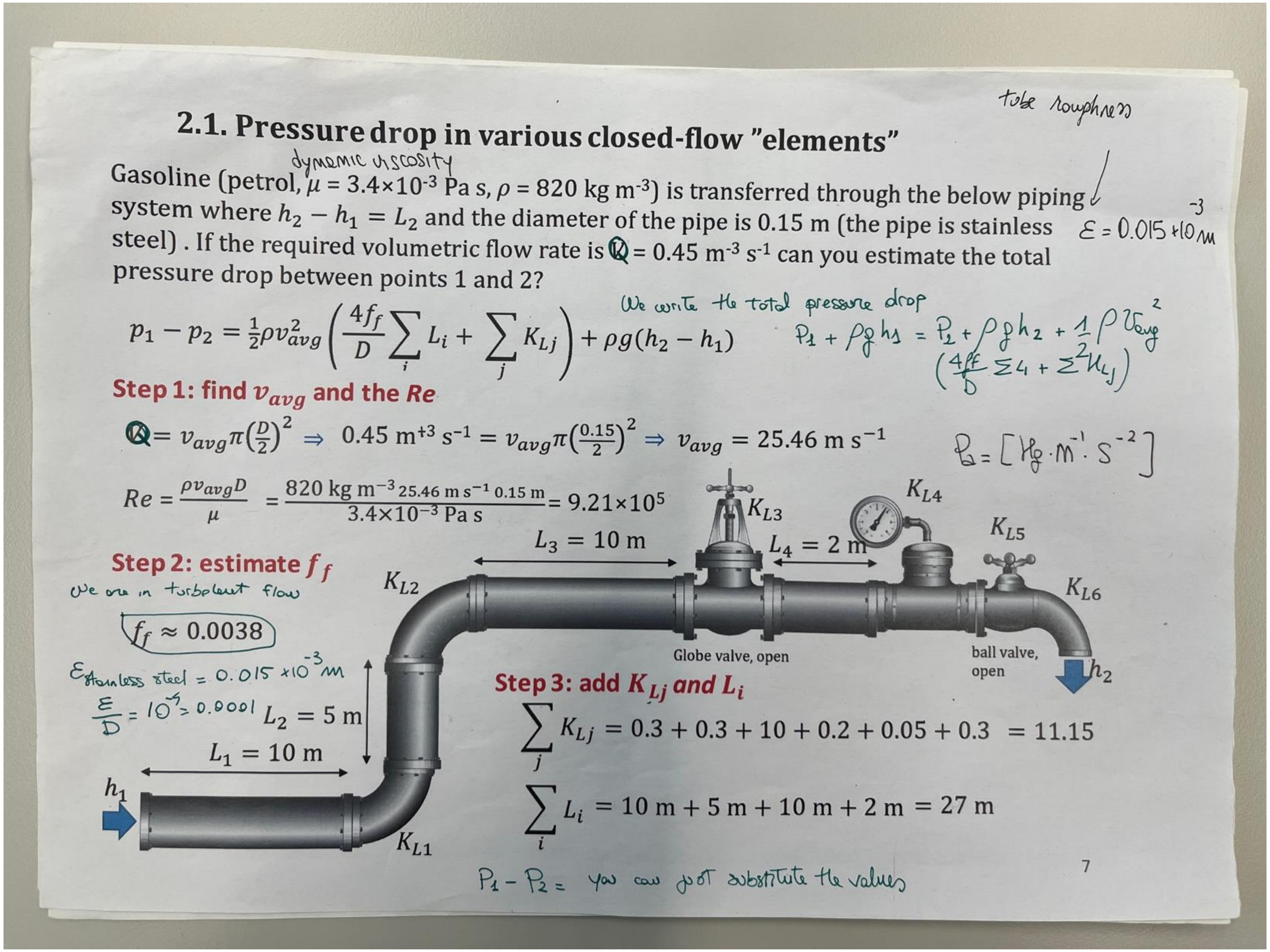
Exercize:

Gasoline (petrol, $\mu = 3.4 \times 10^{-3} Pa\ s, \rho = 820\ Kg\ m^{-3}$)is transferred through the below piping system where $h_2 - h_1 = L_2$and the diameter of the pipe is 0.15 m (the pipe is stainless steel). If the required volumetric flow rate is 0.45 $m^3\ s^{-1}$, can you estimate the total pressure drop between points 1 and 2?



