Photomedicine

Tentative Syllabus

1. **Introduction**
2. **History**
3. **Radiometry / photometry**
4. **Optics review**
    - Ray optics
    - Electromagnetic / wave optics
    - Quantum description of light

Review of selected concepts in optics

4.1 Ray Optics

Light travels in different optical media in accordance with a set of geometrical rules.

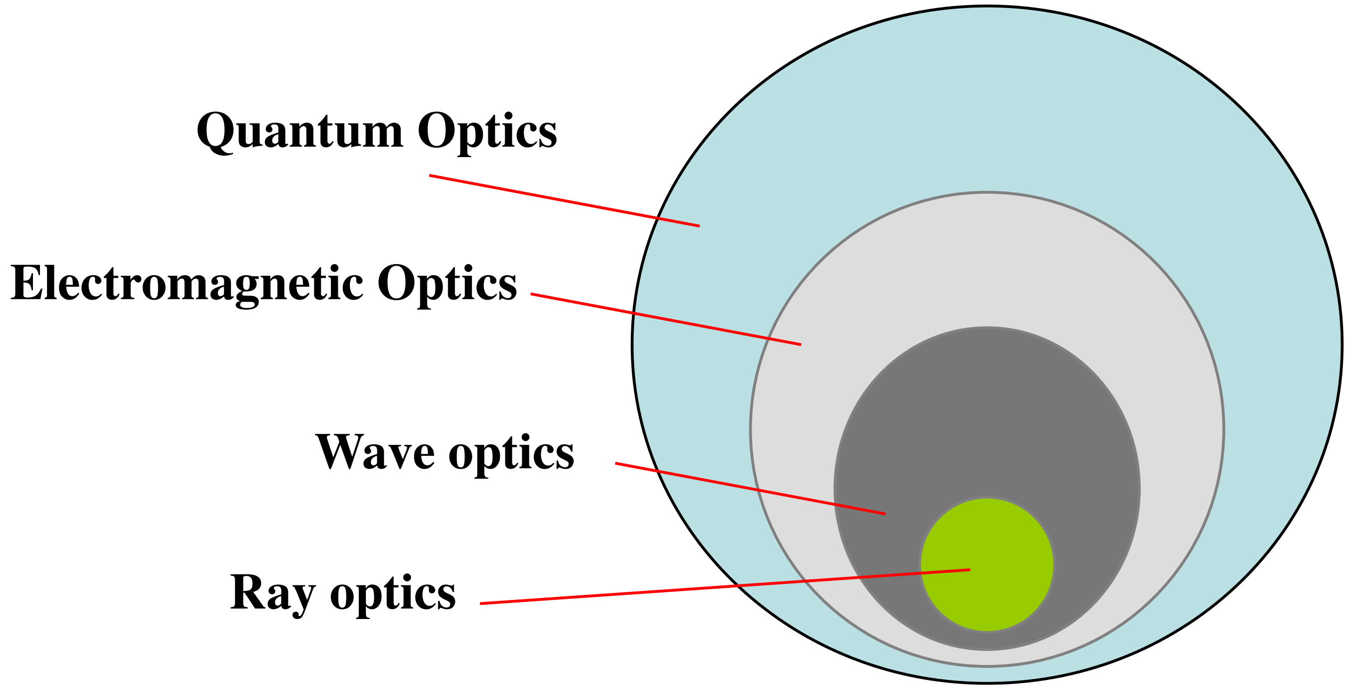
4.2 Classical (Wave) Description

Light is an EM wave

4.3 Quantum (Particle) Description

Localized, massless quanta of energy – photons

Descriptions of Light:

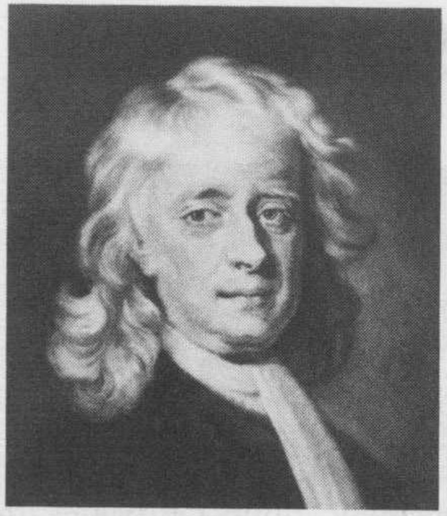


All descriptions are considered in photomedicine !

Quantum Optics	⇒ **Explanation of virtually all optical phenomena.**
Electromagnetic Optics	⇒ **Most complete treatment of light within the confines of classic optics (Maxwell's equations).**
Wave optics (Fourier)	⇒ **Scalar approximation of EM optics. This scalar wavefunction represents any component of the electric or magnetic fields (no physical meaning).**
Ray optics	⇒ **Limit of wave optics when wavelength is very short.**

4.1 Ray Optics

Pioneers in Ray Optics



Sir Isaac Newton (1642–1727) set forth a theory of optics in which light emissions consist of collections of corpuscles that propagate rectilinearly.

Pierre de Fermat (1601–1665) developed the principle that light travels along the path of least time.

Postulates of Ray Optics

Basic concepts

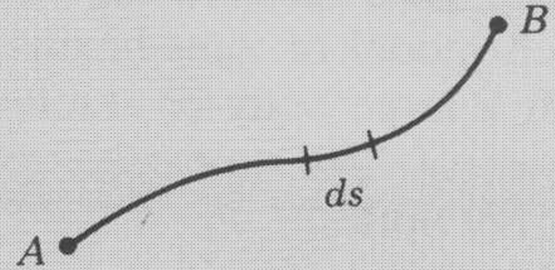
- Light travels in the form of rays. The rays are emitted by light sources and can be observed when they reach an optical detector.
- An optical medium is characterized by a quantity $n \geq 1$, called the **refractive index**. The refractive index is the ratio of the speed of light in free space c_o to that in the medium c. Therefore, the time taken by light to travel a distance d equals $d/c = nd/c_o$. It is thus proportional to the product nd, known as the **optical path length**.

Postulates of Ray Optics

Basic concepts

- In an inhomogeneous medium the refractive index $n(\mathbf{r})$ is a function of the position $\mathbf{r} = (x, y, z)$. The optical path length along a given path between two points A and B is therefore

$$\text{Optical path length} = \int_A^B n(\mathbf{r})\, ds,$$



where ds is the differential element of length along the path. The time taken by light to travel from A to B is proportional to the optical path length.

Postulates of Ray Optics

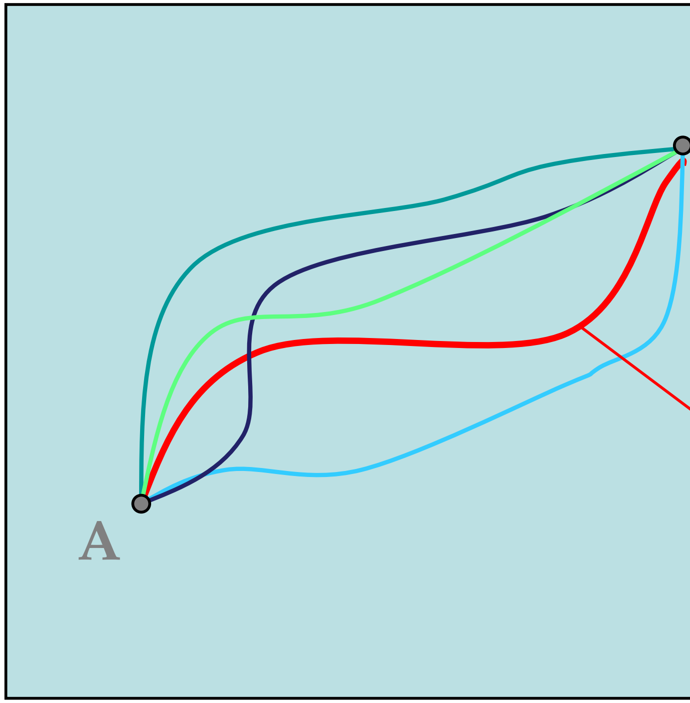
- **Fermat's Principle**. Optical rays traveling between two points, A and B, follow a path such that the time of travel (or the optical path length) between the two points is an extremum relative to neighboring paths. An extremum means that the rate of change is zero, i.e.,

$$\delta \int_A^B n(\mathbf{r})\, ds = 0.$$

The extremum may be a minimum, a maximum, or a point of inflection. It is, however, usually a minimum, in which case

light rays travel along the path of least time.

