Condensé de mécanique statistique

$$S = -k_{\mathrm{B}} \sum_{\mathcal{C}} \mathbb{P}(\mathcal{C}) \log \mathbb{P}(\mathcal{C})$$

1 Ensemble microcanonique

$$\mathbb{P}(\mathcal{C}) = \frac{1}{\Omega(U, \mathbf{X})} \qquad S = k_{\mathrm{B}} \ln \Omega$$

$$\mathrm{d}U = T\,\mathrm{d}S + \underbrace{\sum_i J_i\,\mathrm{d}X_i}_{-P\,\mathrm{d}V + \mu\,\mathrm{d}N}$$

2 Ensemble canonique

$$\mathbb{P}(\mathcal{C}) = \frac{\mathrm{e}^{-\beta E(\mathcal{C})}}{Z} \qquad Z = \sum_{\mathcal{C}} \mathrm{e}^{-\beta E(\mathcal{C})}$$

$$F = -k_{\mathrm{B}} T \ln Z = U - TS \qquad U = \langle E \rangle = -\frac{\partial \ln Z}{\partial \beta}$$

3 Ensemble grand canonique

$$\mathbb{P}(\mathcal{C}) = \frac{\mathrm{e}^{-\beta(E(\mathcal{C}) - \mu N(\mathcal{C}))}}{\Xi} \qquad \Xi = \sum_{\mathcal{C}} \mathrm{e}^{-\beta(E(\mathcal{C}) - \mu N(\mathcal{C}))}$$

$$Y = -k_{\mathrm{B}} T \ln \Xi = U - TS - \mu N \qquad \langle N \rangle = \frac{1}{\beta} \frac{\partial \ln \Xi}{\partial \mu}$$

4 Formules mathématiques utiles

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \sum_{n=0}^{\infty} n\, x^{n-1} = \frac{1}{(1-x)^2}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \mathrm{e}^x \qquad \ln(N!) = N \ln N - N$$

$$\int_{-\infty}^{+\infty} \mathrm{d}X\, \mathrm{e}^{-X^{\mathrm{T}} A X + B^{\mathrm{T}} X} = \sqrt{\frac{\pi^N}{\det A}} \exp\left(\frac{1}{4} B^{\mathrm{T}} A^{-1} B\right)$$