Optical methods in chemistry
or
Photon tools for chemical sciences

Session 3:

Course layout – contents overview and general structure

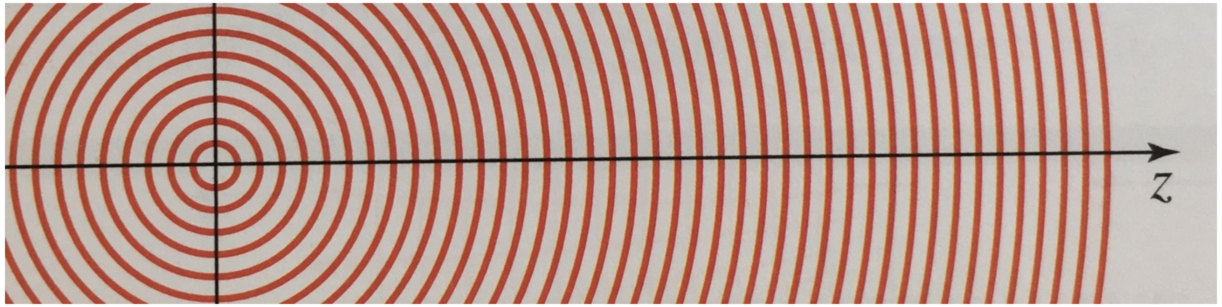
- Introduction and ray optics
- Wave optics
- Beams
- From cavities to lasers
- More lasers and optical tweezers
- From diffraction and Fourier optics
- Microscopy
- Spectroscopy
- Electromagnetic optics
- Absorption, dispersion, and non-linear optics
- Ultrafast lasers
- Introduction to x-rays
- X-ray diffraction and spectroscopy
- Summary

Todays learning goal:

Introduction to beam optics

Change the way you think about light propagation, foci, etc

Special case: Fresnel approximation

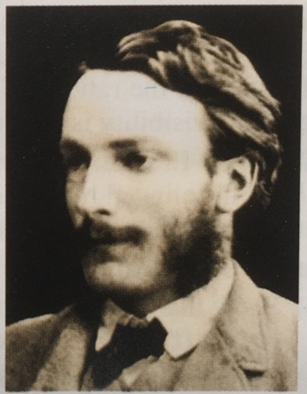


- Spherical wave close to z-axis but far away from origin

Beam optics

- Paraxial beams with intensity distribution
- Gaussian beam is solution for Helmholtz equation

The Gaussian beam takes the name of the celebrated German mathematician **Carl Friedrich Gauss (1777–1855)**, who established what is now known as the Gaussian distribution.



Lord Rayleigh (John William Strutt) (1842–1919) contributed to many areas of optics. The Rayleigh range of a Gaussian beam characterizes its depth of focus.

Gaussian beam I

- Consider paraxial beam with (slowly) varying complex amplitude

- One possible solution for the complex envelope is

Gaussian beam II

- Separat phase and amplitude in terms of real and imaginary part
- Further use substitution:

- So that you obtain:

Gaussian beam III

- U(r) is called a Gaussian beam

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

- With the following parameters

$$W(z) = W_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$$

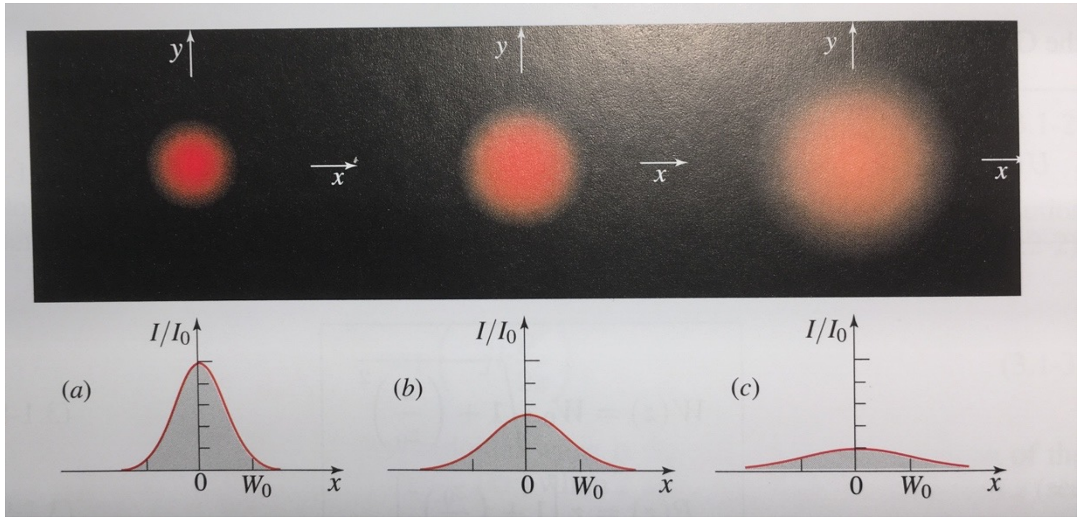
$$R(z) = z\left[1+\left(\frac{z_0}{z}\right)^2\right]$$

$$\zeta(z) = \tan^{-1}\frac{z}{z_0}$$

$$W_0 = \sqrt{\frac{\lambda z_0}{\pi}}.$$

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

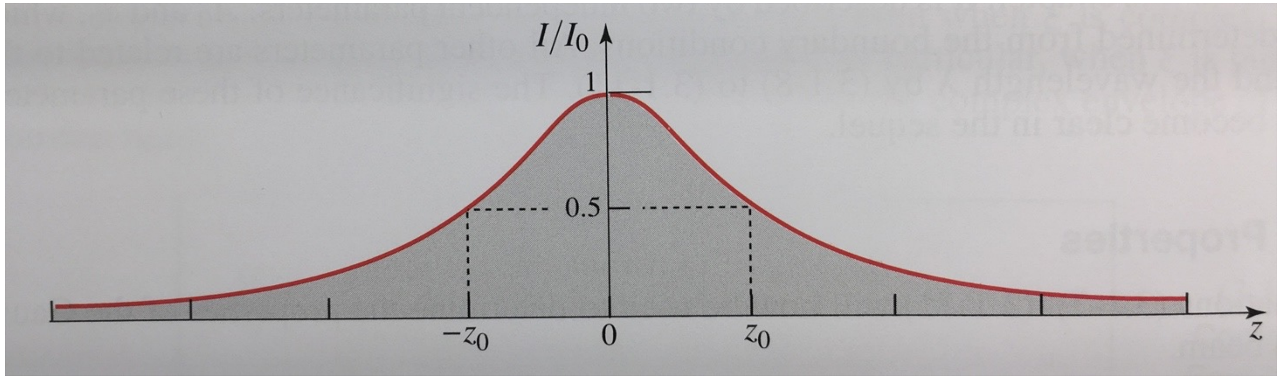
Parameter: Intensity as function of radial distance



Once a Gaussian – always a Gaussian!

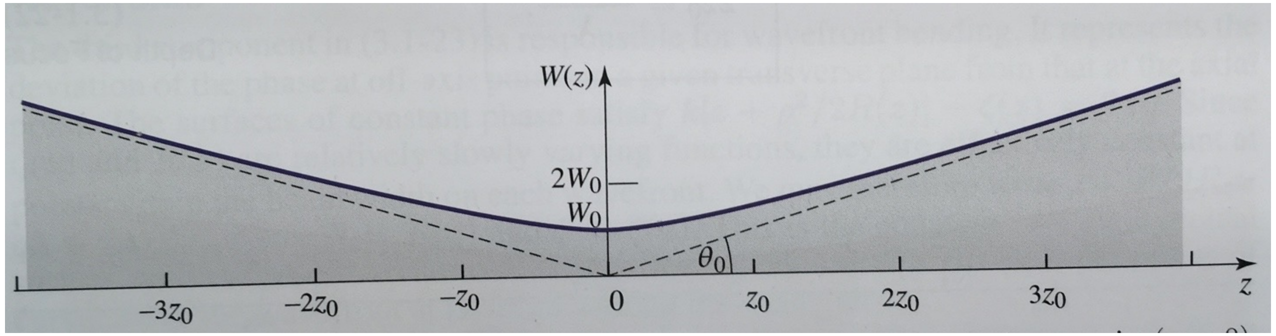
$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