Postulates of Ray Optics



$$\delta \int_A^B n(\mathbf{r})\, ds = 0.$$

Postulates of Ray Optics

All rules governing the propagation of light rays in homogenous or inhomogenous optical media can be determined by the Postulates of Ray Optics.

Hero's Principle

Propagation in a Homogeneous Medium

In a homogeneous medium the refractive index is the same everywhere, and so is the speed of light. The path of minimum time, required by Fermat's principle, is therefore also the path of minimum distance. The principle of the *path of minimum distance* is known as **Hero's principle**. The path of minimum distance between two points is a straight line so that *in a homogeneous medium, light rays travel in straight lines* (Fig. 1.1-1).

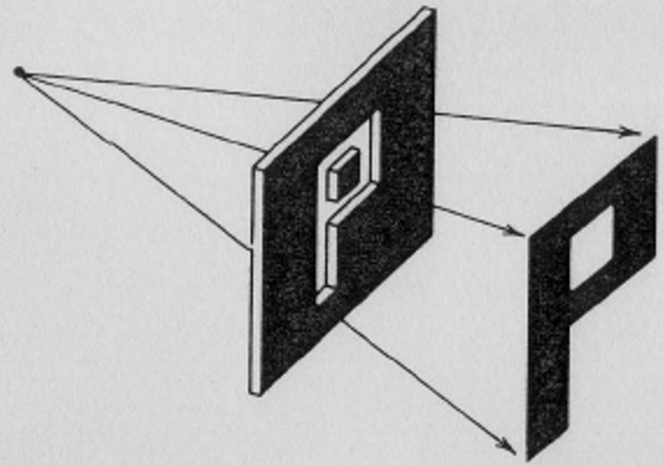


Figure 1.1-1 Light rays travel in straight lines. Shadows are perfect projections of stops.

The Law of Reflection

The reflected ray lies in the plane of incidence; the angle of the reflection equals the angle of incidence.

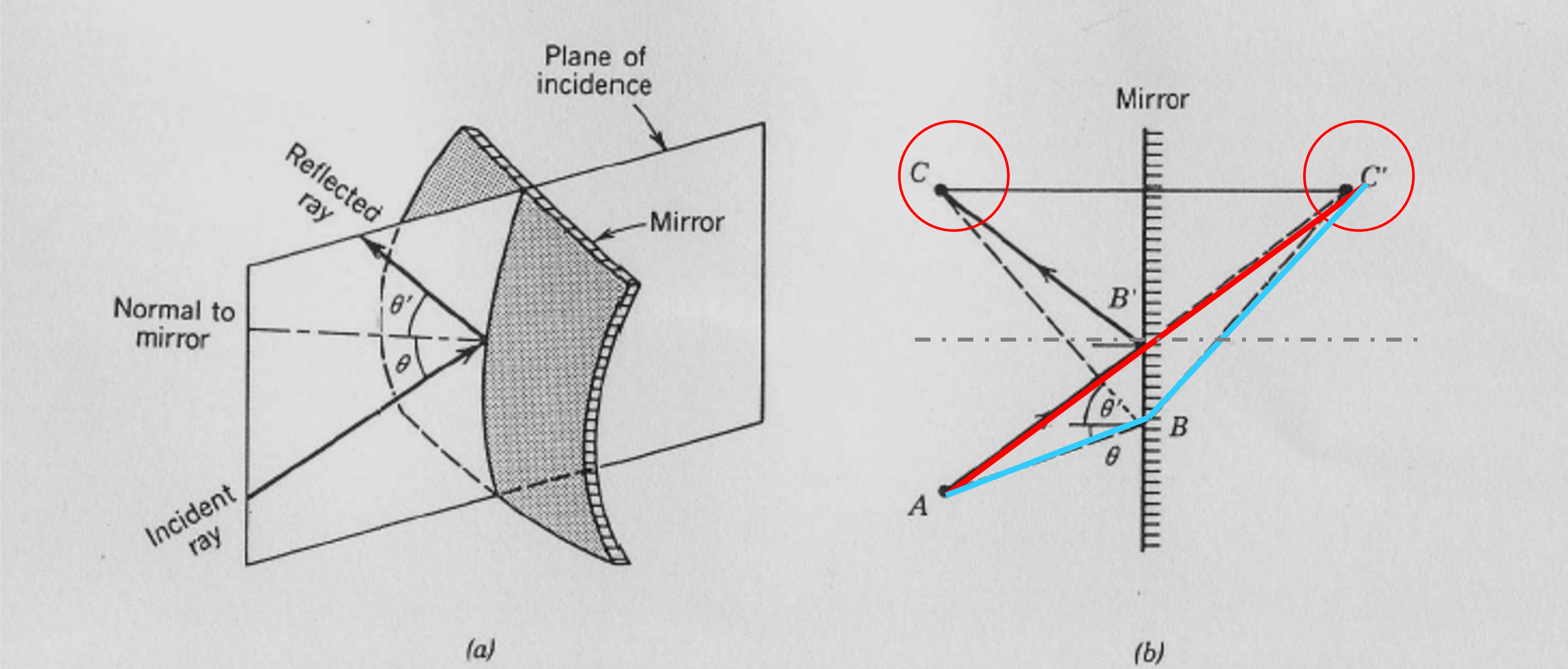


Figure 1.1-2 (*a*) Reflection from the surface of a curved mirror. (*b*) Geometrical construction to prove the law of reflection.

Simple Optical Components

Imaging Equation (Paraxial rays)

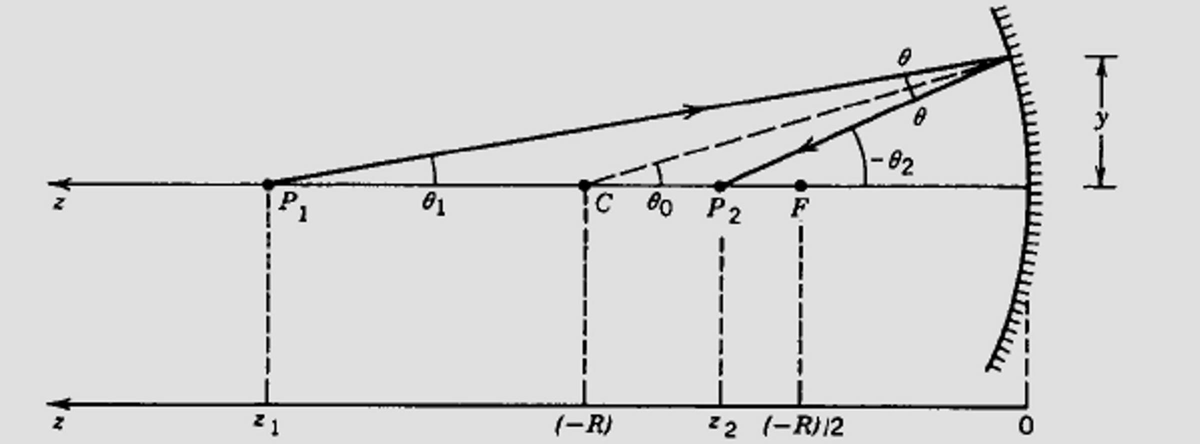


Figure 1.2-6 Reflection of paraxial rays from a concave spherical mirror of radius $R < 0$.

