Series 8 (18 April 2025)

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6.21. A two dimensional, incompressible flow is given by $u = -y$ and $v = x$. Show that the streamline passing through the point $x = 10$ and $y = 0$ is a circle centered at the origin.

6.22. In a certain steady, two-dimensional flow field the fluid density varies linearly with respect to the coordinate x: that is, $\rho = Ax$ where A is a constant. If the x component of velocity u is given by the equation $u = y$, determine an expression for v.

6.25. The stream function for an incompressible flow field is given by the equation:

$$\psi = 3x^2y - y^3$$

where the stream function has the units of m^2/s with x and y in meters. (**a**) Sketch the streamline(s) passing through the origin. (**b**) Determine the rate of flow across the straight path AB shown in Fig. P6.25.

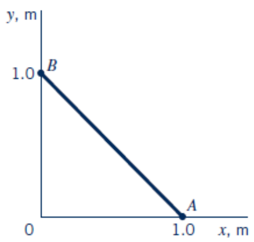

Figure P6.25

6.33. A two-dimensional flow field for a non-viscous, incompressible fluid described by the velocity components:

$$u = U_0 + 2y$$

$$v = 0$$

where U_0 is a constant. If the pressure at the origin (Fig. P6.33) is p_0, determine an expression for the pressure at (**a**) point A, and (**b**) point B. Explain clearly how you obtained your answer. Assume that the units are consistent and body forces may be neglected.

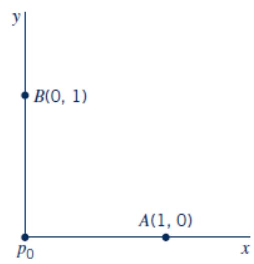

Figure P6.33

6.41. The velocity potential for a certain inviscid, incompressible flow field is given by the equation:

$$\varphi = 2x^2y - (2/3)y^3$$

where φ has the units of m^2/s when x and y are the in meters. Determine the pressure at the point $x = 2m, y = 2m$ if the pressure at $x = 1m, y = 1m$ is 200 kPa. Elevation changes can be neglected, and the fluid is water.

6.44. The velocity potential

$$\varphi = -k(x^2 - y^2) \ \ (k: constant)$$

may be used to represent the flow against an infinite plane boundary, as illustrated in Fig. P6.44. For flow in the vicinity of a stagnation point, it is frequently assumed that the pressure gradient along the surface is of the form:

$$\frac{\partial p}{\partial x} = Ax$$

where A is a constant. Use the given velocity potential to show that this is true.

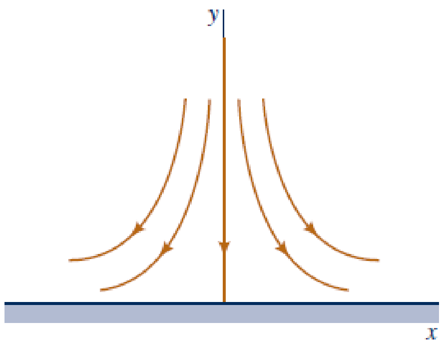

Figure P6.44