Parameter: Intensity on beam axis



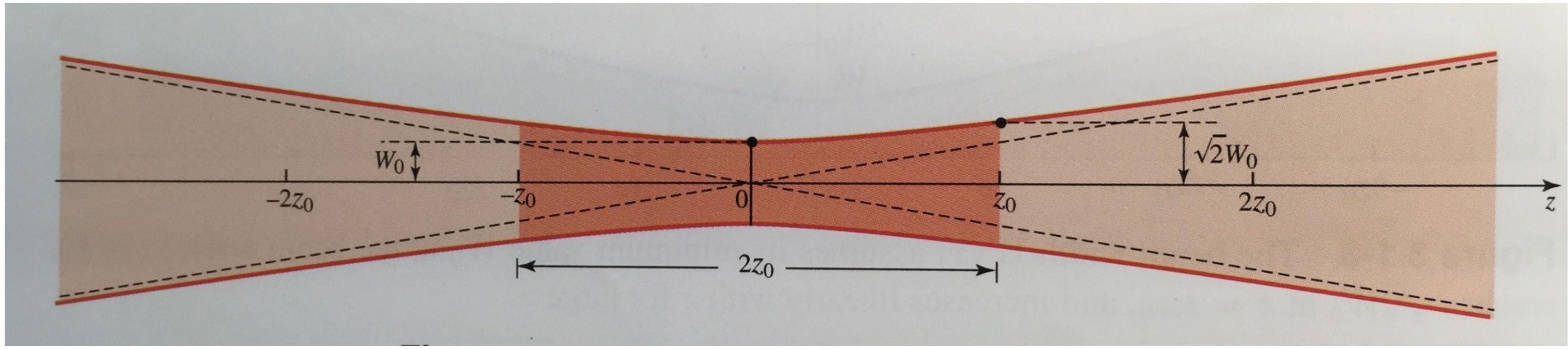
Parameter: Beam waist

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$



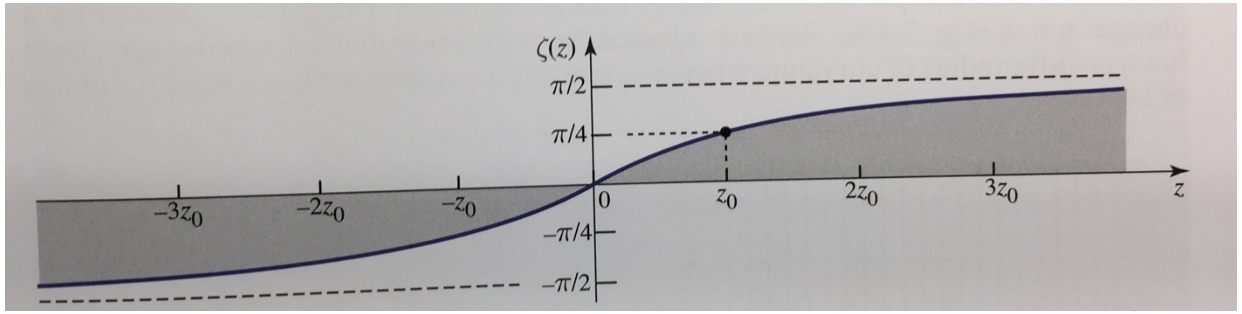
Parameter: Depth of focus