If θ_1 and θ_2 are small

$$(-\theta_2) + \theta_1 \approx \frac{2y}{-R} \rightarrow \frac{1}{z_1} + \frac{1}{z_2} \approx \frac{2}{-R} \rightarrow \boxed{\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}} \qquad f = \frac{-R}{2}$$

Simple Optical Components

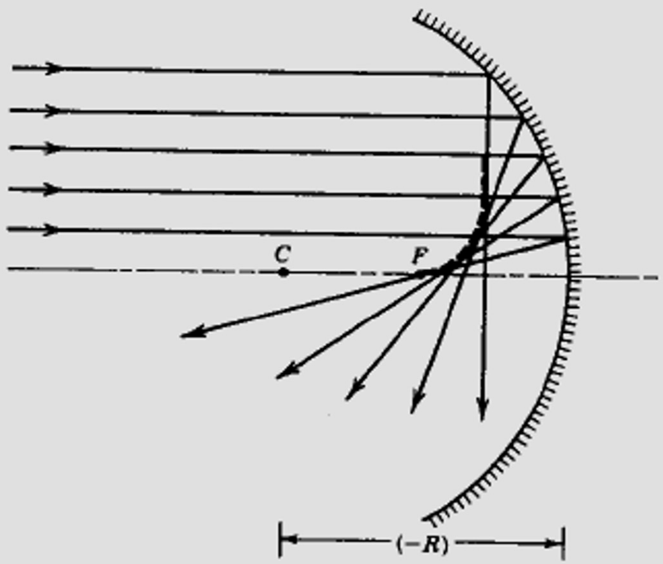


Figure 1.2-4 Reflection of parallel rays from a concave spherical mirror.

Simple Optical Components

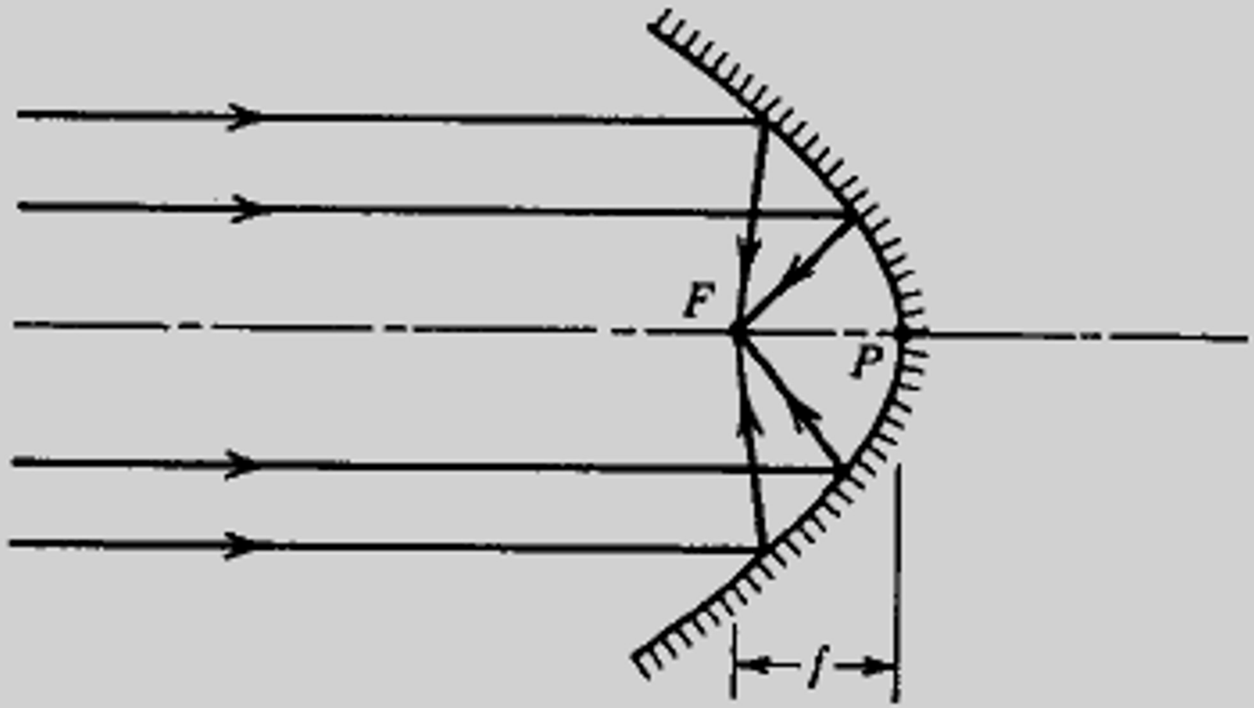


Figure 1.2-2 Focusing of light by a paraboloidal mirror.

The Law of Refraction (Snell's law)

The refracted ray lies in the plane of incidence; the angle of refraction θ_2 is related to the angle of incidence θ_1 by

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Reflection and Refraction at the Boundary Between Two Media

At the boundary between two media of refractive indices n_1 and n_2 an incident ray is split into two—a reflected ray and a refracted (or transmitted) ray (Fig. 1.1-3). The

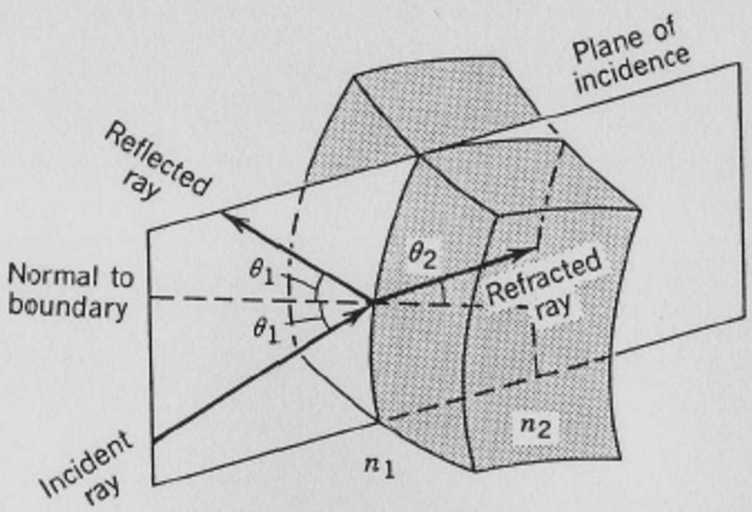


Figure 1.1-3 Reflection and refraction at the boundary between two media.