1.6. Pressure drop in various closed-flow ”elements”

Solution: (This exercize will be solved during the lecture)



tube roughness

2.1. Pressure drop in various closed-flow ”elements”

dynamic viscosity

Gasoline (petrol, $\mu = 3.4\times10^{-3}$ Pa s, $\rho = 820$ kg m^{-3}) is transferred through the below piping system where $h_2 - h_1 = L_2$ and the diameter of the pipe is 0.15 m (the pipe is stainless steel) . If the required volumetric flow rate is $\textcircled{Q} = 0.45$ m^{-3} s^{-1} can you estimate the total pressure drop between points 1 and 2?

$\varepsilon = 0.015 \times 10^{-3}$ m

$$p_1 - p_2 = \frac{1}{2}\rho v_{avg}^2\left(\frac{4f_f}{D}\sum_i L_i + \sum_j K_{Lj}\right) + \rho g(h_2 - h_1)$$

We write the total pressure drop

$$P_1 + \rho g h_1 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_{avg}^2 \left(\frac{4f_f}{D}\sum L_i + \sum K_{Lj}\right)$$

Step 1: find v_{avg} and the Re

$$\textcircled{Q} = v_{avg}\pi\left(\frac{D}{2}\right)^2 \Rightarrow 0.45\ \text{m}^{+3}\ \text{s}^{-1} = v_{avg}\pi\left(\frac{0.15}{2}\right)^2 \Rightarrow v_{avg} = 25.46\ \text{m s}^{-1}$$

$P_a = [\text{Kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}]$

$$Re = \frac{\rho v_{avg} D}{\mu} = \frac{820\ \text{kg m}^{-3}\, 25.46\ \text{m s}^{-1}\, 0.15\ \text{m}}{3.4\times10^{-3}\ \text{Pa s}} = 9.21\times10^5$$

Step 2: estimate f_f

We are in turbulent flow

$$\boxed{f_f \approx 0.0038}$$

$\varepsilon_{\text{stainless steel}} = 0.015 \times 10^{-3}$ m

$\frac{\varepsilon}{D} = 10^{-4} = 0.0001$

Step 3: add K_{Lj} and L_i

$$\sum_j K_{Lj} = 0.3 + 0.3 + 10 + 0.2 + 0.05 + 0.3 = 11.15$$

$$\sum_i L_i = 10\ \text{m} + 5\ \text{m} + 10\ \text{m} + 2\ \text{m} = 27\ \text{m}$$

$P_1 - P_2 =$ you can just substitute the values

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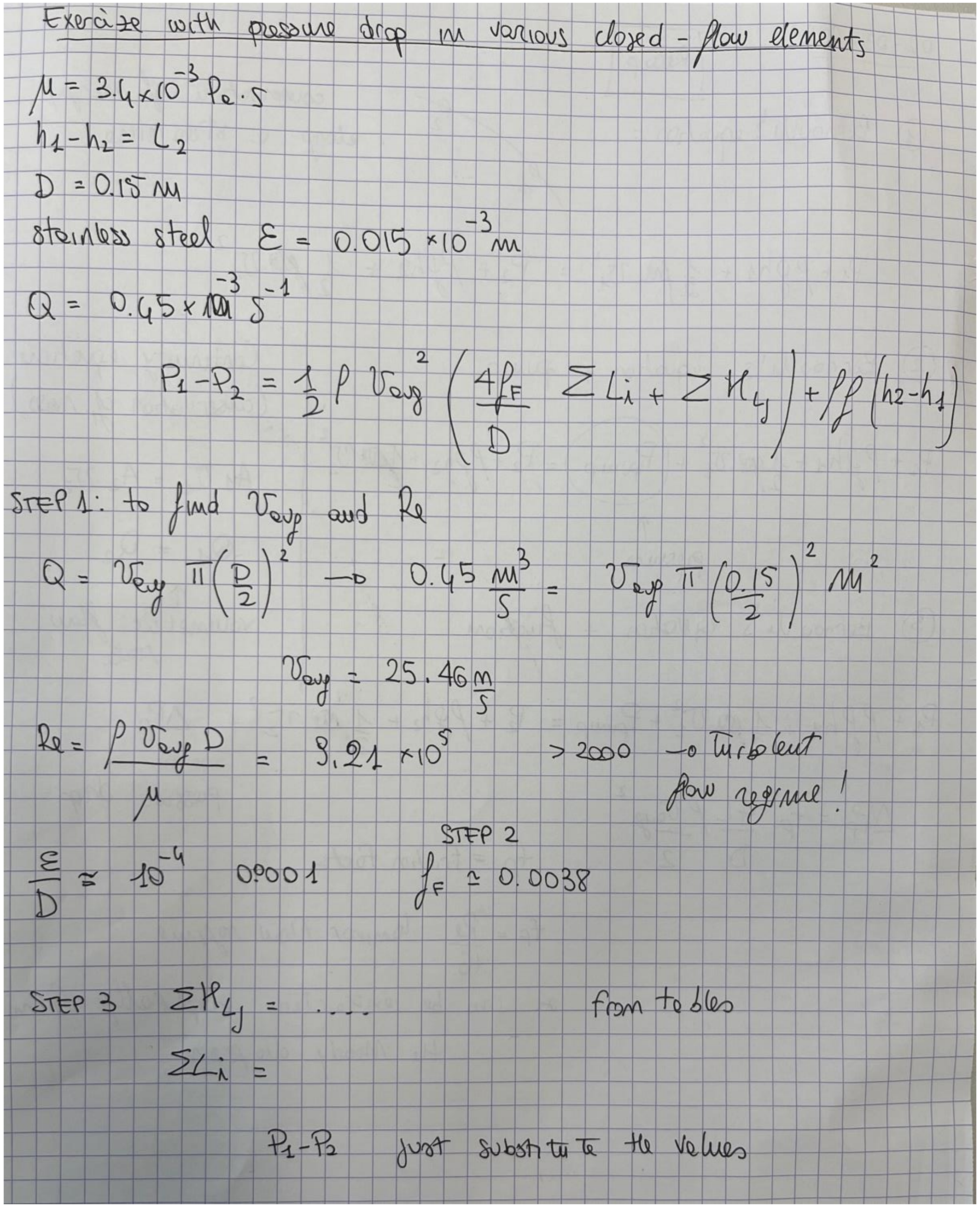
1.6. Pressure drop in various closed-flow ”elements”

