

4. Optics review

4.1 Ray Optics

Light travels in different optical media in accordance with a set of geometrical rules

4.2 Classical (Wave) Description

Light is an EM wave

4.3 Quantum (Particle) Description

Localized, massless quanta of energy – photons

**The most useful and complete treatment of light:
Electromagnetic Theory of Light and Wave Optics**

- Light is an **electromagnetic wave phenomena** described by the same theoretical principles that govern all forms of electromagnetic radiation.
- An electromagnetic field is described by two related vector fields: the **electrical field $E(\mathbf{r}, t)$** and the **magnetic field $H(\mathbf{r}, t)$** .
- $E(\mathbf{r}, t)$ and $H(\mathbf{r}, t)$ must satisfy a set of coupled equations known as **Maxwell's equations**.

Differential form of the Maxwell's Equations

(See appendix 1 of: "Optics", E. Hecht, Addison Wesley, 2000, for their derivation)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}, \quad \mathbf{J} = \sigma \mathbf{E}$$

\mathbf{E} = electric field intensity,

\mathbf{B} = magnetic flux density,

\mathbf{D} = electric flux density,

\mathbf{H} = magnetic field intensity,

\mathbf{J} = current density,

ρ = charge density,

ϵ = permittivity of medium,

μ = permeability of medium,

σ = conductivity of medium.

Maxwell's Equations

For dielectric medium (air, glass, etc.) $\sigma = 0, \quad \rho = 0, \quad \frac{\mu}{\mu_0} = 1$

De-couple $E(r, t)$ and $H(r, t)$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = 0$$

$$\mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu_0 \mathbf{H},$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial}{\partial t} (\nabla \times \mathbf{H})$$

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} = -\mu_0 \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

general form of wave equation:

$$\nabla^2 \mathbf{E} - \mu_0 \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

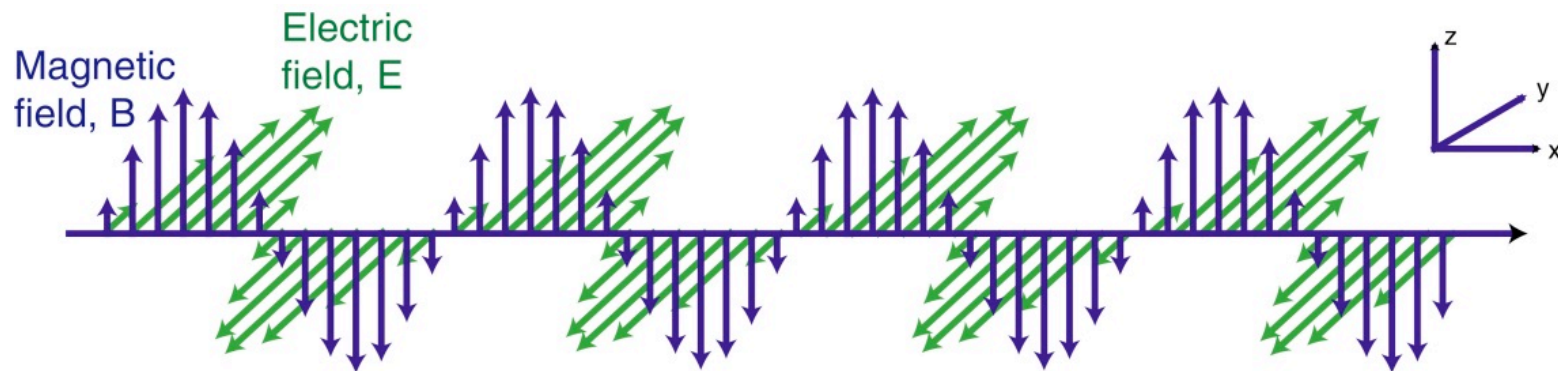
\mathbf{D} = electric flux density \mathbf{B} = magnetic flux density

Maxwell's Equations

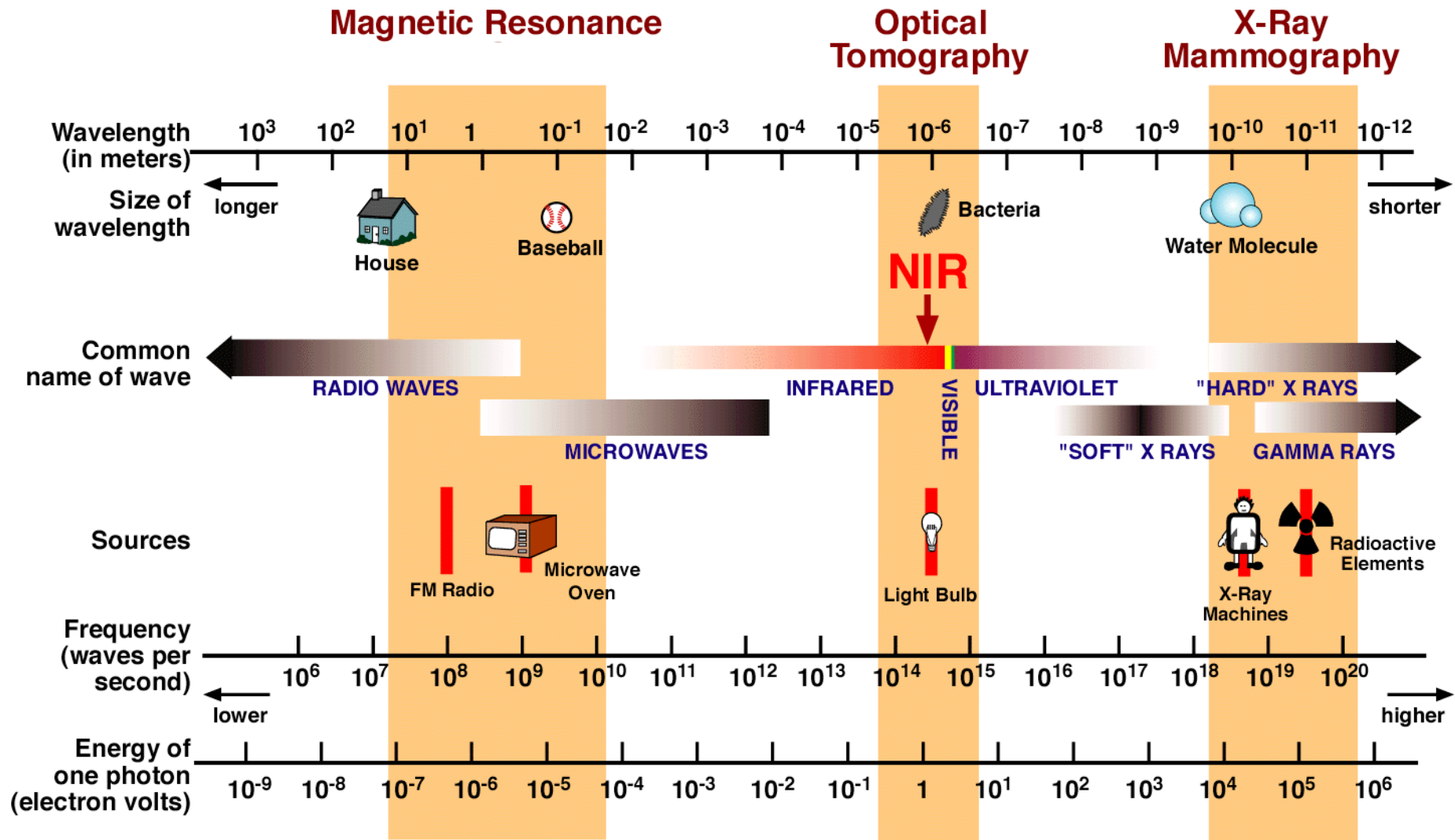
A general solution to the wave equation has this form:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 \exp\left[i\left(\vec{k} \cdot \vec{r} - \omega t\right)\right]$$

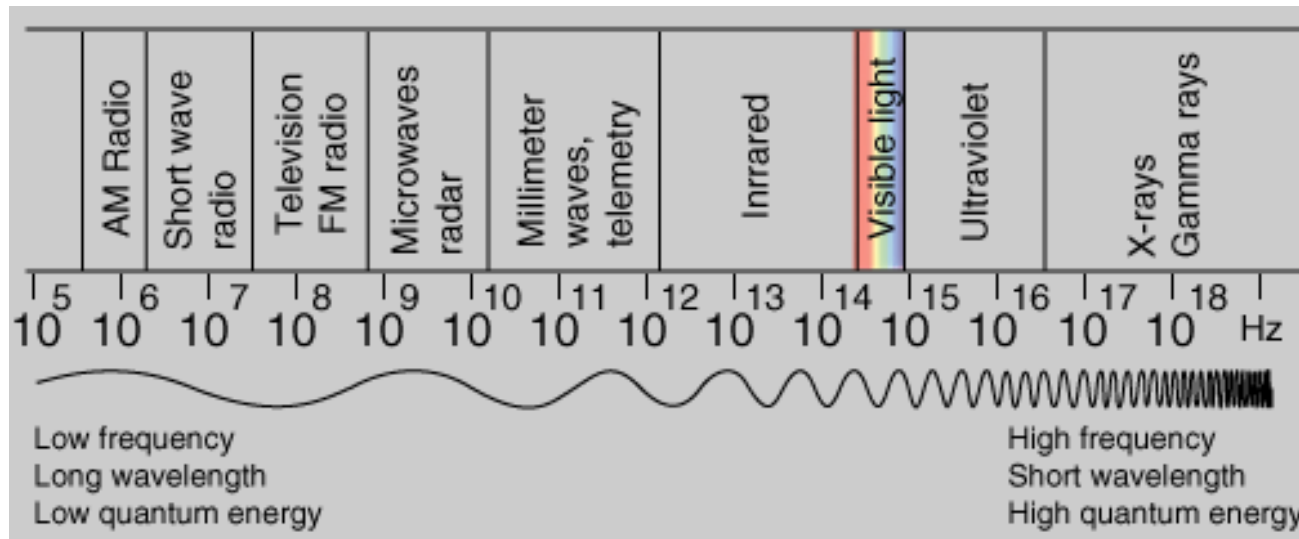
In empty space, plane electromagnetic field patterns look like this transverse ($\vec{E} \times \vec{B} \propto \vec{k}$) wave, with \vec{E} and \vec{B} in phase ...



