

# Computational quantum dynamics: Exercise series 2

## Problem 1: Normalization of the momentum wavefunction

In class, we have discussed the free particle wavepacket using the  $k$ -representation (the same as momentum representation in atomic units with  $\hbar = 1$ ). Show that in general (i.e., not necessarily for a free particle), if the position state wavefunction  $\psi(q)$  is normalized, so is the  $k$ -representation wave function  $a(k)$ . I.e., prove that

$$\int_{-\infty}^{\infty} |a(k)|^2 = 1.$$

*Hint:* Use the definition of  $a(k)$  from class and the orthonormality of position states expressed in the  $k$ -representation,

$$\langle q' | q'' \rangle = \int_{-\infty}^{\infty} \langle q' | k \rangle \langle k | q'' \rangle dk = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikq'} e^{-ikq''} dk = \delta(q' - q'').$$

Read Chapter 2 (except for Section 2.2.2) of Tannor.