Solution:



Exercize with pressure drop in various closed - flow elements

$\mu = 3.4 \times 10^{-3}\ Pa \cdot s$

$h_1 - h_2 = L_2$

$D = 0.15\ m$

stainless steel $\varepsilon = 0.015 \times 10^{-3}\ m$

$Q = 0.45 \times 10^{-3}\ m\ s^{-1}$

$$P_1 - P_2 = \frac{1}{2} \rho v_{avg}^2 \left(\frac{4 f_F}{D} \sum L_i + \sum K_{Lj} \right) + \rho g (h_2 - h_1)$$

STEP 1: to find v_{avg} and Re

$$Q = v_{avg}\, \pi \left(\frac{D}{2}\right)^2 \rightarrow 0.45\ \frac{m^3}{s} = v_{avg}\, \pi \left(\frac{0.15}{2}\right)^2 m^2$$

$$v_{avg} = 25.46\ \frac{m}{s}$$

$$Re = \frac{\rho\, v_{avg}\, D}{\mu} = 9.21 \times 10^5$$

$> 2000 \rightarrow$ Turbolent flow regime!

STEP 2

$\frac{\varepsilon}{D} \simeq 10^{-4}$ 0.0001

$f_F \simeq 0.0038$

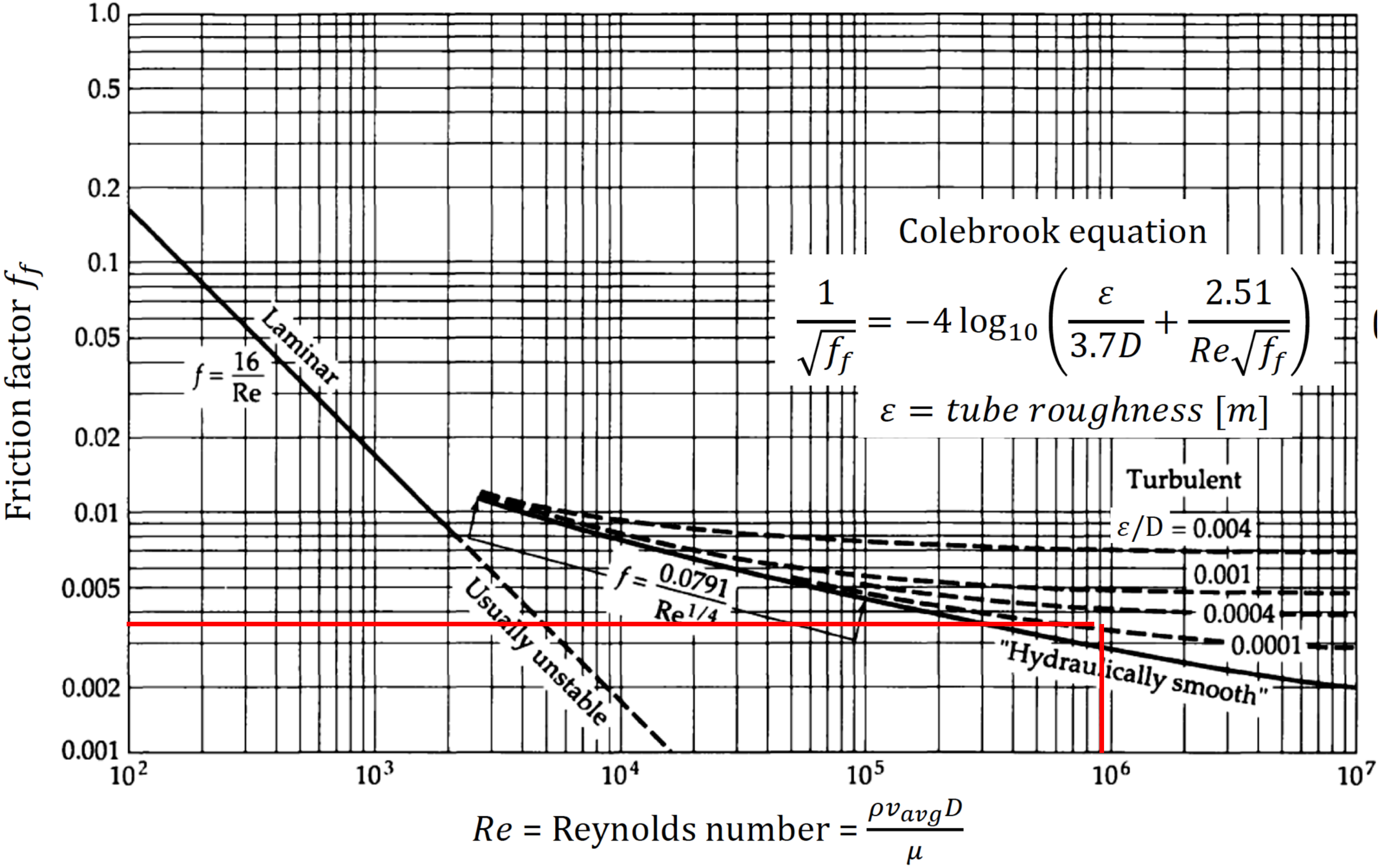
STEP 3 $\sum K_{Lj} = \dots$ from tables

$\sum L_i =$

$P_1 - P_2$ just substitute the values

1.6. Pressure drop in various closed-flow "elements"

Solution: (This exercize will be solved during the lecture)