The Electromagnetic Spectrum



The Electromagnetic Spectrum



INFRARED

Frequencies: $.003 - 4 \times 10^{14}$ Hz

Wavelengths: 1 mm - 750 nm

Quantum energies: 0.0012 - 1.65 eV

VISIBLE

Frequencies: $4 - 7.5 \times 10^{14}$ Hz

Wavelengths: 750 - 400 nm

Quantum energies: 1.65 - 3.1 eV

ULTRAVIOLET

Frequencies: $7.5 \times 10^{14} - 3 \times 10^{16}$ Hz

Wavelengths: 400 nm - 10 nm

Quantum energies: 3.1 - 124 eV

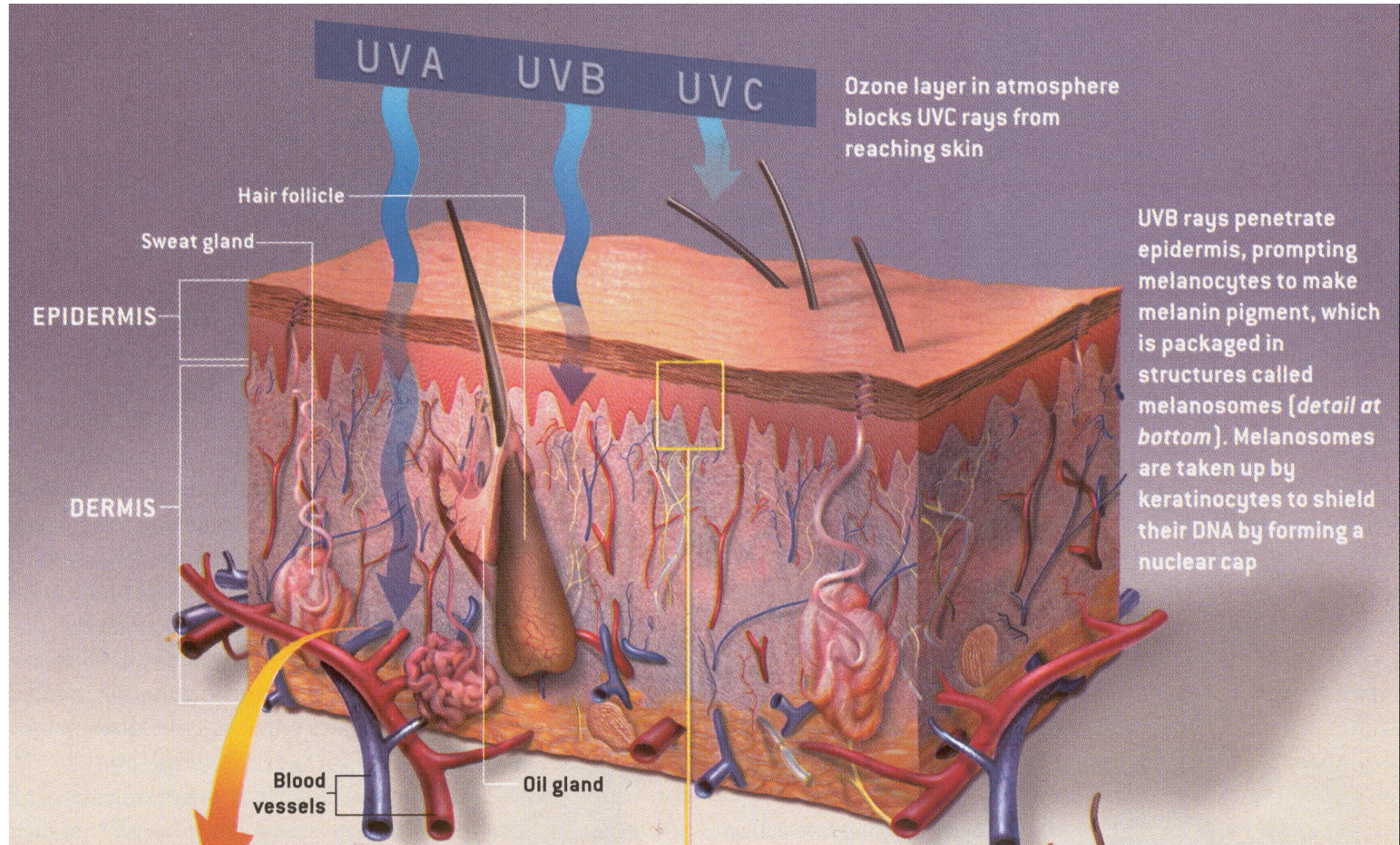
UV-A: 400 – 315 nm (95 % of the UV sunlight at sea level; short term suntan, free radicals, carcinogenic)

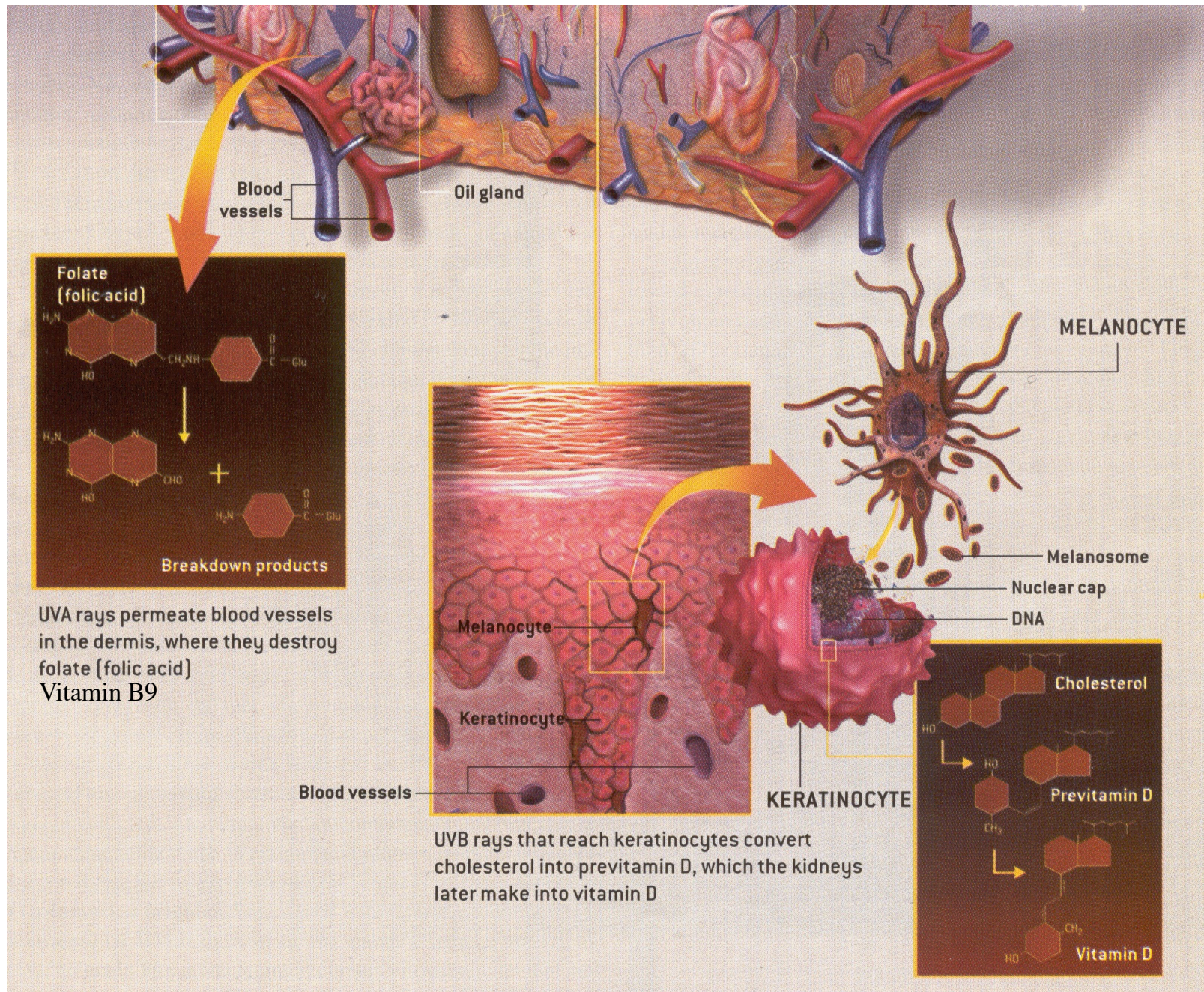
UV-B: 315 – 280 nm (5 % of the UV sunlight at sea level; long term suntan, free radicals, **carcinogenic**)