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$



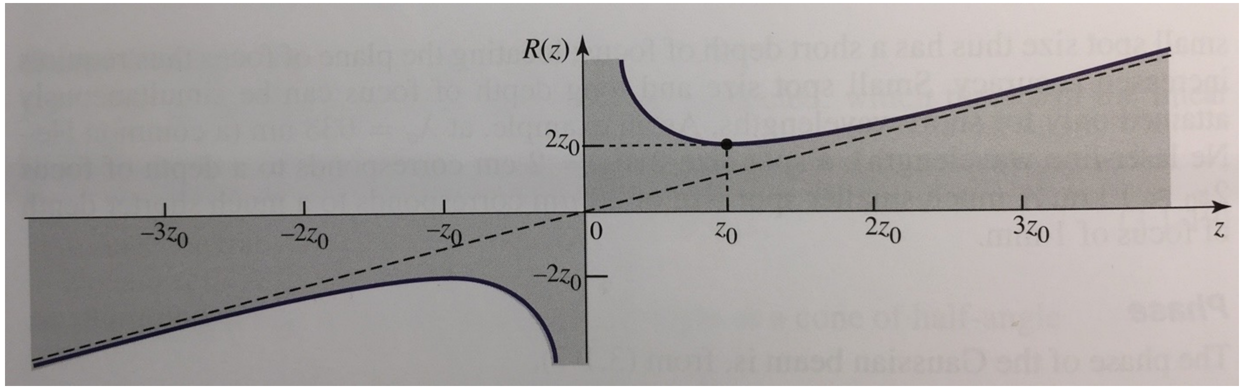
Parameter: Phase

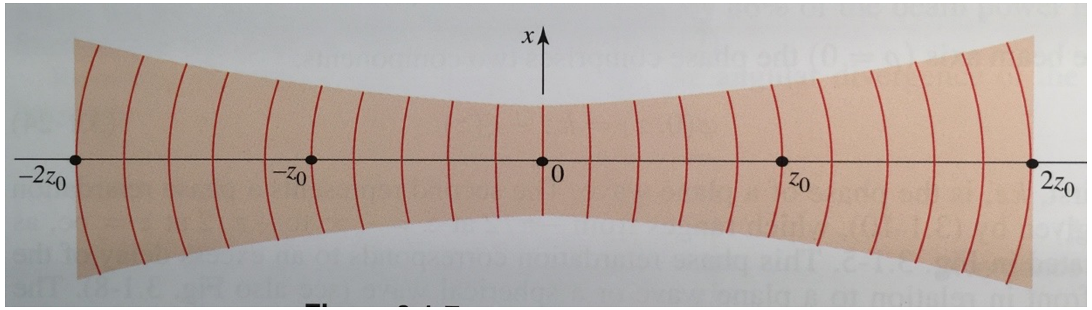
$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$



