Series 7 - Solutions

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6.3

6.3 The velocity in a certain flow field is given by the equation

$$\mathbf{V} = x\hat{\mathbf{i}} + x^2z\hat{\mathbf{j}} + yz\hat{\mathbf{k}}$$

Determine the expressions for the three rectangular components of acceleration.

From expression for velocity, $u = x$, $v = x^2 z$, $w = yz$

Since

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

then

$$a_x = 0 + (x)(1) + (x^2z)(0) + (yz)(0)$$

$$= \underline{\underline{x}}$$

Similarly,

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

and

$$a_y = 0 + (x)(2xz) + (x^2z)(0) + (yz)(x^2)$$

$$= \underline{\underline{2x^2z + x^2yz}}$$

Also,

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

so that

$$a_z = 0 + (x)(0) + (x^2z)(z) + (yz)(y)$$

$$= \underline{\underline{x^2z^2 + y^2z}}$$

6-3

6.4

6.4 The three components of velocity in a flow field are given by

$$u = x^2 + y^2 + z^2$$
$$v = xy + yz + z^2$$
$$w = -3xz - z^2/2 + 4$$

(a) Determine the volumetric dilatation rate, and interpret the results. **(b)** Determine an expression for the rotation vector. Is this an irrotational flow field?

(a) Volumetric dilatation rate $= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ (Eq. 6.9)

Thus, for velocity components given

volumetric dilatation rate $= 2x + (x+z) + (-3x-z) = \underline{\underline{0}}$

This result indicates that there is no change in the volume of a fluid element as it moves from one location to another.

(b) From Eqs. 6.12, 6.13, and 6.14 with the velocity components given:

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(y - 2y\right) = -\frac{y}{2}$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) = \frac{1}{2}\left[0 - (y+2z)\right] = -\left(\frac{y}{2}+z\right)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = \frac{1}{2}\left[2z - (-3z)\right] = \frac{5z}{2}$$

Thus, $\vec{\omega} = \underline{\underline{-\left(\frac{y}{2}+z\right)\hat{i} + \frac{5z}{2}\hat{j} - \frac{y}{2}\hat{k}}}$

Since $\vec{\omega}$ is not zero everywhere the flow field is not irrotational. $\underline{\underline{\text{No.}}}$

6-4

6.6

6.6 Determine an expression for the vorticity of the flow field described by

$$\mathbf{V} = -xy^3\,\hat{\mathbf{i}} + y^4\,\hat{\mathbf{j}}$$

Is the flow irrotational?

$$\vec{\zeta} = 2\vec{\omega} \qquad \text{(Eq. 6.17)}$$

From expression for velocity, $u = -xy^3$, $v = y^4$, and $w = 0$, and with

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \text{(Eq. 6.13)}$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \text{(Eq. 6.14)}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{(Eq. 6.12)}$$

it follows that

$\omega_x = 0$, $\omega_y = 0$, and $\omega_z = \frac{1}{2}\left[0 - (-3xy^2)\right] = \frac{3}{2}xy^2$

Thus,

$$\vec{\zeta} = 2\left(\omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}\right)$$

$$= 2\left[(0)\hat{i} + (0)\hat{j} + \left(\tfrac{3}{2}xy^2\right)\hat{k}\right]$$

$$= \underline{\underline{3xy^2\,\hat{k}}}$$

Since $\vec{\zeta}$ is not zero everywhere the flow is not irrotational. <u>No.</u>

6-6

6.8

6.8 For a certain incompressible, two-dimensional flow field the velocity component in the y direction is given by the equation

$$v = 3xy + x^2y$$

Determine the velocity component in the x direction so that the volumetric dilatation rate is zero

For zero volumetric rate in a two-dimensional flow,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Since

$$\frac{\partial v}{\partial y} = 3x + x^2$$

Then from Eq. (1)

$$\frac{\partial u}{\partial x} = -3x - x^2 \qquad (2)$$

Equation (2) can be integrated with respect to x to obtain

$$\int du = -\int 3x\,dx - \int x^2 dx + f(y)$$

or

$$u = -\frac{3}{2}x^2 - \frac{x^3}{3} + f(y)$$

where $f(y)$ is an undetermined function of y.

6.12

6.12 For a certain incompressible flow field it is suggested that the velocity components are given by the equations

$$u = 2xy \qquad v = -x^2y \qquad w = 0$$

Is this a physically possible flow field? Explain.

Any physically possible incompressible flow field must satisfy conservation of mass as expressed by the relationship

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1)$$

For the velocity distribution given,

$$\frac{\partial u}{\partial x} = 2y \qquad \frac{\partial v}{\partial y} = -x^2 \qquad \frac{\partial w}{\partial z} = 0$$

Substitution into Eq. (1) shows that

$$2y - x^2 + 0 \neq 0$$

Thus, this is not a physically possible flow field. No.

6-12

6.14

6.14 For each of the following stream functions, with units of m^2/s, determine the magnitude and the angle the velocity vector makes with the x-axis at $x = 1$ m, $y = 2$ m. Locate any stagnation points in the flow field.
(a) $\psi = xy$
(b) $\psi = -2x^2 + y$

From the definition of the stream function,

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \qquad \text{(Eqs. 6.37)}$$

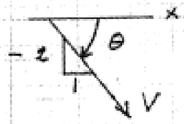
(a) For $\psi = xy$,

$$u = \frac{\partial \psi}{\partial y} = x \qquad v = -\frac{\partial \psi}{\partial x} = -y$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = -2\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2+v^2} = \sqrt{(1m)^2+(-2m)^2} = \underline{\underline{2.24\ \frac{m}{s}}}$$



$$\tan\theta = \frac{-2}{1} \qquad \underline{\underline{\theta = -63.4^\circ}}$$

Since $u = 0$ at $x = 0$ and $v = 0$ at $y = 0$, a stagnation point occurs at $x = y = 0$.

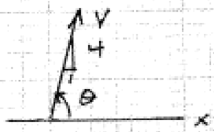
(b) For $\psi = -2x^2 + y$,

$$u = \frac{\partial \psi}{\partial y} = 1\frac{m}{s} \qquad v = -\frac{\partial \psi}{\partial x} = 4x$$

At $x = 1m$, $y = 2m$, it follows that $u = 1\frac{m}{s}$ and $v = 4\frac{m}{s}$

Thus,

$$|V| = \sqrt{u^2+v^2} = \sqrt{(1\tfrac{m}{s})^2+(4\tfrac{m}{s})^2} = \underline{\underline{4.12\ \frac{m}{s}}}$$



$$\tan\theta = \frac{4}{1} \qquad \underline{\underline{\theta = 76.0^\circ}}$$

Since $u \neq 0$, there are no stagnation points.

6-14