UV-C: 280 – 200 nm (Totally absorbed by the atmosphere; strong germicide effect)

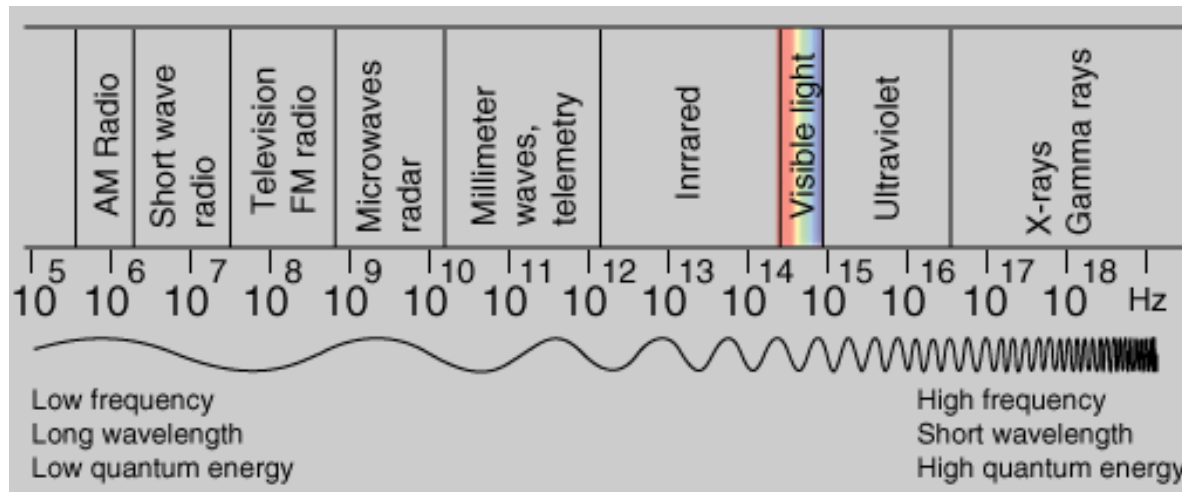
V-UV: 200 – 100 nm (Vacuum UV)

X-UV: 100 – 10 nm (Transition between the UV and the X-rays)





The Electromagnetic Spectrum



X-Rays

Frequencies: : 3×10^{16} Hz upward

Wavelengths: 10 nm - > downward

Quantum energies: 124 eV -> upward

Gamma-Rays

Frequencies: : typically $>10^{20}$ Hz

Wavelengths: typically $< 10^{-12}$ m

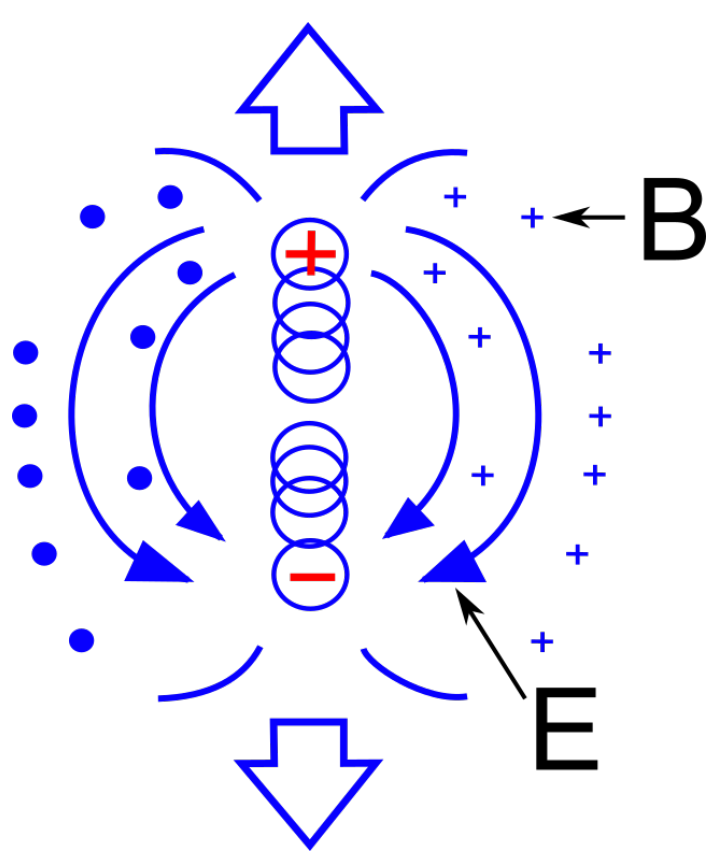
Quantum energies: typically >1 MeV

Sources of EM Radiation

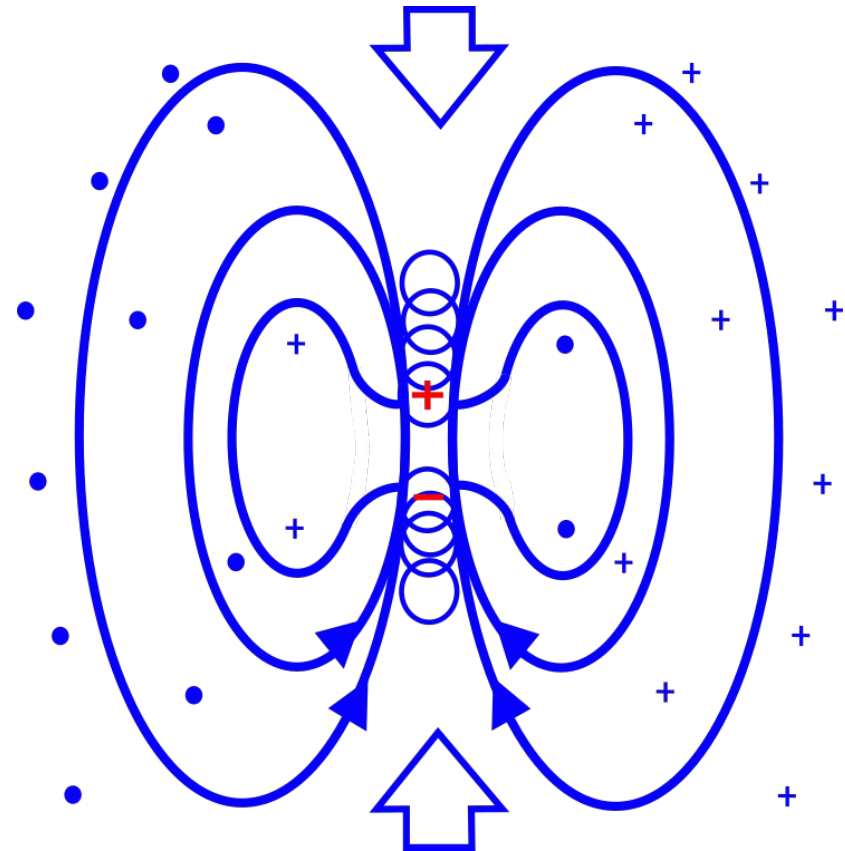
Accelerating charges emit radiations

- Oscillating electric dipole
- Atomic and Molecular Transitions
 - Vibrations and rotations
 - Core and valence electrons
 - Nuclear transitions