Proof of Snell's law

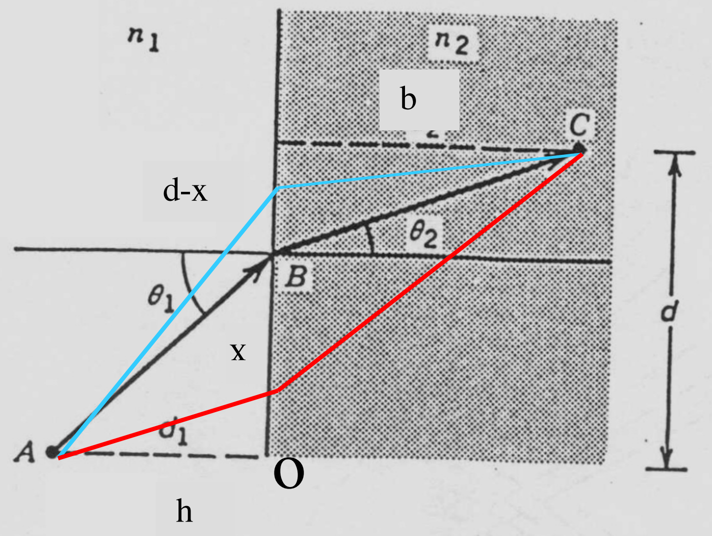


Figure 1.1-4 Construction to prove Snell's law.

$$t = \frac{\overline{AB}}{v_1} + \frac{\overline{BC}}{v_2} = \frac{1}{v_1}\sqrt{h^2 + x^2} + \frac{1}{v_2}\sqrt{b^2 + (d-x)^2}$$

Fermat's Principle ⇒ Minimal Time

Proof of Snell's law

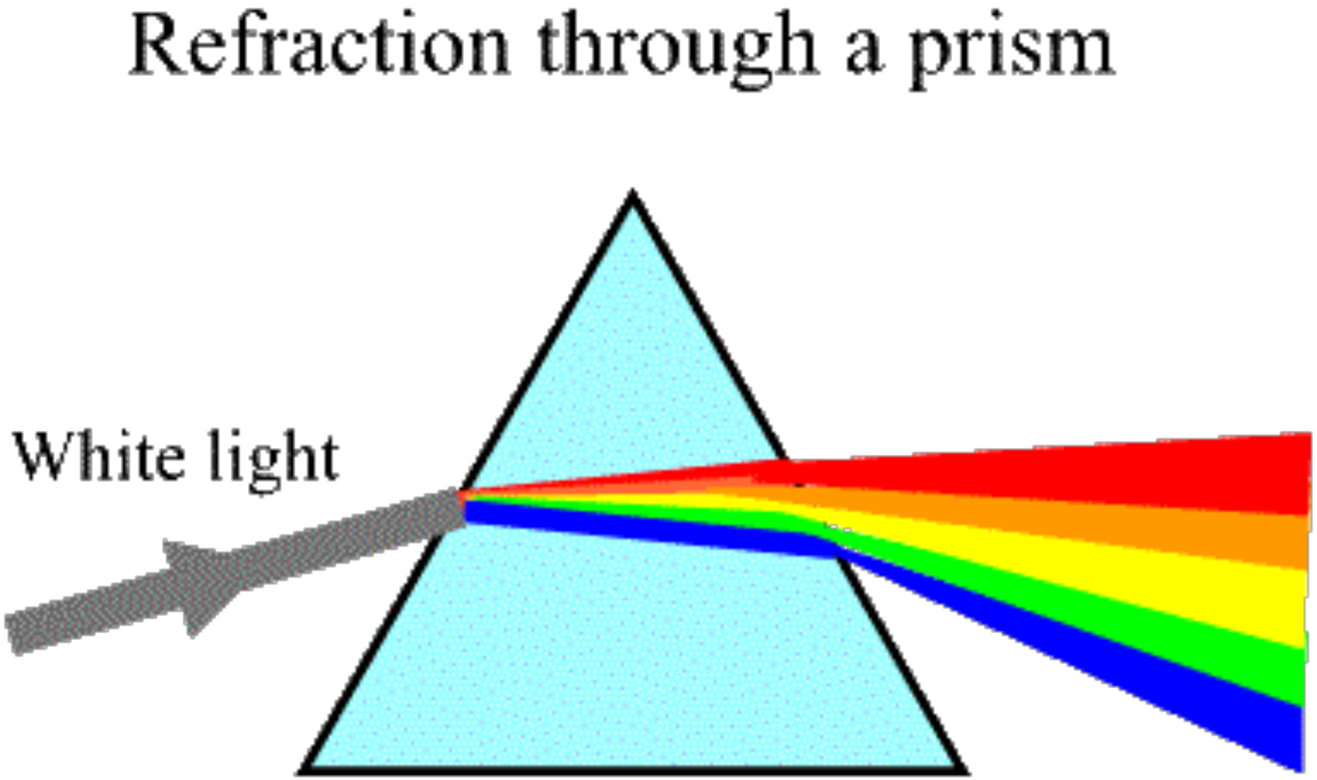
$$\frac{\partial t}{\partial x} = 0$$

$$\frac{\partial t}{\partial x} = \frac{1}{v_1}\underbrace{\frac{1}{\sqrt{h^2 + x^2}}x}_{\sin\theta_1} + \frac{1}{v_2}\underbrace{\frac{-(d-x)}{\sqrt{b^2+(d-x)^2}}}_{-\sin\theta_2}$$

$$= \frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \quad \text{with} \quad n = \frac{c}{v}$$

But “n” is not the same for all wavengths ! Why?



Paraxial Optics/First–order Optics/Gaussian Optics

Thin Lens (aberration-free)