Parameter: Curvature of wavefront

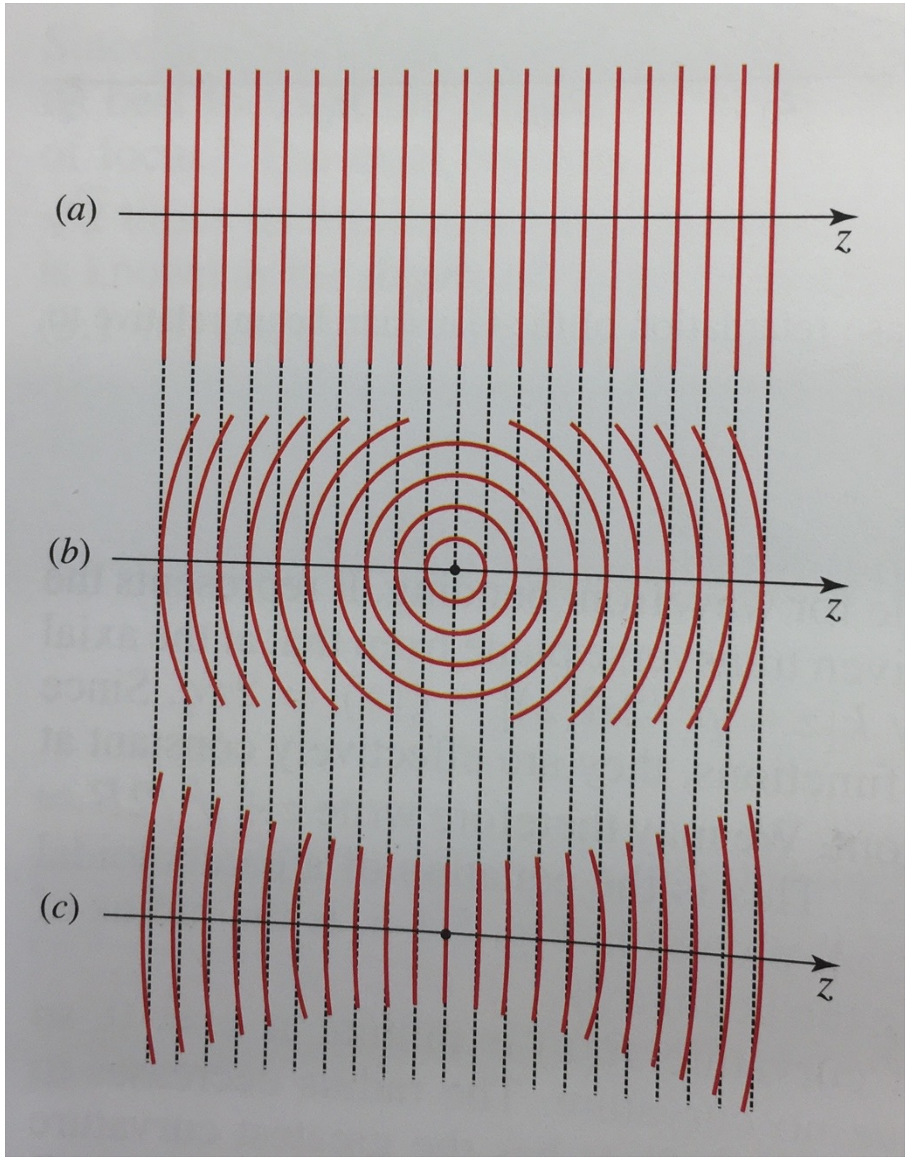
$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$





Comparison: plance wave, spherical wave, Gaussian beam

$$U(\mathbf{r}) = A_0 \frac{W_0}{W(z)} \exp\left[-\frac{\rho^2}{W^2(z)}\right] \exp\left[-jkz - jk\frac{\rho^2}{2R(z)} + j\zeta(z)\right]$$

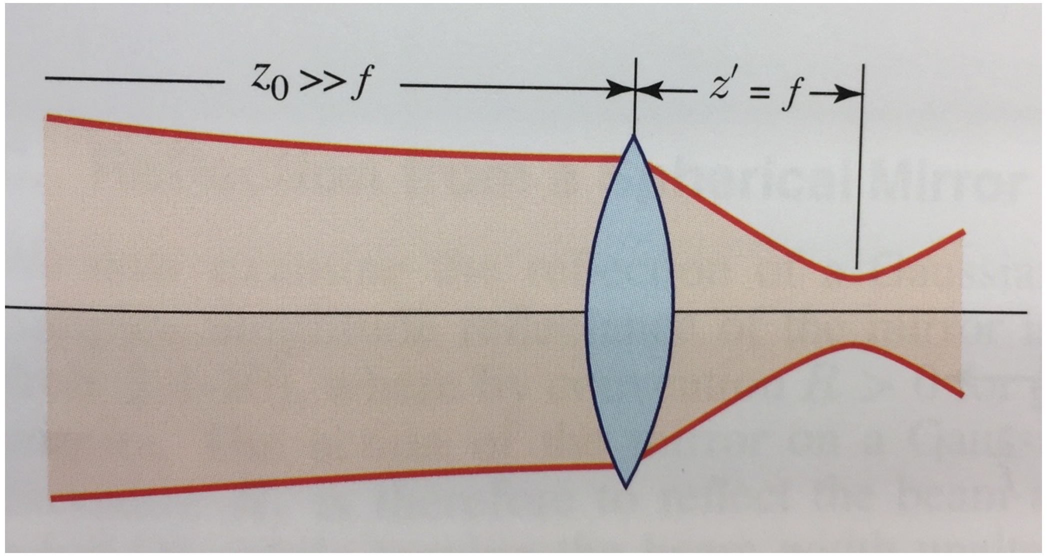


Some statements to remember

- A Gaussian beam transmitted through a circularly symmetric optical component remains a Gaussian beam

