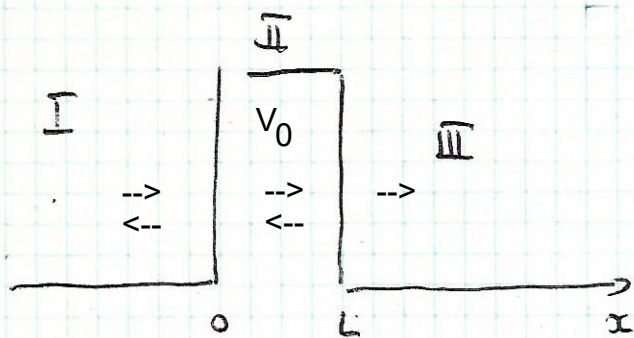




$$\textcircled{\text{I}}: -\frac{\hbar^2}{2m_0} \frac{d^2 \psi_{\text{I}}(x)}{dx^2} = E \psi_{\text{I}}(x) \Rightarrow \frac{d^2 \psi_{\text{I}}(x)}{dx^2} = -\frac{2m_0 E}{\hbar^2} \psi_{\text{I}}(x) \Rightarrow \frac{d^2 \psi_{\text{I}}(x)}{dx^2} = -\Sigma \psi_{\text{I}}(x)$$

$$\Sigma = \frac{2m_0 E}{\hbar^2}$$

$$\frac{d^2 \psi_{\text{I}}(x)}{dx^2} + \Sigma \psi_{\text{I}}(x) = 0 \Rightarrow \text{solution: } \psi_{\text{I}}(x) = A_1 e^{r_1 x} + B_1 e^{r_2 x}$$

$$\text{equation caractéristique: } ar^2 + br + c = 0 \Rightarrow r^2 + \Sigma = 0 \Rightarrow r^2 = -\Sigma$$

$$r^2 = -\Sigma \Rightarrow r_1 = i\sqrt{\Sigma} \text{ and } r_2 = -i\sqrt{\Sigma}$$

$$\psi_{\text{I}}(x) = A_1 e^{i\sqrt{\Sigma}x} + B_1 e^{-i\sqrt{\Sigma}x}$$

$$\Sigma = \frac{2m_0 E}{\hbar^2} = \frac{2m_0 p^2/2m_0}{\hbar^2} = \frac{p^2}{\hbar^2} = k^2 \Rightarrow \sqrt{\Sigma} = k$$

$$\psi_{\text{I}}(x) = A_1 e^{ikx} + B_1 e^{-ikx}$$

$$\textcircled{\text{II}}: -\frac{\hbar^2}{2m_0} \frac{d^2 \psi_{\text{II}}(x)}{dx^2} + V_0 \psi_{\text{II}}(x) = E \psi_{\text{II}}(x) \Rightarrow \frac{d^2 \psi_{\text{II}}(x)}{dx^2} \left(-\frac{2m_0 V_0}{\hbar^2} + \frac{2m_0 E}{\hbar^2} \right) \psi_{\text{II}}(x) = 0$$

$$\Sigma = \frac{2m_0 E}{\hbar^2} \text{ and } V_0 = \frac{2m_0 V_0}{\hbar^2}$$

$$\frac{d^2 \psi_{\text{II}}(x)}{dx^2} + (\Sigma - V_0) \psi_{\text{II}}(x) = 0 \Rightarrow \text{solution } \psi_{\text{II}}(x) = A_2 e^{r_1 x} + B_2 e^{r_2 x}$$

$$\text{equation caractéristique: } r^2 + (\Sigma - V_0) = 0 \Rightarrow r^2 = -(\Sigma - V_0)$$

$$\text{if } \Sigma < V_0 \Rightarrow \Sigma - V_0 < 0 \quad r^2 > 0 \Rightarrow r_1 = +\sqrt{V_0 - \Sigma} \quad r_2 = -\sqrt{V_0 - \Sigma}$$

$$r_1 = k \text{ and } r_2 = -k \Rightarrow \psi_{\text{II}}(x) = A_2 e^{kx} + B_2 e^{-kx}$$

$$\text{if } \Sigma > V_0 \Rightarrow \Sigma - V_0 > 0 \quad r^2 < 0 \Rightarrow \psi_{\text{II}}(x) = A_2 e^{-ikx} + B_2 e^{ikx}$$