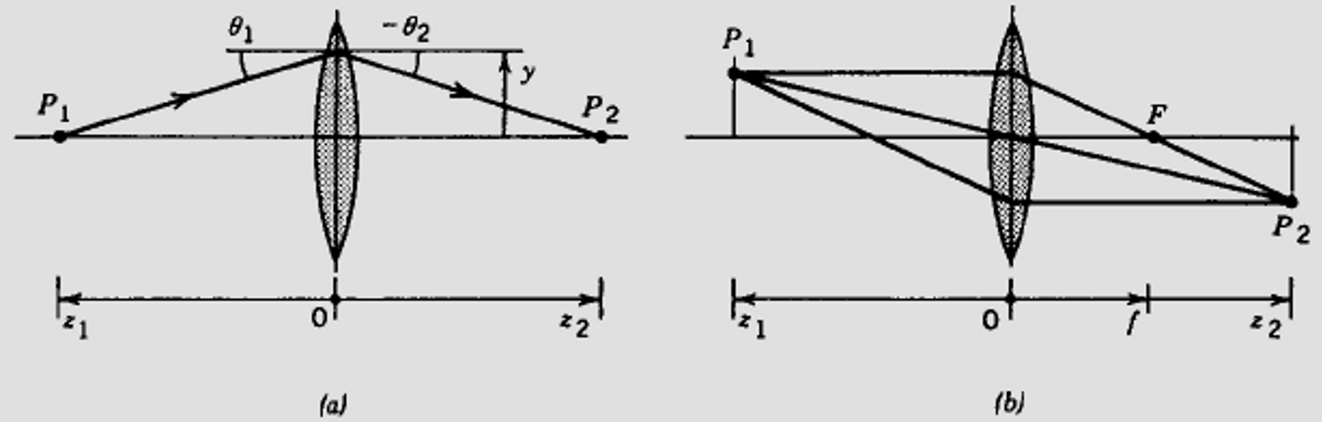


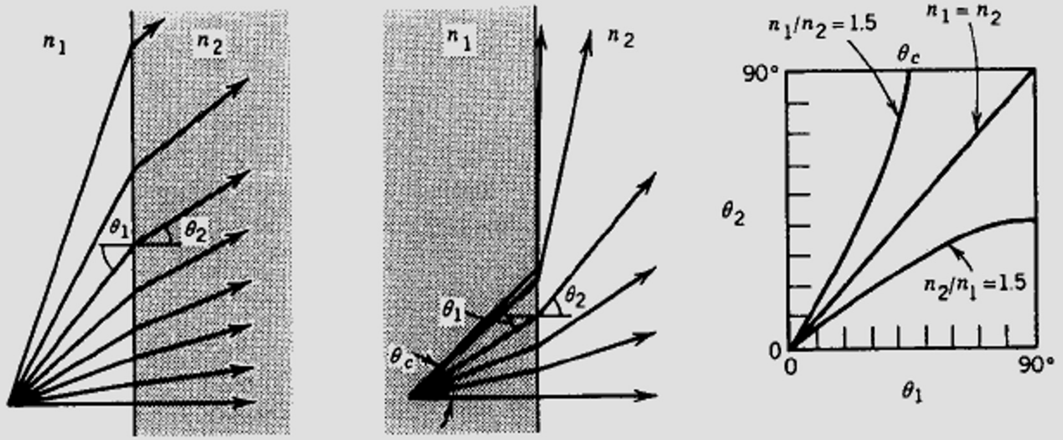
Figure 1.2-14 (*a*) Ray bending by a thin lens. (*b*) Image formation by a thin lens.

Imaging Equation:

$$\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

Total internal reflection



External refraction Internal refraction

Figure 1.2-8 Relation between the angles of refraction and incidence.

Total Internal Reflection

For internal refraction ($n_1 > n_2$), the angle of refraction is greater than the angle of incidence, $\theta_2 > \theta_1$, so that as θ_1 increases, θ_2 reaches 90° first (see Fig. 1.2-8). This occurs when $\theta_1 = \theta_c$ (the **critical angle**), with $n_1 \sin\theta_c = n_2$, so that

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}. \quad (1.2\text{-}5)$$

Critical Angle

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Total internal reflection

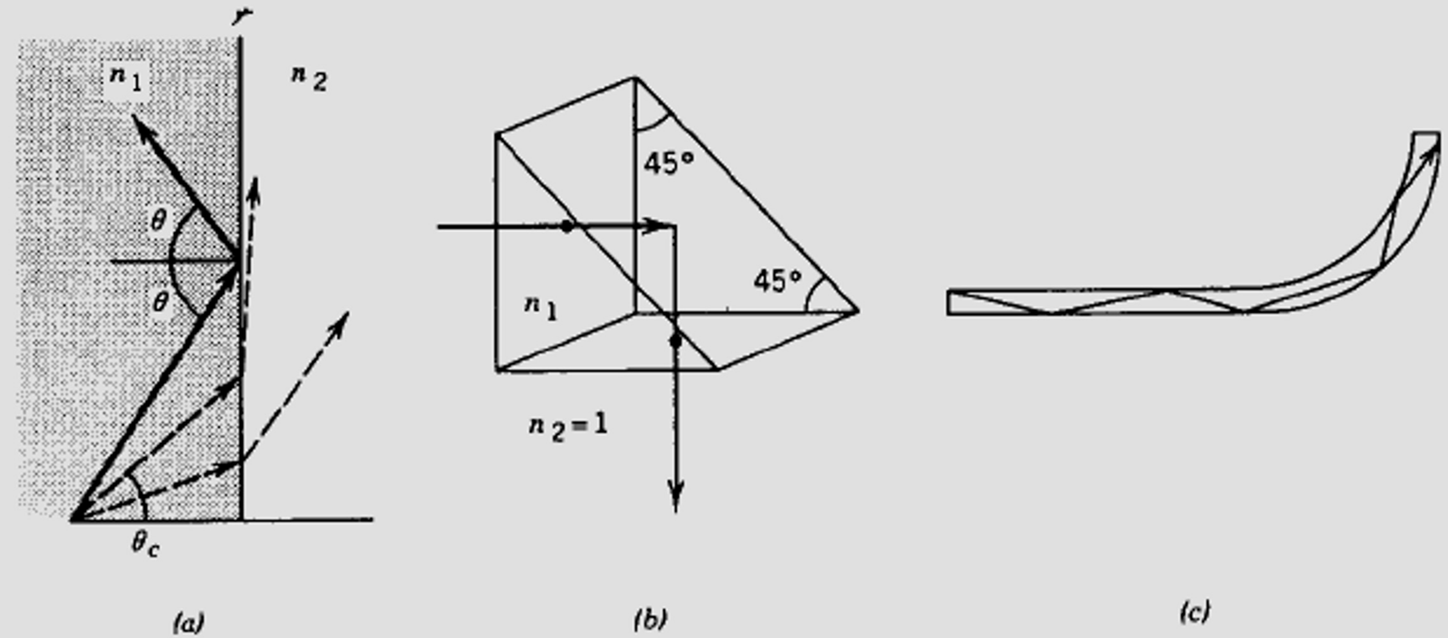


Figure 1.2-9 (*a*) Total internal reflection at a planar boundary. (*b*) The reflecting prism. If $n_1 > \sqrt{2}$ and $n_2 = 1$ (air), then $\theta_c < 45°$; since $\theta_1 = 45°$, the ray is totally reflected. (*c*) Rays are guided by total internal reflection from the internal surface of an optical fiber.

Optical Waveguide

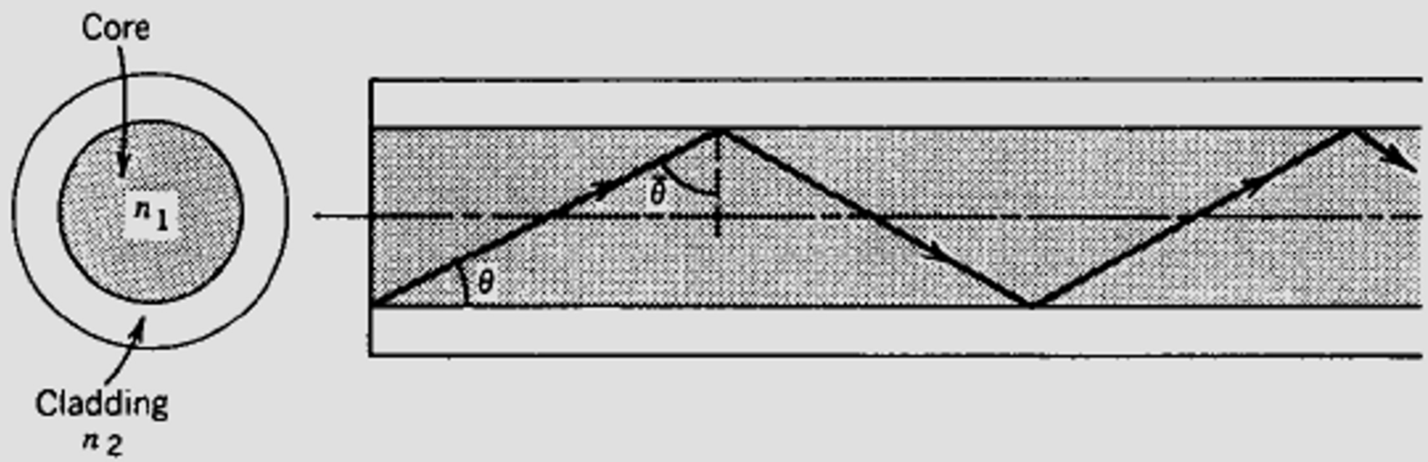


Figure 1.2-17 The optical fiber. Light rays are guided by multiple total internal reflections.