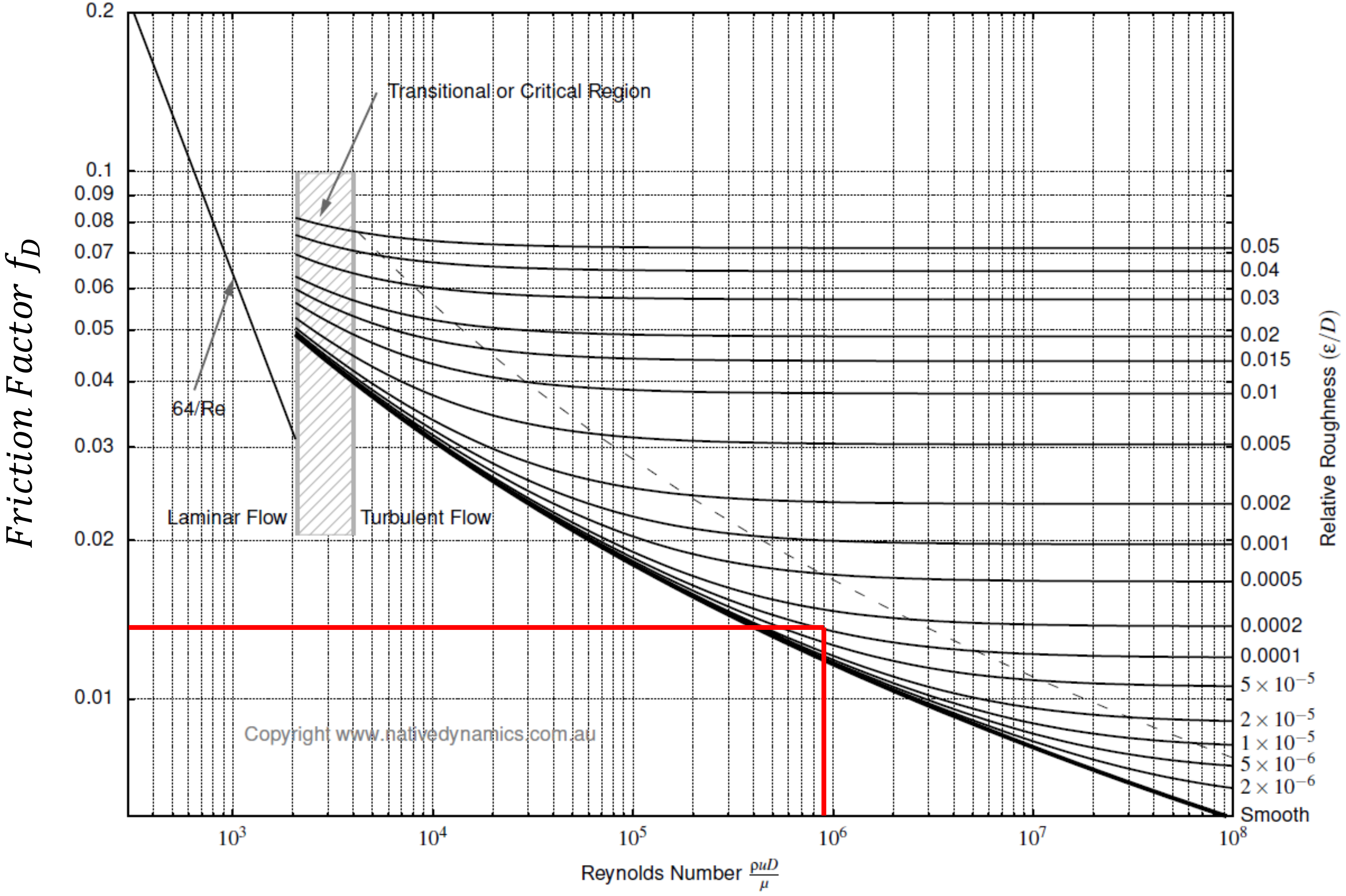
Colebrook equation

$$\frac{1}{\sqrt{f_f}} = -4\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f_f}}\right)$$

$\varepsilon = tube\ roughness\ [m]$

1.6. Pressure drop in various closed-flow "elements"

Solution: (This exercize will be solved during the lecture)