Oscillating Electric Dipole

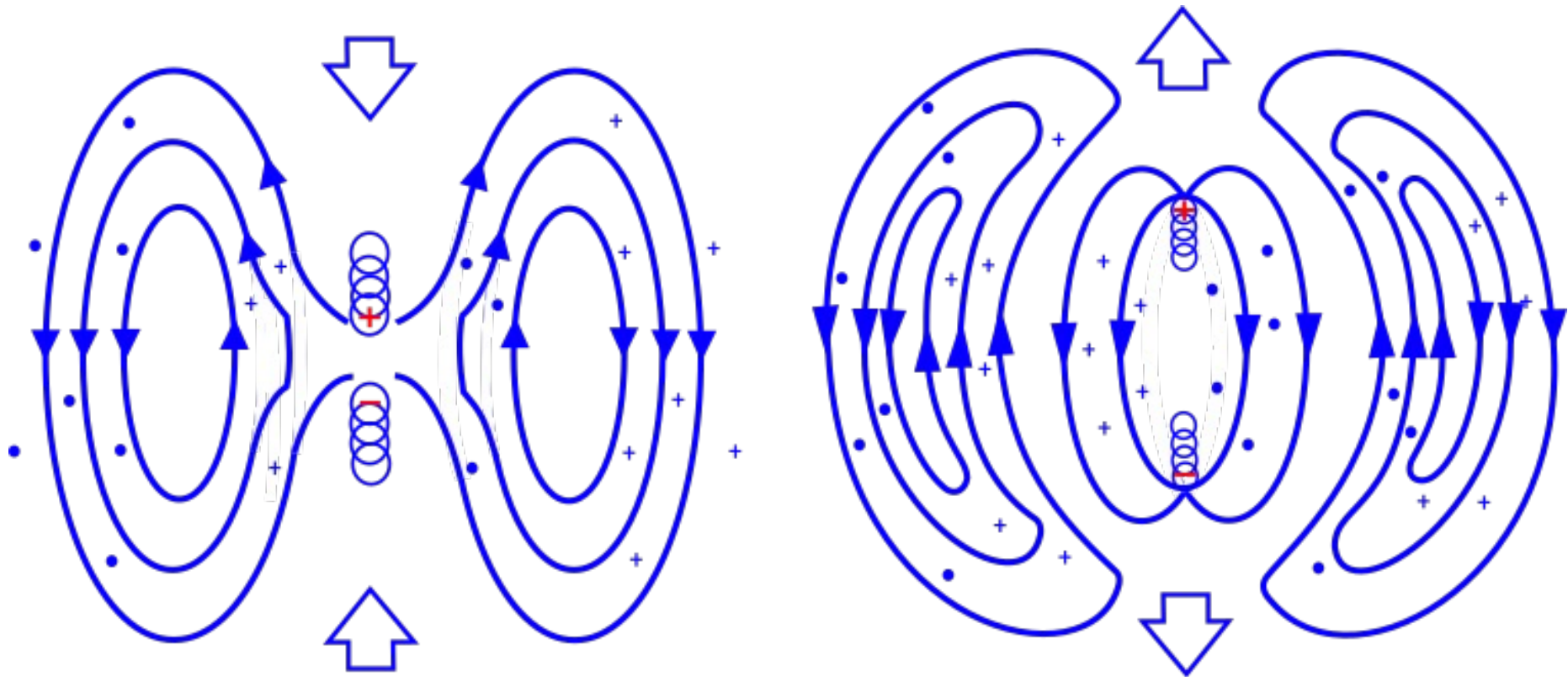


a)

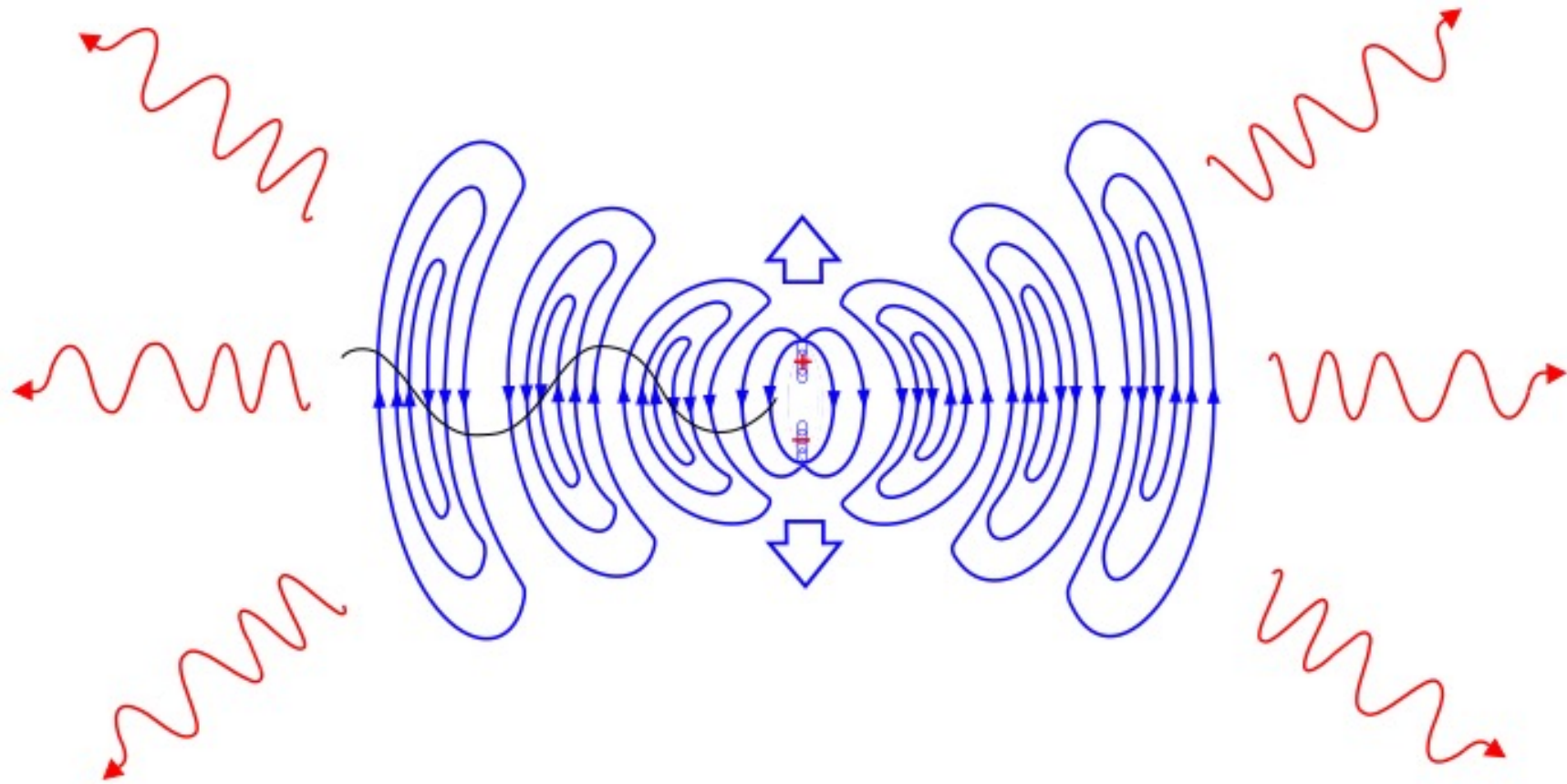


b)

Oscillating Electric Dipole



Oscillating Electric Dipole

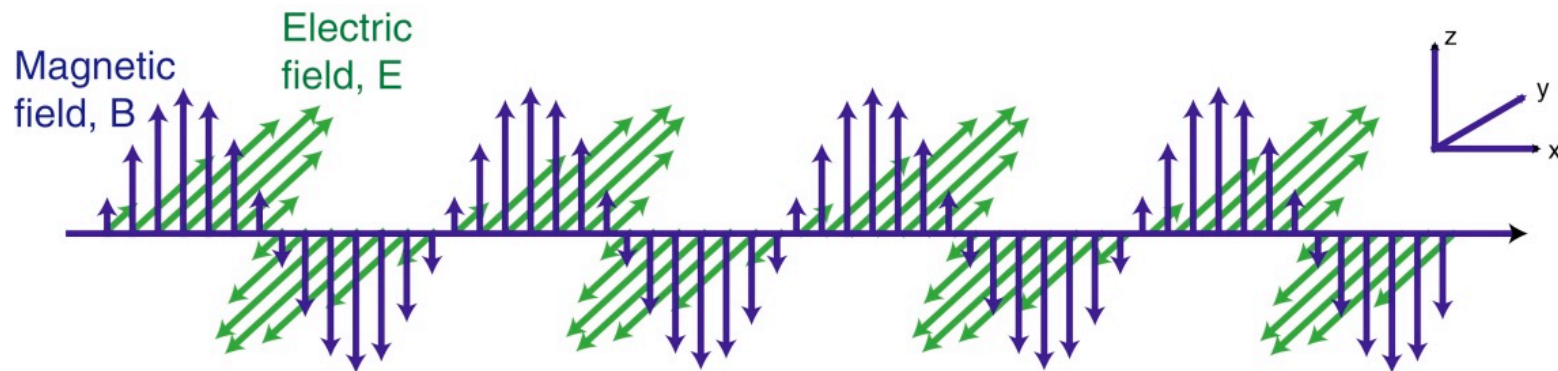


Maxwell's Equations **Quid polarization ?**

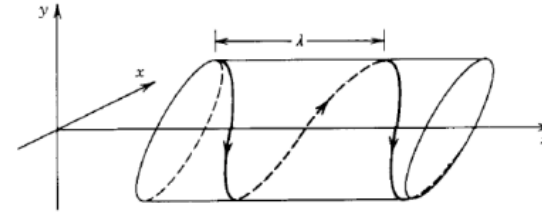
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In empty space, plane electromagnetic field patterns look like this transverse ($\vec{E} \times \vec{B} \propto \vec{k}$) wave, with \vec{E} and \vec{B} in phase ...



Polarization of Light



Consider a monochromatic plane wave of frequency ν travelling in the z direction with velocity c . The electric field lies in the x - y plane and is generally described by

$$\vec{E}(z,t) = E_x \hat{x} + E_y \hat{y}$$

To describe the polarization of this wave, we trace the endpoint of the vector $E(z,t)$ at each position z as a function of time.