Optical Waveguide

Numerical Aperture and Angle of Acceptance of an Optical Fiber. An optical fiber is illuminated by light from a source (e.g., a light-emitting diode, LED). The refractive indices of the core and cladding of the fiber are n_1 and n_2, respectively, and the refractive index of air is 1 (Fig. 1.2-18).

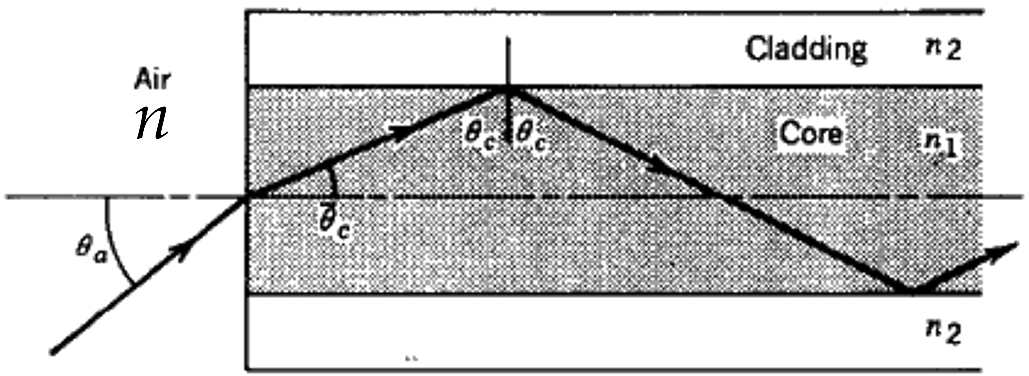
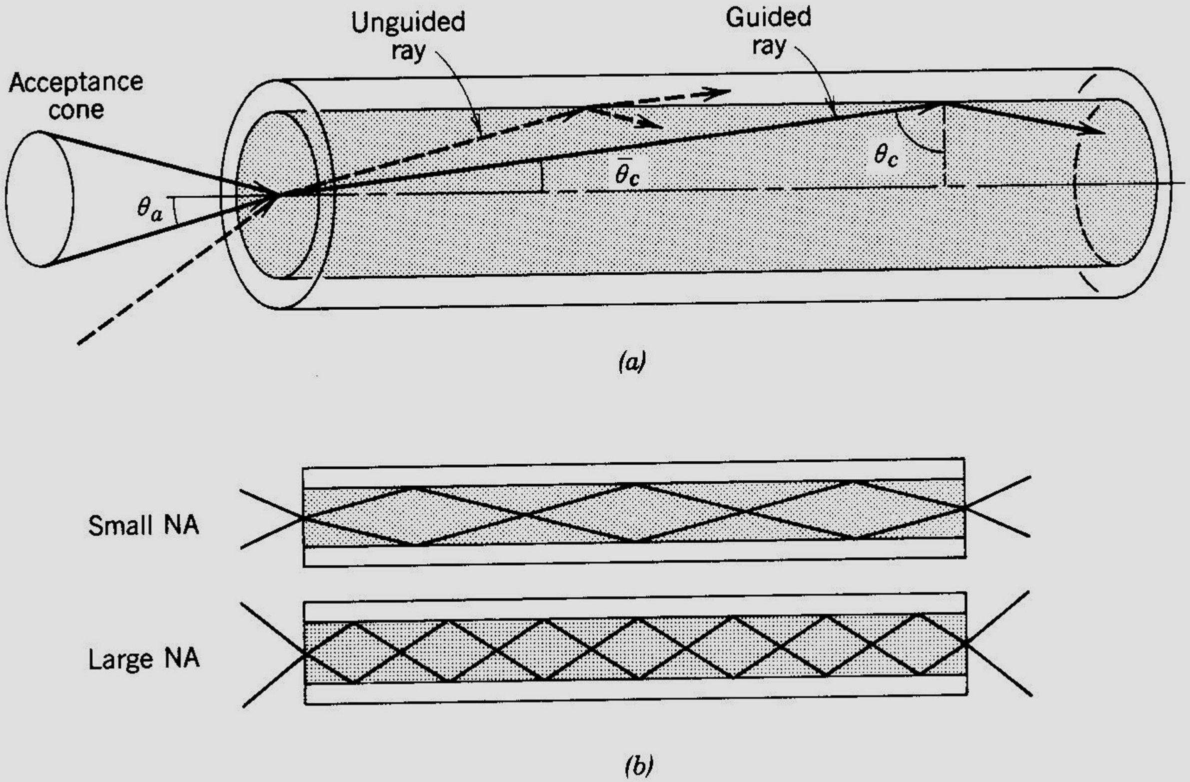


Figure 1.2-18 Acceptance angle of an optical fiber.

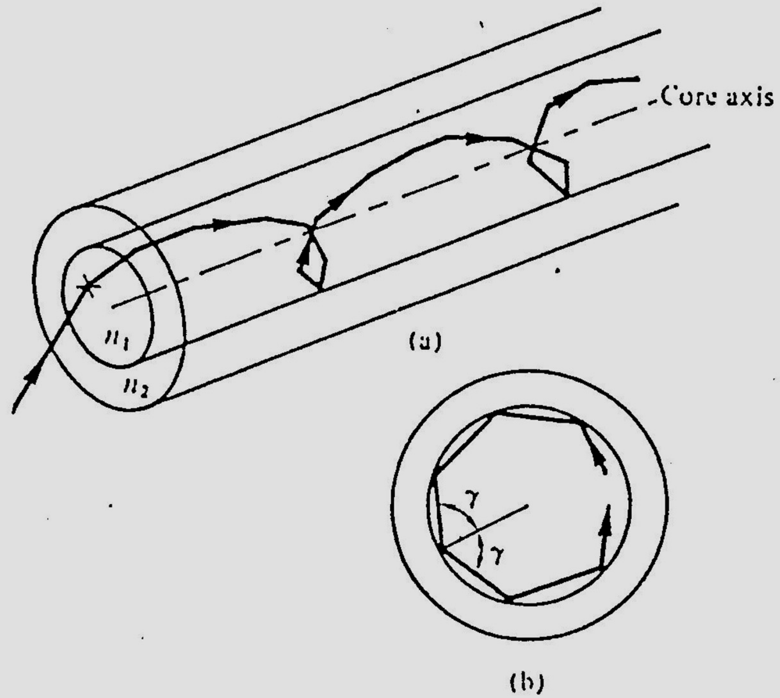
$$\mathrm{NA} = n \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$$

Numerical Apperture of Optical Waveguide
(step-index)



