BIOENG 455 Test 1 Answers

1a) The following equation describes the surface energy (E) of an oil droplet in terms of its radius (R): what are the dimensions (M, L, T) of the constant σ?

$$E = 4\pi R^2 \sigma$$

$[\sigma]=[E] / [R^2]=M.L^2.T^{-2} / L^2 = M/T^2$

1b) The equation of state of DPD "water" is:

$$P = \rho\, k_B T + 2\alpha\, a\, \rho^2 / k_B T$$

where P = pressure, ρ = number density, k_B = Boltzmann's constant and T = temperature, a is the conservative force parameter, and α is a constant. What are the dimensions of (αa)? Given that [a] = [Force], what are the dimensions of α?

$[\alpha a] = [\text{Pressure}]\, [k_B T] / [\rho^2] = (ML^2T^{-2} / L^3) \times (ML^2T^{-2}) \times L^6 = M^2 L^7 / T^4$

Given $[a] = MLT^{-2}$, we get $[\alpha] = (M^2 L^7 / T^4) / (M\, L\, T^{-2}) = M\, L^6 / T^2$.

2a) Assuming that a cell is filled with synaptic vesicles, how many vesicles could fit inside the cell (i.e., have the same volume, ignore the missing pieces of volume between the spheres)?

Cell radius = 10 microns
Vesicle radius = 30 nm

and their volumes are: $V_{cell} = 4\pi/3\, R_{cell}^3$ $V_{vesicle} = 4\pi/3\, R_{ves}^3$

So, $N = V_{cell} / V_{ves} = (R_{cell} / R_{ves})^3 = (10 \text{ microns} / 30 \text{ nm})^3 \sim 3.7\ 10^7$

NB Note that the factors of $4\pi/3$ cancel in the ratio.

2b) What is the ratio of the surface area of the cell to the combined surface area of all these vesicles (as a fraction)?

Area of cell / area of all vesicles = area of cell / (N x area of 1 vesicle)

$= (10 \text{ micron} / 30 \text{ nm})^2 / 3.7.10^7 = 0.003.$

So the plasma membrane of the cell is only 0.3% of the vesicles' area. For comparison, the real ratio is 2%.

3) State the Equipartition theorem (making clear the key assumptions)

Each **additive**, **quadratic** degree of freedom in the Hamiltonian of a system contributes **1/2 k_BT** to its internal energy **in equilibrium**; the energy is shared among all accessible degrees of freedom. If the temperature is such that some d.o.f cannot be excited, they do not contribute to the internal energy.

4) Consider a real polymer as a linear chain of **N** spherical monomers of diameter **a:**

a) If the real polymer contains N = 1000 monomers of diameter a = 1.5 nm and has an end-to-end length of $\sqrt{\langle R_{ee}^2\rangle}$ = 122.5 nm, what is the Kuhn length and how many Kuhn lengths are in the equivalent phantom chain?

From $l_k = \langle L_{ee}^2\rangle / L$, we get L = 1500 nm, so

$l_k = \langle L_{ee}^2\rangle / L = (122,5 \text{ nm})^2 / 1500 \text{ nm} \sim 10 \text{ nm}$

From $N_k = L / l_k = 1500 / 10 \text{ nm} = 150$

NB. It's interesting to note that if you only have the two lengths related to a polymer: its contour length L, and its mean square end-to-end distance $\langle L_{ee}^2\rangle$, and you know that the Kuhn length has to be smaller than the contour length (it can't be bigger than the stretched out length!), then pretty much the only formula for the Kuhn length that has the right units is $l_k = \langle L_{ee}^2\rangle / L$. The number of Kuhn lengths must then be $N_k = L / l_k$.

This is another example of dimensional analysis guiding you to the right answer.

b) Given the expression for the end-to-end length squared of a real polymer of length N:

$\langle R_{ee}^2\rangle = N.a^2 + \langle \sum r_i . r_j \rangle$

where the sum over i, j is taken over all distinct monomers, write down the expression for the Characteristic Ratio C_N of the polymer. Can it be less than 1? If so, what does it tell you about the monomers in the polymer?

Yes. The characteristic ratio is the ratio of its end-to-end length squared to that of a freely-jointed chain with the same number of monomers: $C_N = \langle R_{ee}^2\rangle / N.a^2$

We have $C_N = \langle R_{ee}^2\rangle / N a^2$

and from Lecture 2, slide 28, $\langle R_{ee}^2\rangle = N.a^2 + \langle \sum r_i . r_j \rangle$, so

$C_N = 1 + \langle \sum r_i . r_j \rangle / N a^2$

and if $C_N < 1$ it means the sum is negative, or that there is an attraction between monomers that makes the monomers more likely to be close together instead of being stretched out.

5) Give two examples of entropic forces in a cell and two examples of membrane-mediated forces. Do these forces get stronger or weaker as the temperature is reduced?

Hydrophobic effect, depletion force, fluctuation-induced force

Capillary force, depletion force, composition or thickness mismatch, curvature force, fluctuation-induced force

They get weaker as the entropy decreases with decreasing temperature.
cp Pressure of an ideal gas: $PV = Nk_BT$, so it drops with temperature.

6) What are two uses of Dimensional analysis?

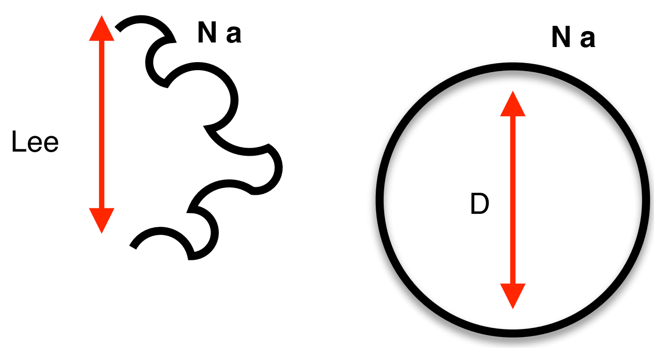
1. Verifies that equations or relations between quantities are physically correct
2. Predicts new relations between physical quantities

7) What are four reasons that biological polymers are more "interesting" than industrial polymers like polyethylene?

1. monomers carry information
2. spontaneously fold into a 3d shape
3. non-covalently self-assemble into 1d, 2d, and 3d aggregates
4. respond to their environment by changing shape

(Lecture 2, slide 25)

8) Which is larger: the diameter of a circular polymer with N monomers of size a or the end-to-end length of a phantom chain with N such monomers? Assume N >> 1 and express your answer as the ratio of the end-to-end length to diameter.



Circumference = $2\pi R = Na$

and diameter $D = 2R$,

$\langle L_{ee}^2 \rangle = Na^2$

So, $\sqrt{\langle L_{ee}^2 \rangle} / D = \sqrt{N}\, a / (N a / \pi) = \pi / \sqrt{N}$

and if N >> 1, the circle has a much larger diameter than the polymer's end-to-end length.