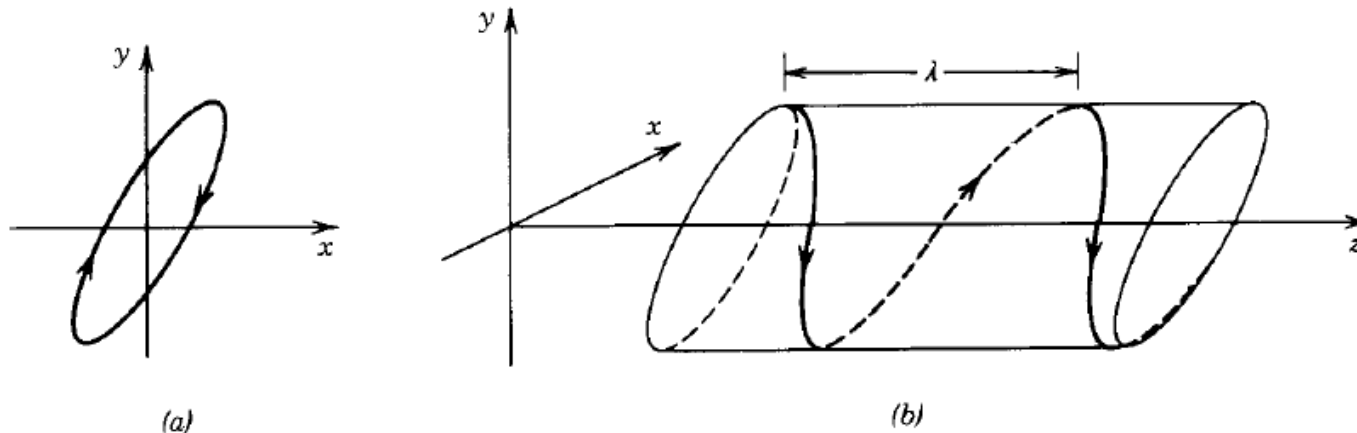
The components E_x and E_y are periodic functions of $(t-z/c)$ oscillating at frequency ν :

$$E_x = a_x \cos \left[2\pi\nu \left(t - \frac{z}{c} \right) + \varphi_x \right]$$

$$E_y = a_y \cos \left[2\pi\nu \left(t - \frac{z}{c} \right) + \varphi_y \right]$$

$$\varphi = \varphi_y - \varphi_x \quad \text{is the phase difference.}$$

Polarization of Light



(a) Rotation of the endpoint of the electric-field vector in the x - y plane at a fixed position z . (b) Snapshot of the trajectory of the endpoint of the electric-field vector at a fixed time t .

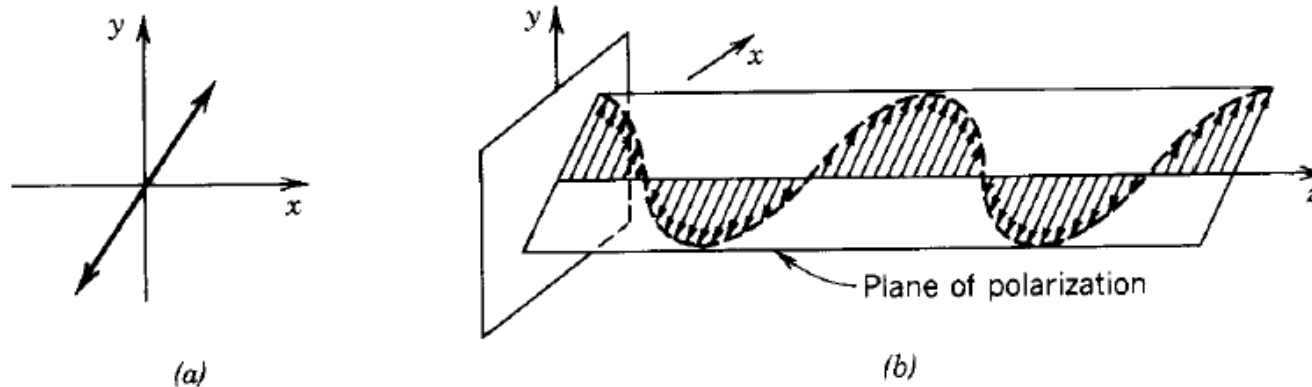
At a fixed value of z , the tip of the electric-field vector rotates periodically in the x - y plane, tracing out this ellipse.

At a fixed time t , the locus of the tip of the electric-field vector follows a helical trajectory in space lying on the surface of an elliptical cylinder.

Polarization of Light

Linearly Polarized Light

If one of the components vanishes ($a_x=0$, for example), the light is linearly polarized in the direction of the other component (the y direction). The wave is also linearly polarized if the phase difference $\varphi=0$ or π , since $E_y = \pm (a_y/a_x)E_x$

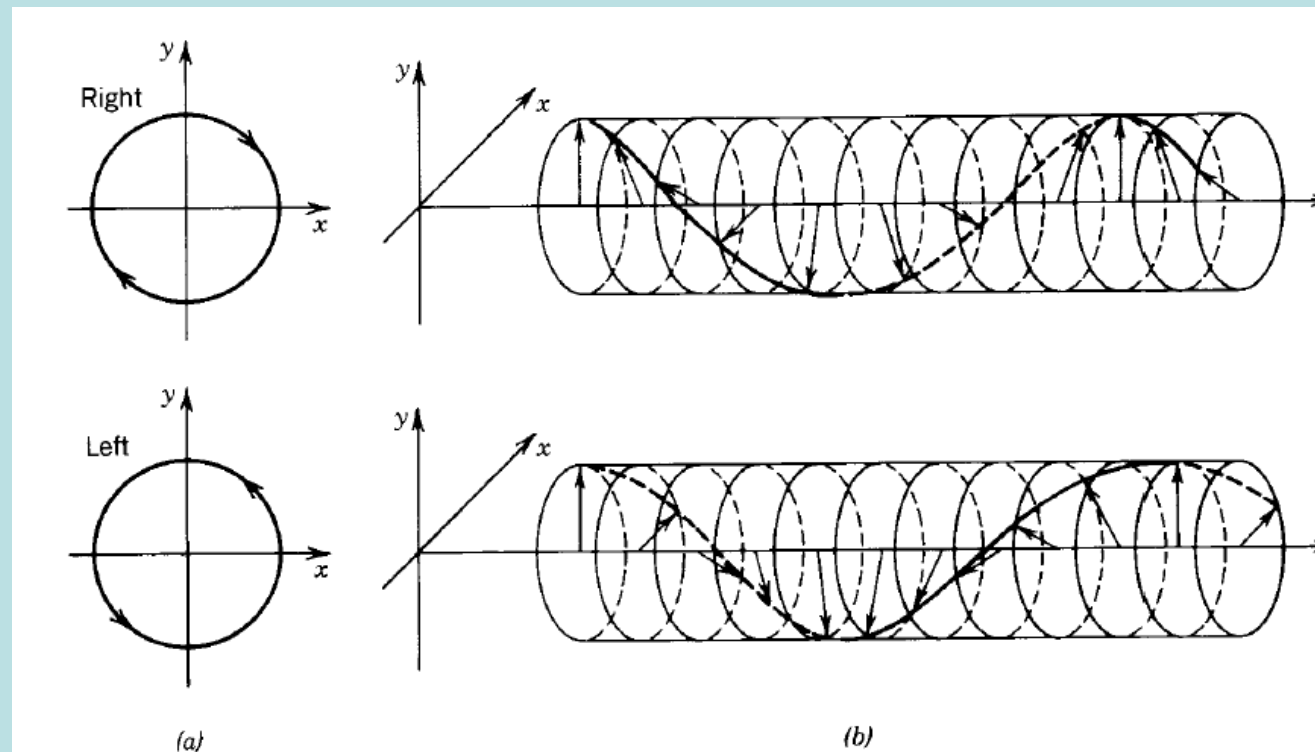


Linearly polarized light. (a) Time course at a fixed position z . (b) A snapshot (fixed time t).