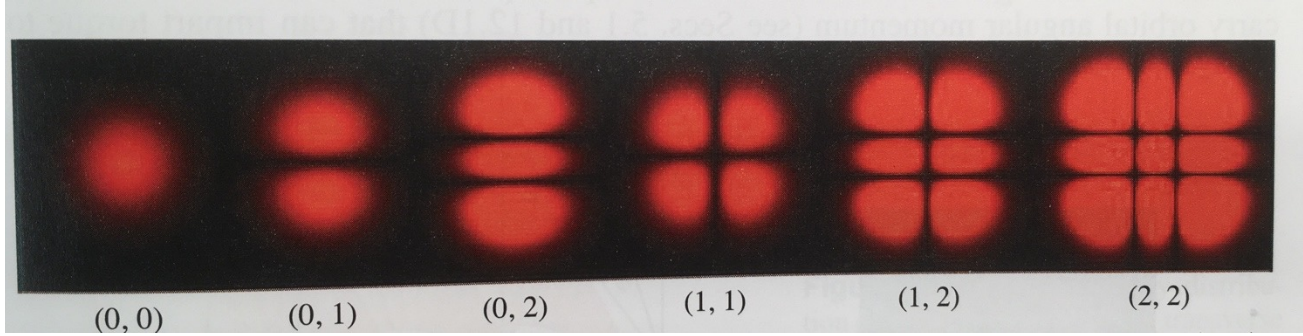
- Such optical components reshape the beam, i.e., its waist and curvature

- Focusing a collimated Gaussian beam:



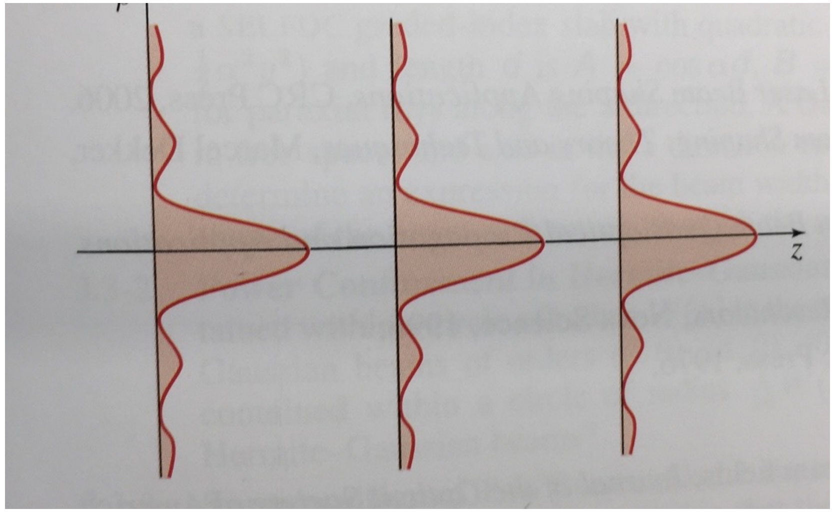
Note: There exist other solutions for the paraxial Helmholtz equation

- Hermite Gaussian beams
  - Non-Gaussian intensity distribution
  - Same wavefront as Gaussian beams
  - Can match curvature of spherical mirrors, ideal for building resonators



- Laguerre Gaussian beams
  - Solution of Helmholtz equation in cylindrical coordinates
  - Separation of variables in r and f (not x and y)

- Bessel beams
  - Solution of 2D Helmholtz equation in polar coordinates
  - For Bessel beams the intensity distribution is independent of z
  - Asymptotic behavior different from Gaussian beam



The end.