Numerical methods in chemistry. Exercises 1

Problem 1

Determine, directly from the definition (i.e., not using various properties of $\mathcal{L}$ derived in the lecture), the Laplace transform of the following functions:

(a) e^{kt}

(b) t^2

(c) $\cosh(t)$

Problem 2

Find the Laplace transforms of the following functions (now you can and *should* use various properties of $\mathcal{L}$ derived in the lecture to simplify the calculations):

(a) $t^2 e^{-3t}$

(b) $4t + 6e^{4t}$

(c) $e^{-4t}\sin(5t)$

Problem 3

Use the relation

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s)$$

[property (2) in the lecture, Theorem 1.3 from the book] to determine the following Laplace transforms:

(a) te^{2t}

(b) $t\cos(t)$

(c) $t^2\cos(t)$

Problem 4

If $f(t) = \cos(at)$, use the derivative formula

$$\mathcal{L}[f'(t)] = sF(s) - f(0)$$

[property (5) in the lecture, Theorem 2.1 in the book] to re-establish the Laplace transform of $\sin(at)$.

Problem 5

Prove the following change of scale result:

$$\mathcal{L}[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right).$$

Hence evaluate the Laplace transform of the function $t\cos(6t)$.