Polarization of Light

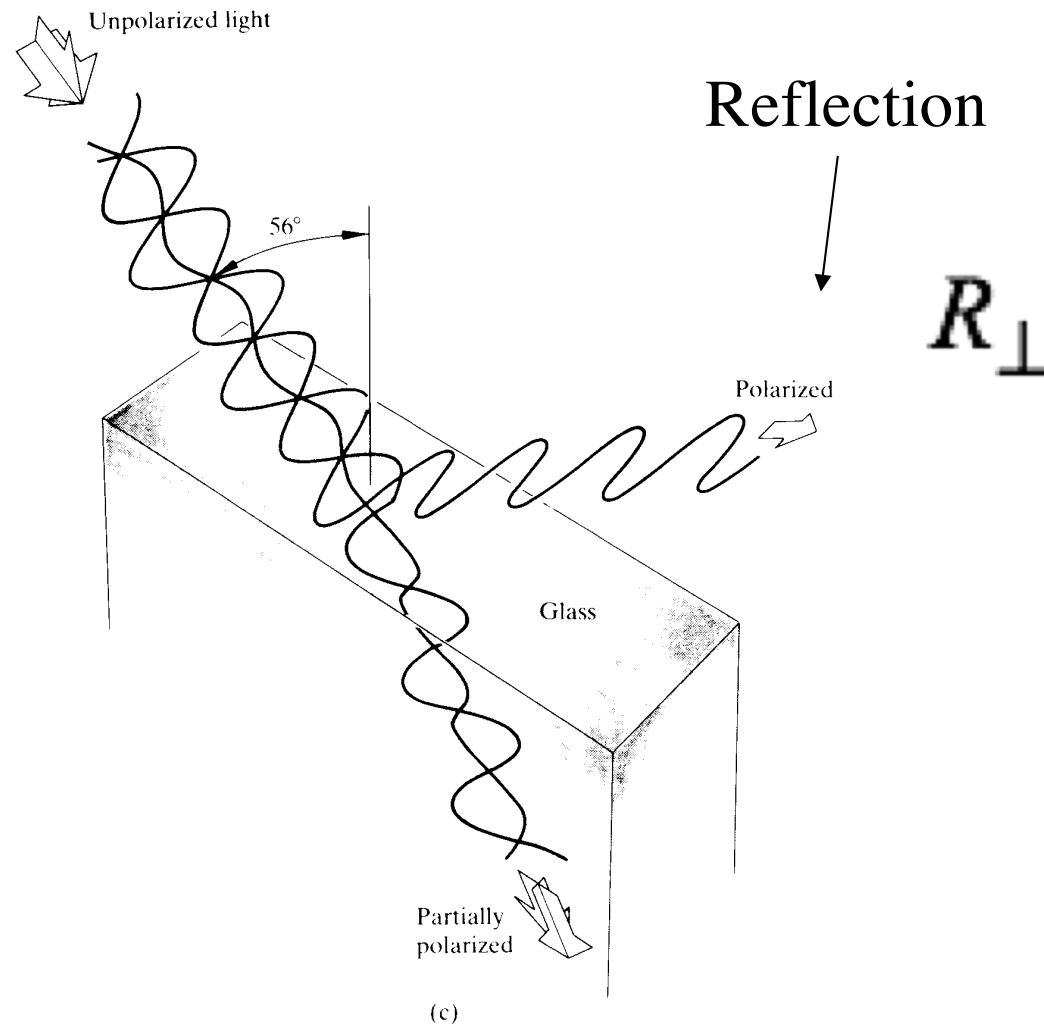
Circularly Polarized Light

If $\varphi = \pm \pi/2$ and $a_x = a_y = a_0$, $E_x^2 + E_y^2 = a_0^2$



Trajectories of the endpoint of the electric-field vector of a circularly polarized plane wave. (a) Time course at a fixed position z . (b) A snapshot (fixed time t). The sense of rotation in (a) is opposite that in (b) because the traveling wave depends on $t - z/c$.

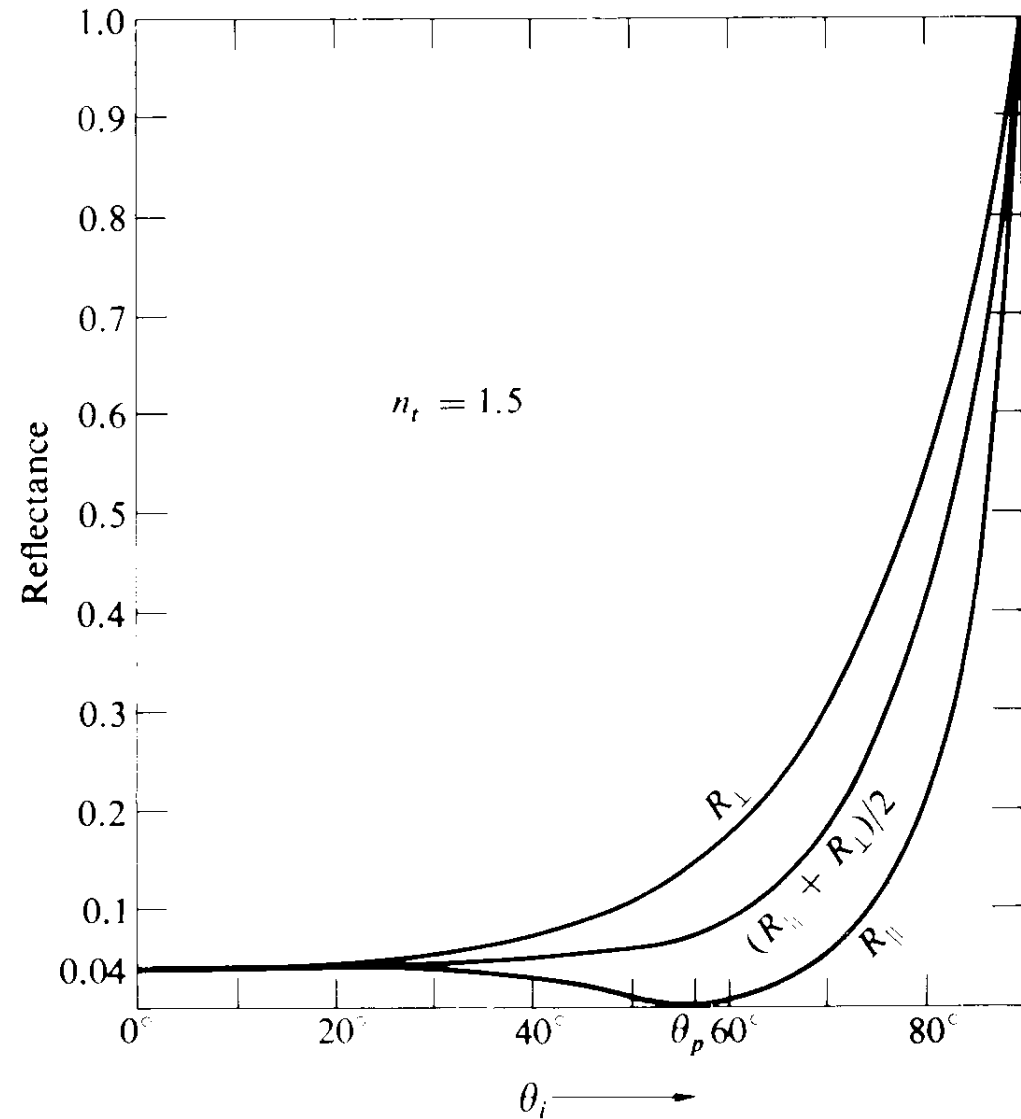
The Brewster angle



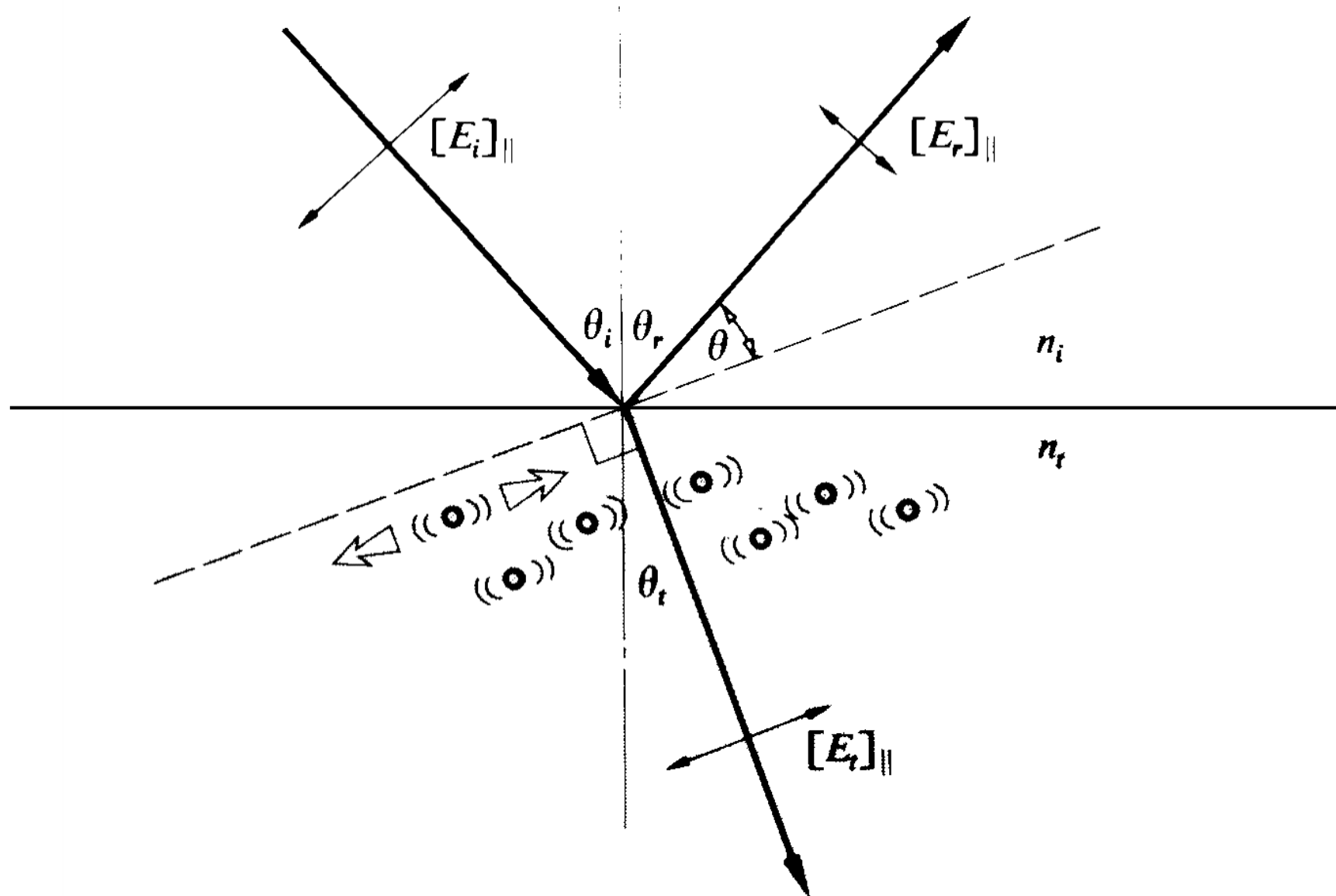
The Brewster angle

θ_p = the Brewster angle

$$\tan \theta_p = \frac{n_{\text{trans}}}{n_{\text{incid}}}$$



Reflectance versus incident angle.