1.6. Pressure drop in various closed-flow ”elements”

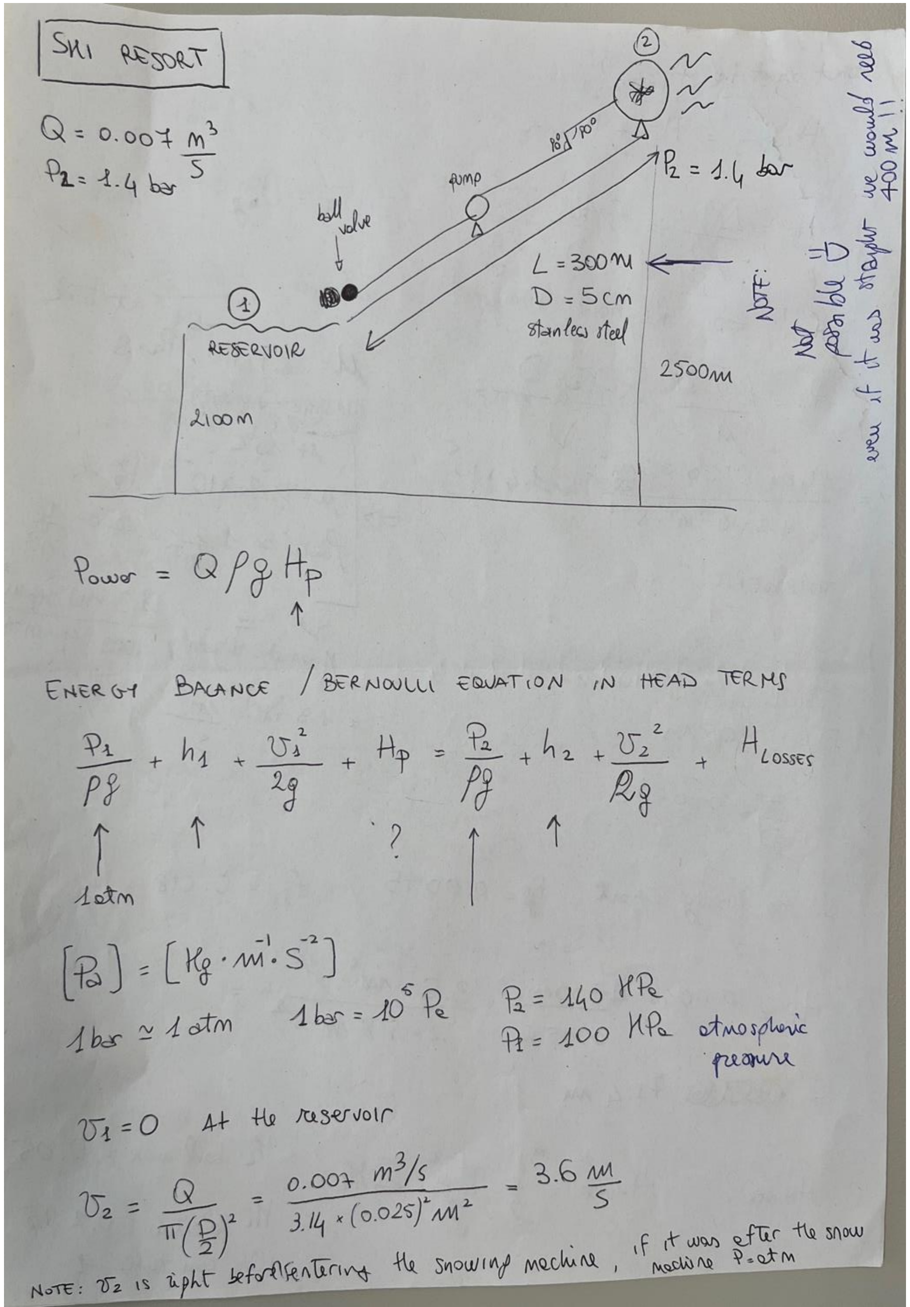
Exercize

A ski resort needs to pump water at 10ºC through 300 meters of 5 cm diameter steel pipe from a pond at 2100 meters to a snow making machine at 2500 meters. The volumetric flow rate has to be 0.007 m^3/s and pressure into the machine at 1.4 bar. Determine the power added by the pump. There is one ball valve controlling the inlet to the steel pipe and two 90º bends.

Solution: (This exercize will be solved during the lecture)

1.6. Pressure drop in various closed-flow ”elements”

Solution:



$Q = 0.007 \frac{m^3}{s}$

$P_2 = 1.4$ bar

$$\text{Power} = Q \rho g \underset{\uparrow}{H_p}$$

ENERGY BALANCE / BERNOULLI EQUATION IN HEAD TERMS

$$\underset{\substack{\uparrow \\ 1atm}}{\frac{P_1}{\rho g}} + \underset{\uparrow}{h_1} + \frac{v_1^2}{2g} + \underset{?}{H_p} = \underset{\uparrow}{\frac{P_2}{\rho g}} + \underset{\uparrow}{h_2} + \frac{v_2^2}{2g} + H_{LOSSES}$$