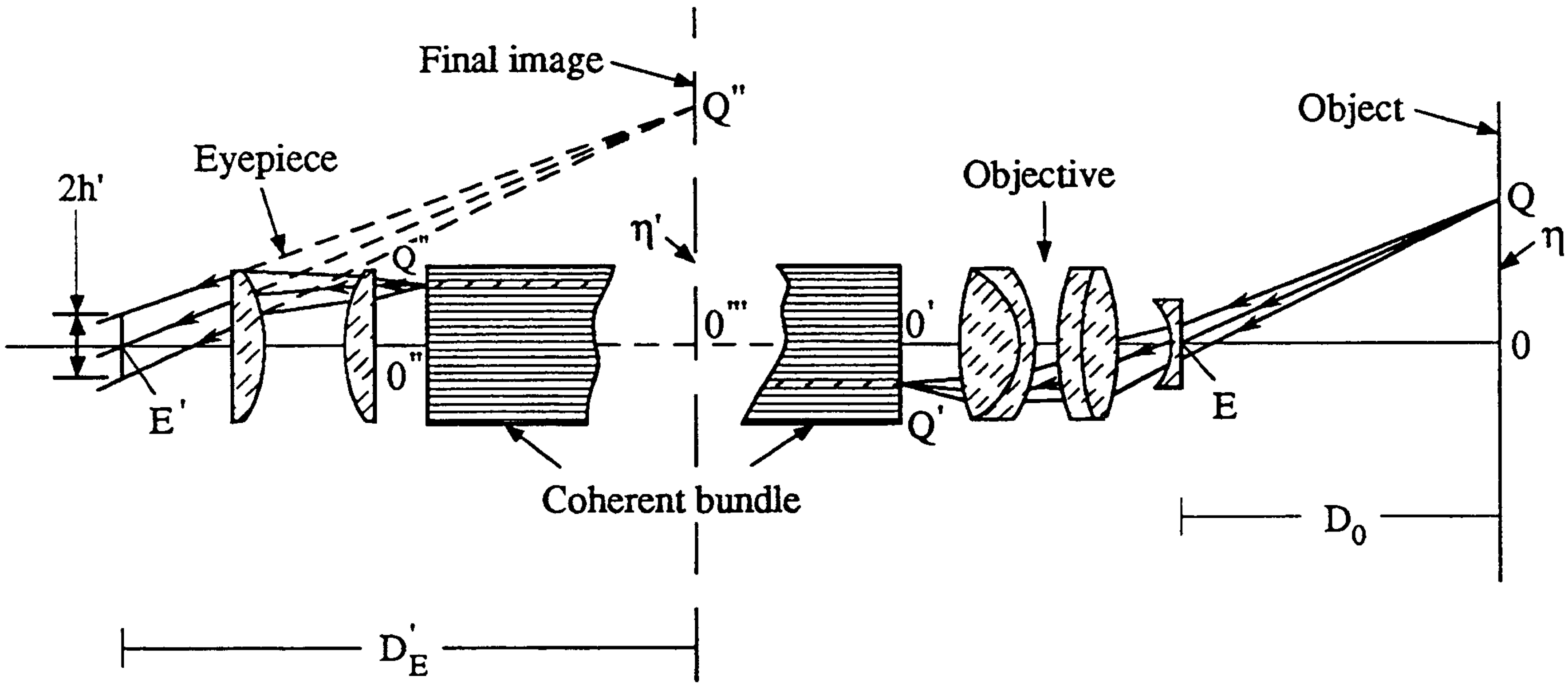
(*a*) The acceptance angle θ_a of a fiber. Rays within the acceptance cone are guided by total internal reflection. The numerical aperture NA = sin θ_a. (*b*) The light-gathering capacity of a large NA fiber is greater than that of a small NA fiber. The angles θ_a and $\bar{\theta}_c$ are typically quite small; they are exaggerated here for clarity.

Optical Waveguide (step-index)



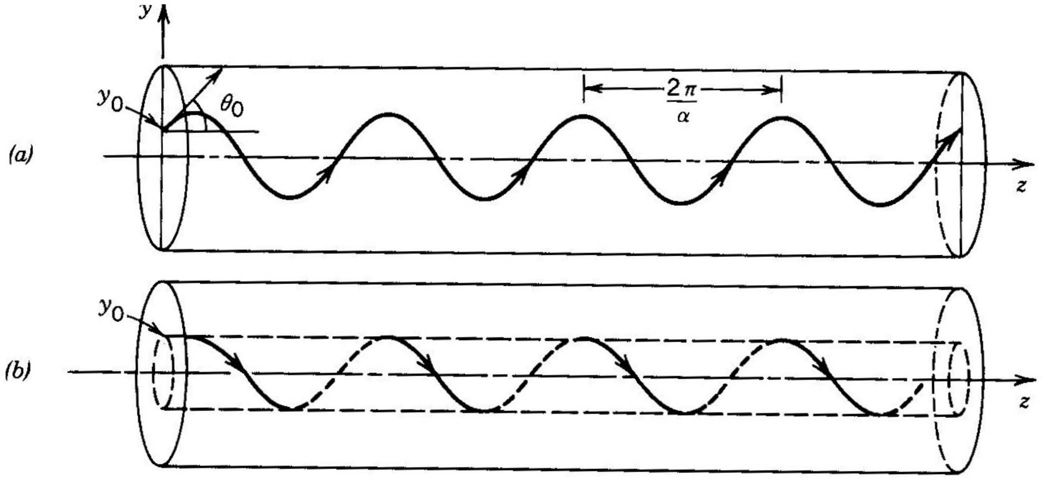
The helical path taken by a skew ray in an optical fiber: (a) skew ray path down the fiber; (b) cross-sectional view of the fiber.

Ray Optics enables to explain the transfer of an image by a fiberoptics endoscope



Graded-Index (GRIN) Optics

- Material has a refractive inex varying with the position ***n(r)***.
- Optical rays follow curved trajectories ⇐ Fermat's Principle.
- By appropriate choice of ***n(r)***, a GRIN plate can have the same effect on light as convention optical components.



(*a*) Meridional and (*b*) helical rays in a graded-index fiber with parabolic index profile.

Graded-Index Slab

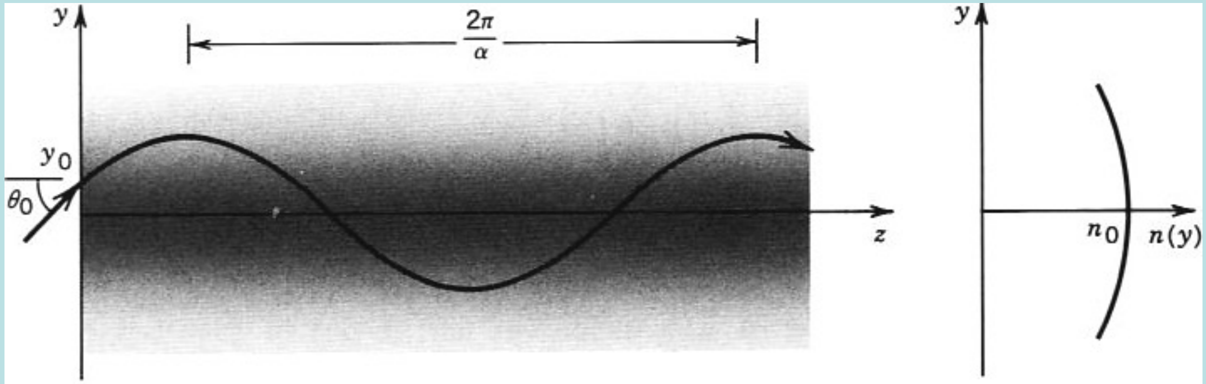


Figure 1.3-4 Trajectory of a ray in a GRIN slab of parabolic index profile (SELFOC).