(b)

E. Brewster windows in a laser cavity

Brewster windows are used in laser cavities to ensure that the laser light—after bouncing back and forth between the cavity mirrors—emerges as linearly polarized light. Figure 4-30 shows the general arrangement of the windows—thin slabs of glass with parallel sides—mounted on the opposite edges of the gas laser tube—in this case a helium-neon gas laser.

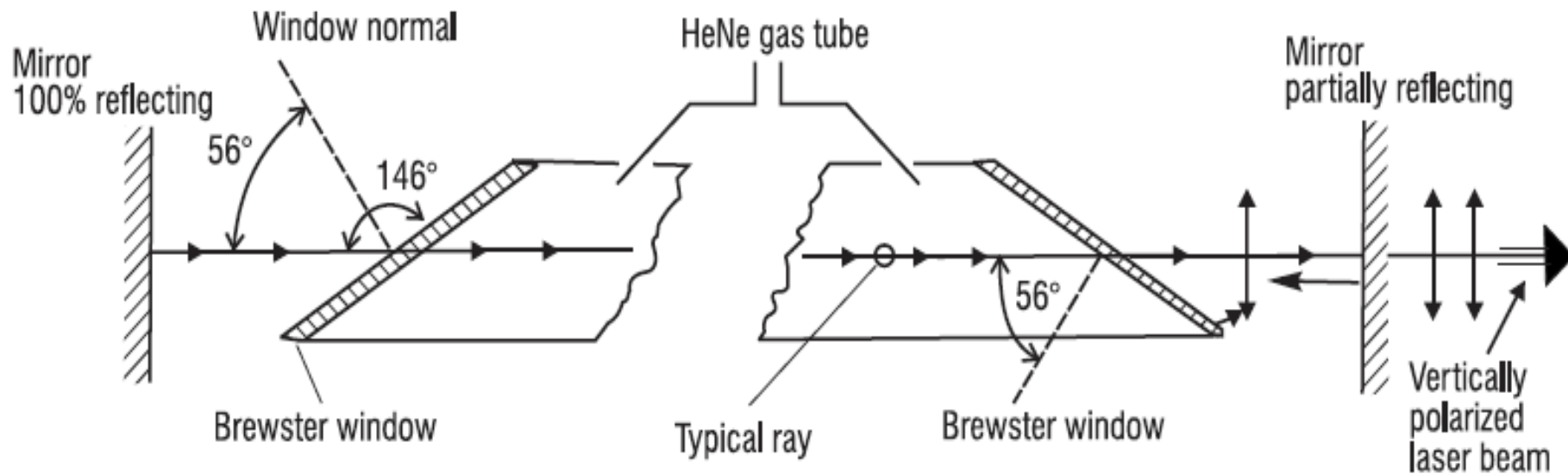


Figure 4-30 *Brewster windows in a HeNe gas laser*

Special case of Normal incidence $\theta_i = 0$

Normal incidence: $\theta_i = 0$

R = percentage of the light reflected at the interface

$$R = R_{\perp} = R_{\parallel}$$

$$R = \left(\frac{n_t - n_i}{n_t + n_i} \right)^2$$

Special case of Normal incidence $\theta_i = 0$

- For an **air-glass** interface ($n_i = 1$ and $n_t = 1.5$),

$$R = 4\% \text{ and } T = 96\%$$

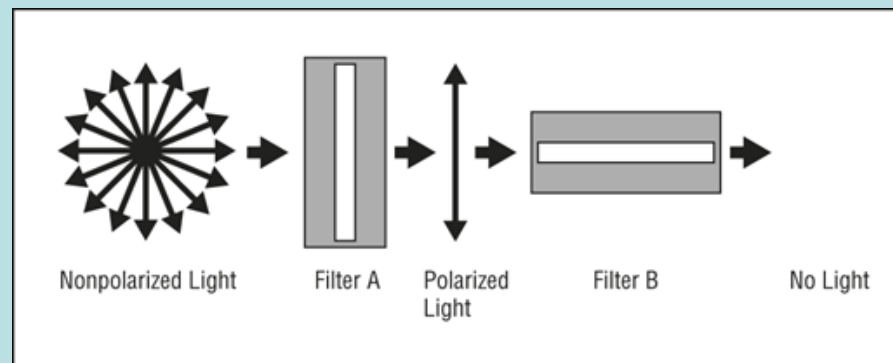
(The values are the same, whichever direction the light travels, from air to glass or from glass to air.)

- For an air-water (**air-tissue**) interface ($n_i = 1$ and $n_t = 1.33$),

$$R = 2\% \text{ and } T = 98\%$$

Cross-polarization

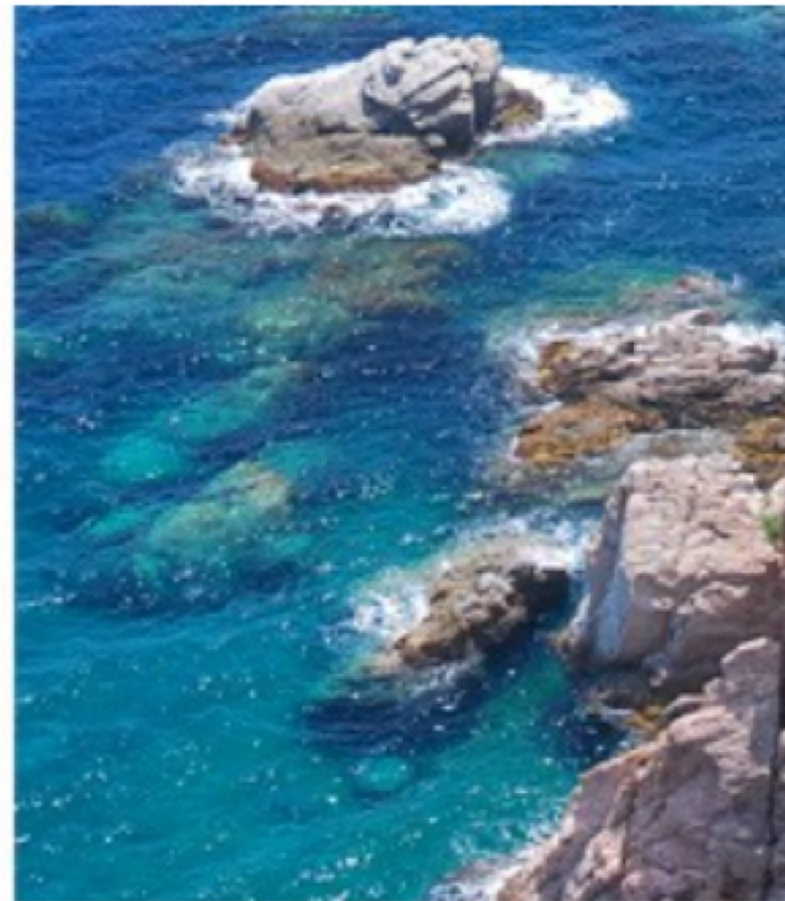
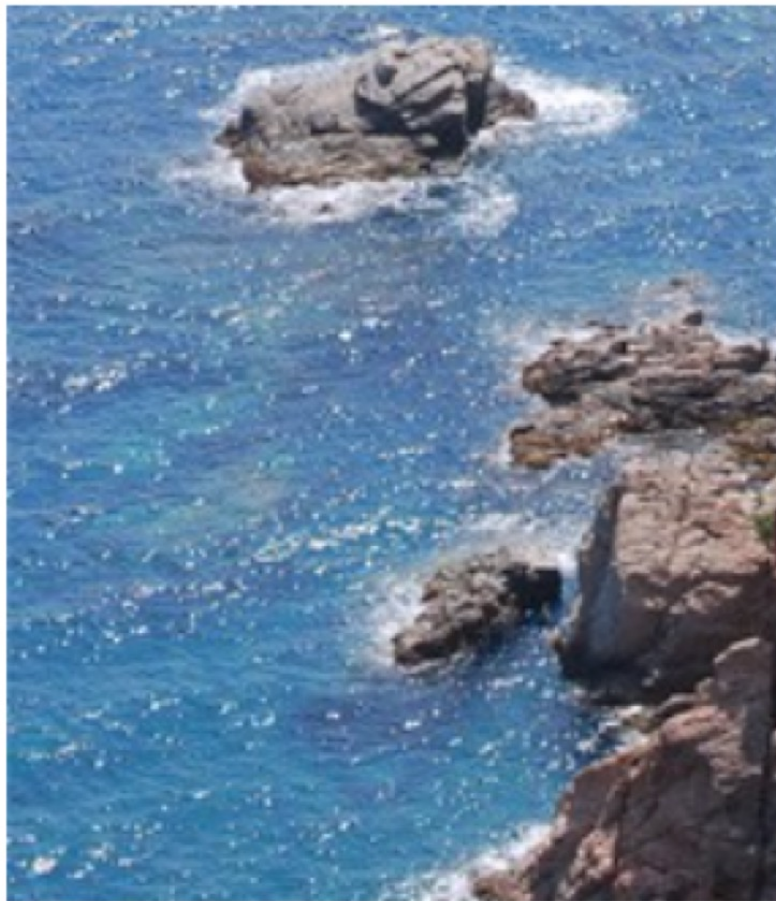
- Concept: Use of two perpendicular polarizer to filter out light whose polarization was not changed by the sample (specular reflections; single or few scatterings).
- Multiple scattering events randomize the polarization state of the photons:
 - Few scatterings (surface of sample) \rightarrow polarization is conserved
 - Multiple scatterings (deep in the sample) \rightarrow loss of polarization