$[Pa] = [Kg \cdot m^{-1} \cdot s^{-2}]$

1 bar ≃ 1 atm

1 bar = 10^5 Pa

$P_2 = 140$ KPa

$P_1 = 100$ KPa atmospheric pressure

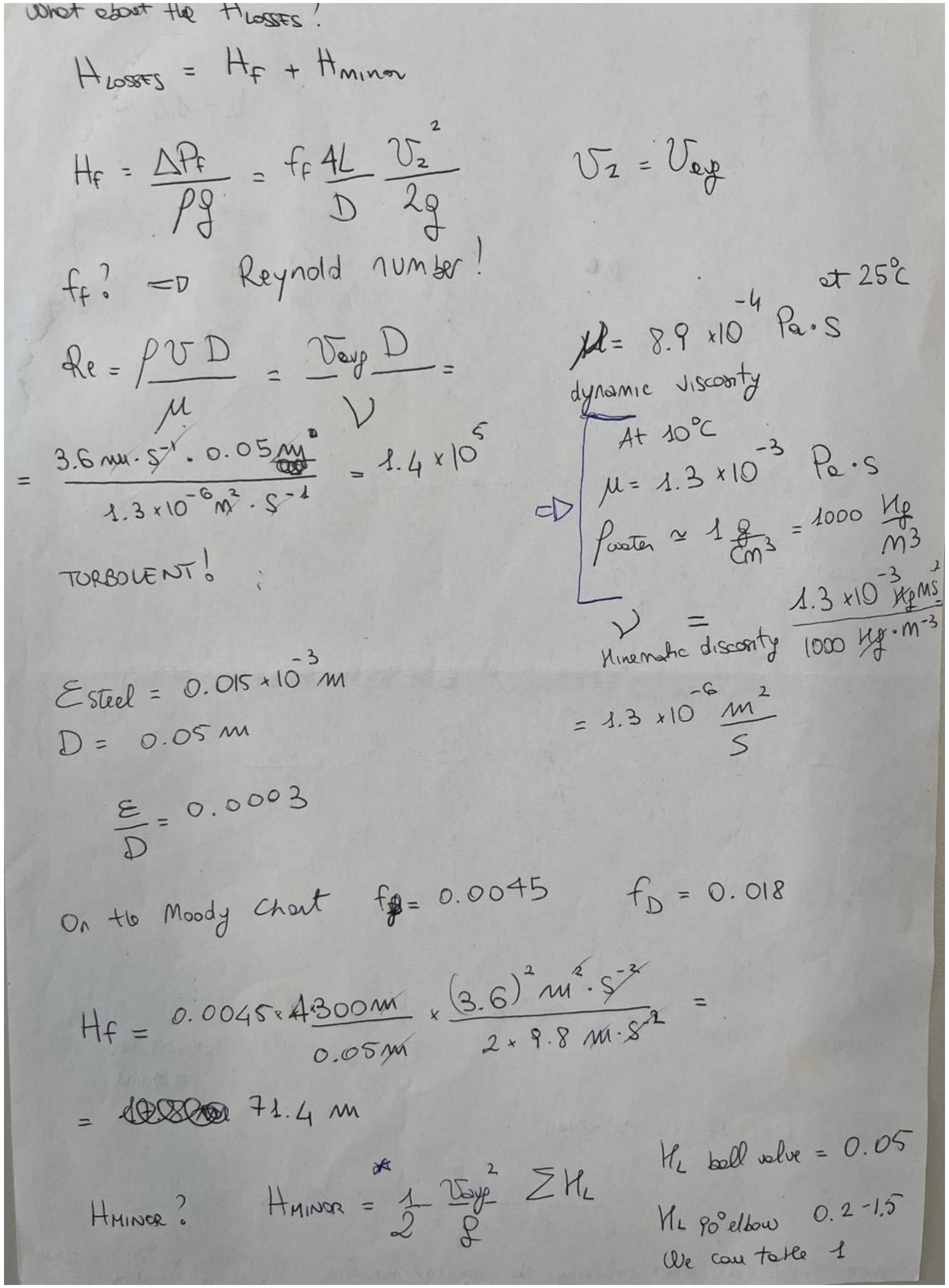
$v_1 = 0$ At the reservoir

$$v_2 = \frac{Q}{\pi \left(\frac{D}{2}\right)^2} = \frac{0.007 \; m^3/s}{3.14 \times (0.025)^2 \, m^2} = 3.6 \frac{m}{s}$$

NOTE: v_2 is right before entering the snowing machine, if it was after the snow machine P = atm

1.6. Pressure drop in various closed-flow "elements"

Solution:



What about the H_{LOSSES}?

$$H_{LOSSES} = H_F + H_{minor}$$

$$H_f = \frac{\Delta P_f}{\rho g} = f_F \frac{4L}{D} \frac{v_2^2}{2g}$$

$$v_2 = v_{avg}$$

f_F? $\Rightarrow$ Reynold number!

$$Re = \frac{\rho v D}{\mu} = \frac{v_{avg} D}{\nu} = $$

$$= \frac{3.6\, m \cdot s^{-1} \cdot 0.05\, m}{1.3 \times 10^{-6}\, m^2 \cdot s^{-1}} = 1.4 \times 10^5$$

TORBOLENT!

at 25°C

$\mu = 8.9 \times 10^{-4}$ Pa·S

dynamic viscosity

$\Rightarrow$ At 10°C

$\mu = 1.3 \times 10^{-3}$ Pa·s

$\rho_{water} \simeq 1 \frac{g}{cm^3} = 1000 \frac{kg}{m^3}$

$$\nu = \frac{1.3 \times 10^{-3}\, kg\, m\, s^{-2}}{1000\, kg \cdot m^{-3}}$$

kinematic viscosity

$$= 1.3 \times 10^{-6} \frac{m^2}{s}$$

$\varepsilon_{steel} = 0.015 \times 10^{-3}\, m$

$D = 0.05\, m$

$$\frac{\varepsilon}{D} = 0.0003$$

On the Moody chart $f_F = 0.0045$ $f_D = 0.018$

$$H_f = 0.0045 \times \frac{4300\, m}{0.05\, m} \times \frac{(3.6)^2\, m^2 \cdot s^{-2}}{2 \times 9.8\, m \cdot s^{-2}} =$$

$$= 71.4\, m$$

H_{MINOR}?