Pitch = $2\pi/\alpha$

$$n(y)=n_0\,(1-\alpha^2y^2)^{1/2}$$

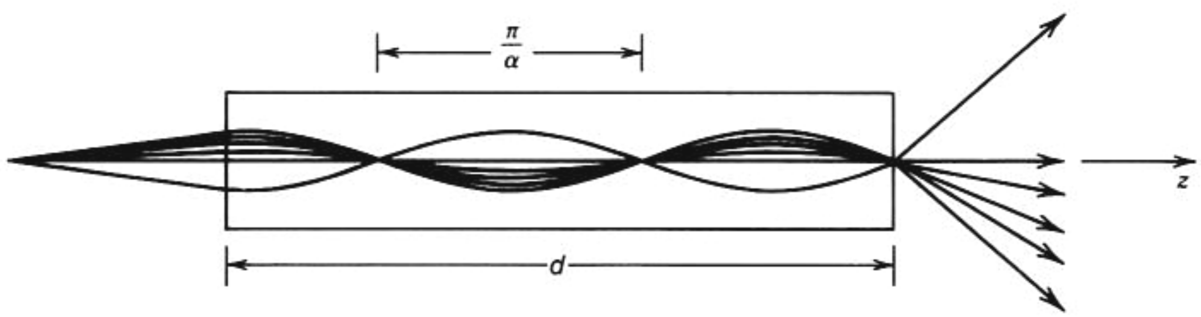


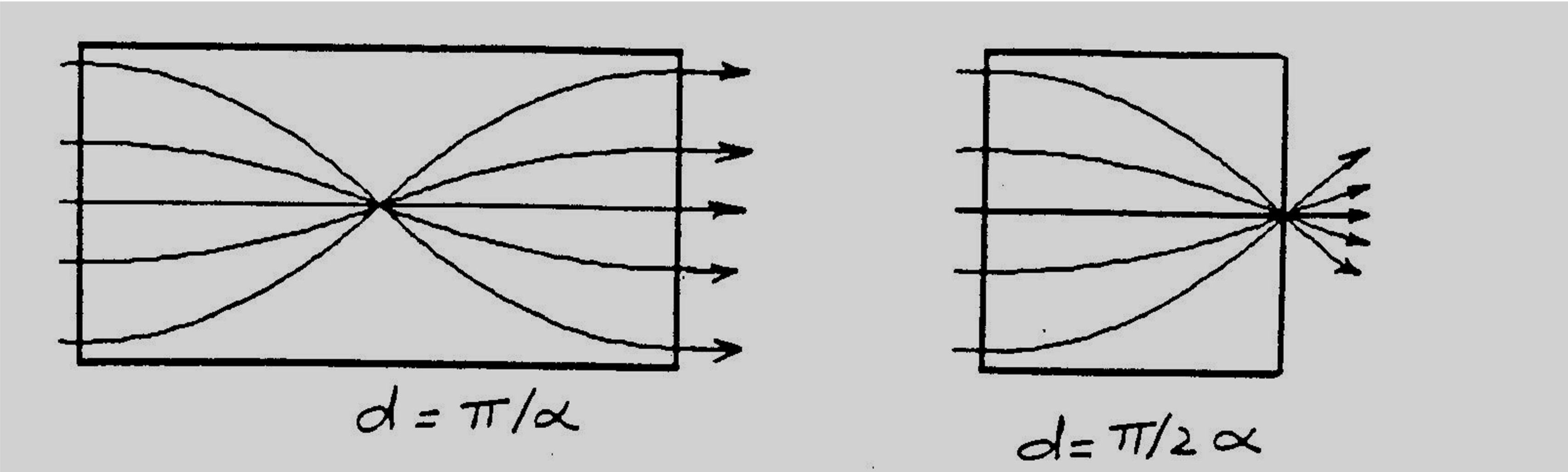
Figure 1.3-5 Trajectories of rays in a SELFOC slab.

Maximum excursion of the ray:

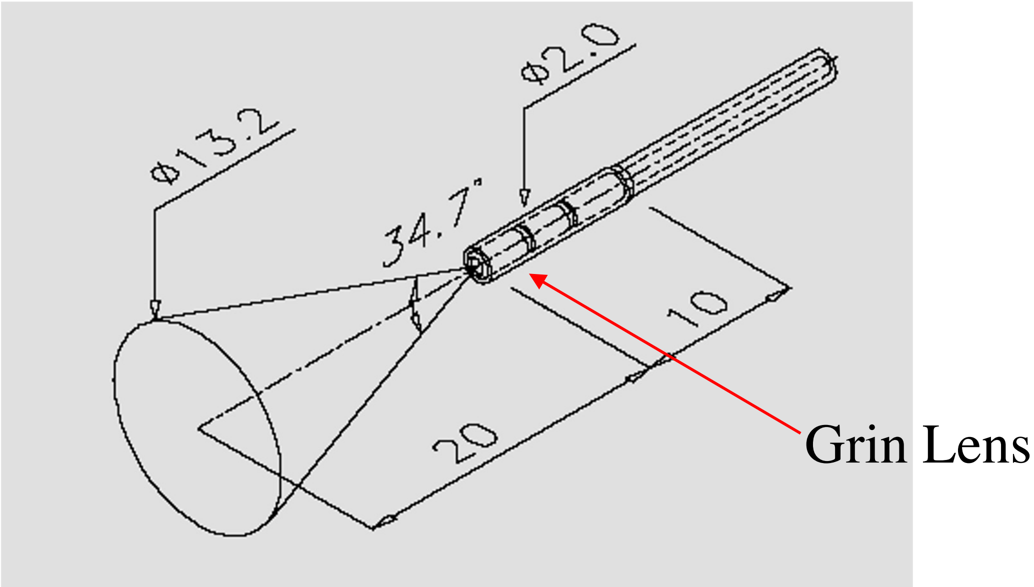
$$y_{max} = [y_0^2 + (\theta_0/\alpha)^2]^{1/2}$$

$$\theta_{max} = \alpha\, y_{max}$$

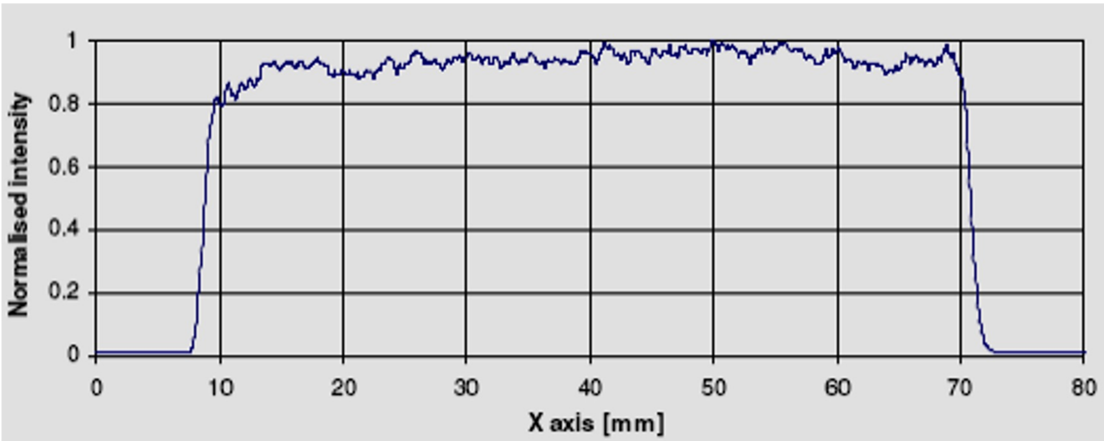
Graded-Index Lens



Frontal Light Distributor



Courtesy from Medlight SA



Typical Light Intensity Profile

(FD1, distance to screen : 100 mm)

Frontal light Distributor inserted through the biopsy channel of a flexible bronchoscope