Source: Benvenuto-Andrade C, Dusza SW, Agero AC, et al



Reduction of glares!

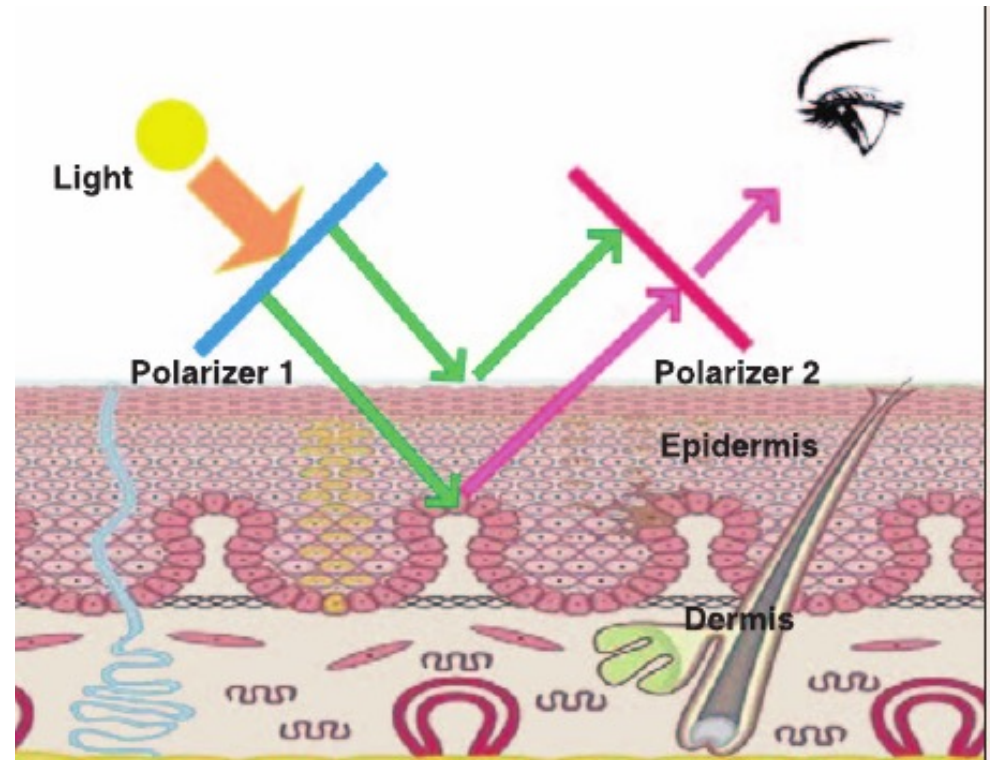


Cross-polarized dermoscope (1)

- Skin is important to shield deeper tissues from damage by the external environment. → Difficult to see deeper layers because of the shielding effect.
- Cross polarization can be used to obtain dermal images from deep layers of the skin. → Removes glistering appearance of the skin.
- Based on the principle that surface reflection maintains the same polarization as incident light whereas scattering from deep tissues induces a change in polarization.

Cross-polarized dermoscope (2)

- Polarizer 1 and 2 are placed perpendicularly.
- Light reflected directly at the surface is stopped by polarizer 2.
- Part of the scattered light from deep tissues can go through polarizer 2.



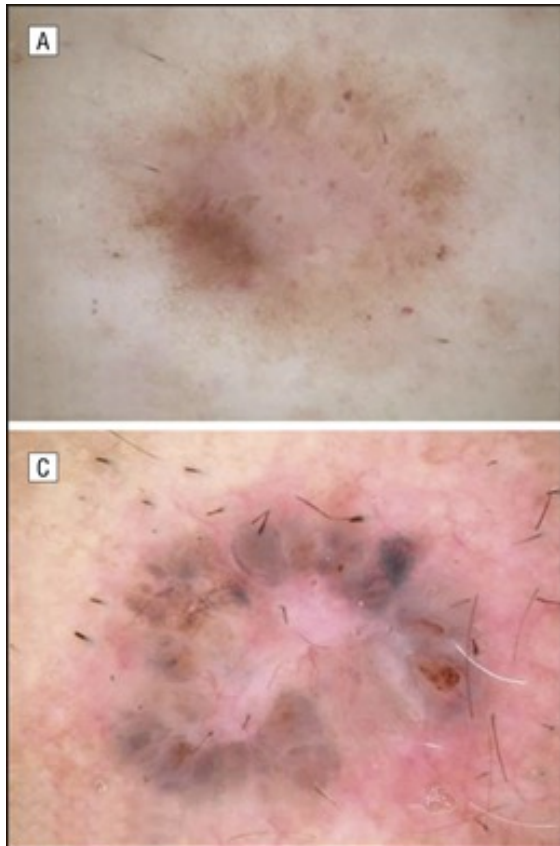
Source: Hening Wang et al., Systematic Design of a Cross-Polarized Dermoscope for Visual Inspection and Digital Imaging, 2012

Cross-polarized dermoscope (3)

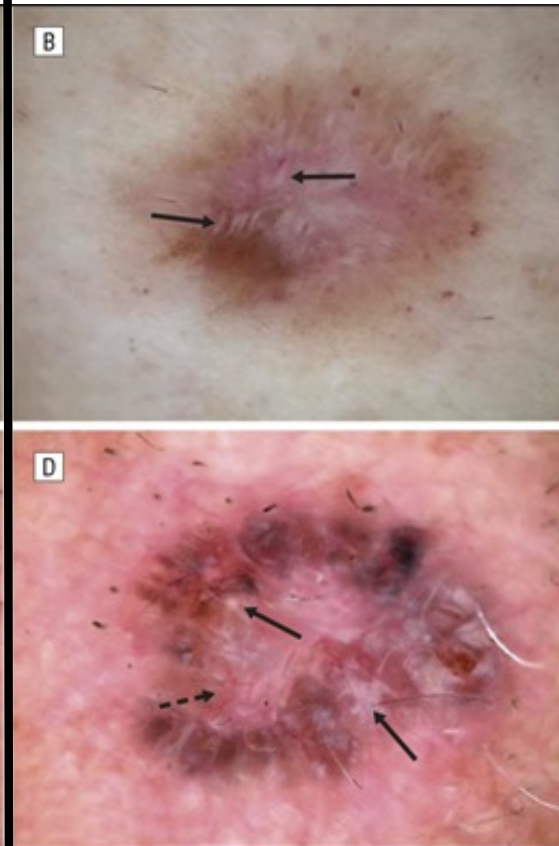
- Used mostly in dermatology to find boundaries of melanoma.
 - Better blocking of superficially reflected light than non-polarized dermoscopy → better contrast and deep structure visualization.
 - Better visualization of vascular patterns and pigmentation.
 - Limitations:
 - Color distortions compared to non polarized light imaging → this can influence clinical decision.
 - Surface lesions are less visible (ex: milia-like cysts).

Cross-polarized dermoscope (4)

Non-polarized imaging:



Cross-polarization imaging:



Appearance of dermatofibroma (A and B) and basal cell carcinoma (C and D) under nonpolarized (A and C) and polarized (B and D) light dermoscopy. Shiny-white streaks are seen in polarized images but not in nonpolarized images. These streaks (arrows in B and D) may be due to polarized light being reflected by collagen present in the fibrotic stroma. Also, the brown colors appear darker and the blood vessels are more prominent under polarized light (dashed arrow in D). The central portion of the dermatofibroma shows a pink veil (vascular blush) and the basal cell carcinoma also has a pink vascular blush that is not visible in the nonpolarized images.

Source: Benvenuto-Andrade C, Dusza SW, Agero AC, et al. Differences Between Polarized Light Dermoscopy and Immersion Contact Dermoscopy for the Evaluation of Skin Lesions. *Arch Dermatol*. 2007