$$H_{MINOR} = \frac{1}{2} \frac{v_{avg}^2}{g} \sum K_L$$

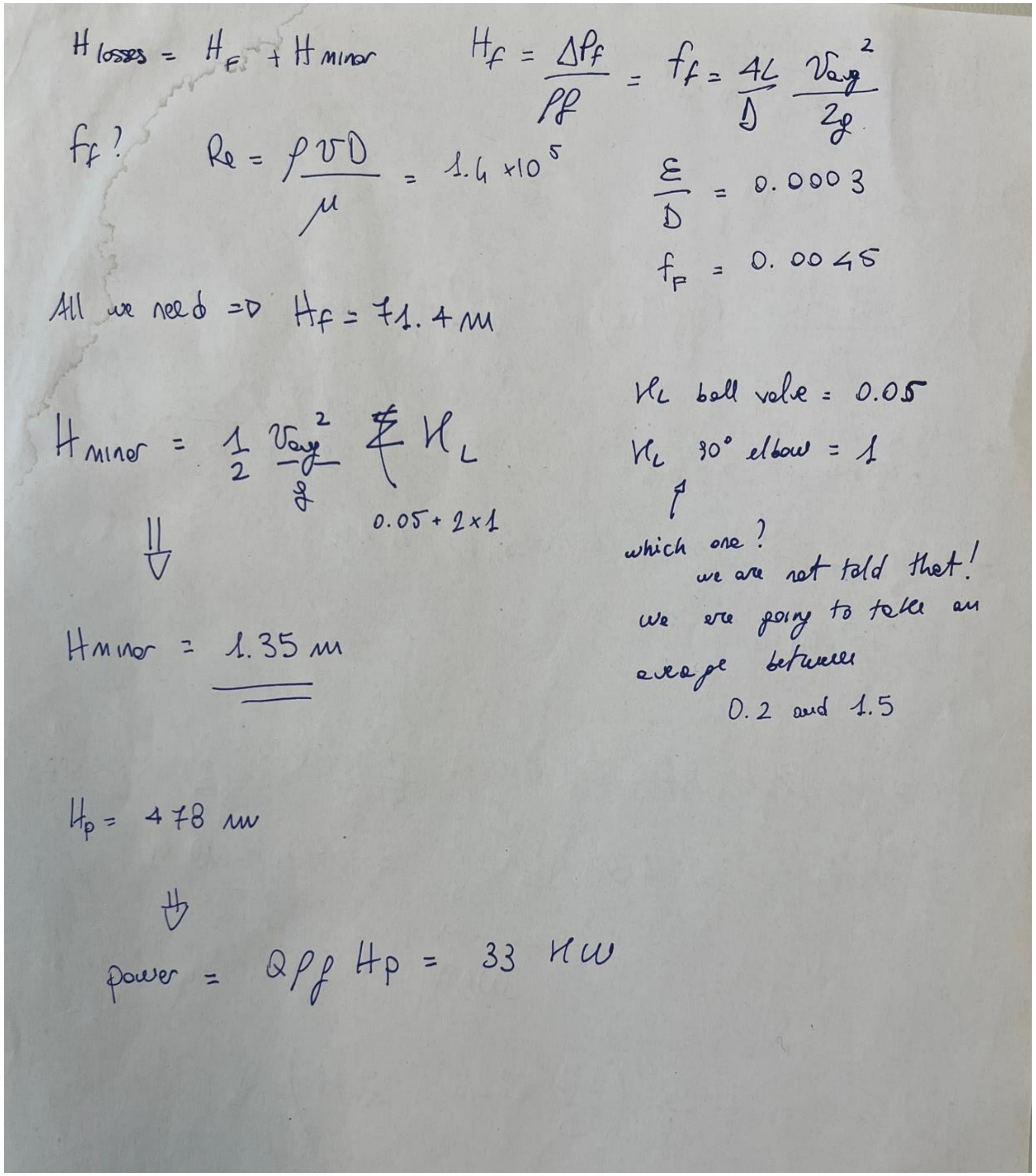
K_L ball valve = 0.05

K_L 90° elbow 0.2 - 1,5

We can take 1

1.6. Pressure drop in various closed-flow ”elements”

Solution:



$H_{losses} = H_F + H_{minor}$ $\quad H_f = \frac{\Delta P_f}{\rho g} = f_f = \frac{4L}{D} \frac{V_{avg}^2}{2g}$

f_f ? $\quad Re = \frac{\rho v D}{\mu} = 1.4 \times 10^5$

$\frac{\varepsilon}{D} = 0.0003$

$f_F = 0.0045$

All we need $\Rightarrow$ $H_f = 71.4$ m

K_L ball valve = 0.05

K_L 90° elbow = 1

↑

which one ? we are not told that! we are going to take an average between 0.2 and 1.5

$H_{minor} = \frac{1}{2} \frac{V_{avg}^2}{g} \sum K_L$

0.05 + 2×1

$\Downarrow$

$H_{minor} = 1.35$ m

$H_p = 478$ m

$\Downarrow$

power = $Q \rho g H_p$ = 33 kW