

Computational quantum dynamics: Exercise series 2

Problem 1: Normalization of the momentum wavefunction

In class, we have discussed the free particle wavepacket using the k -representation (the same as momentum representation in atomic units with $\hbar = 1$). Show that in general (i.e., not necessarily for a free particle), if the position state wavefunction $\psi(q)$ is normalized, so is the k -representation wave function $a(k)$. I.e., prove that

$$\int_{-\infty}^{\infty} |a(k)|^2 dk = 1.$$

Hint: Use the definition of $a(k)$ from class and the orthonormality of position states expressed in the k -representation,

$$\langle q' | q'' \rangle = \int_{-\infty}^{\infty} \langle q' | k \rangle \langle k | q'' \rangle dk = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikq'} e^{-ikq''} dk = \delta(q' - q'').$$

Read Chapter 2 (except for Section 2.2.2) of Tannor.