Series 6 - Solutions

TAs: *Cemre Celikbudak, Soroush Rafiei, Sokratis Anagnostopoulos, Ramin Mohammadi, Ellen Jamil Dagher, Veronika Pak, El Ghali Jaidi, Coline Jeanne Leteurtre*

5.102

5.102 Oil ($SG = 0.9$) flows downward through a vertical pipe contraction as shown in Fig. P5.102. If the mercury manometer reading, h, is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.

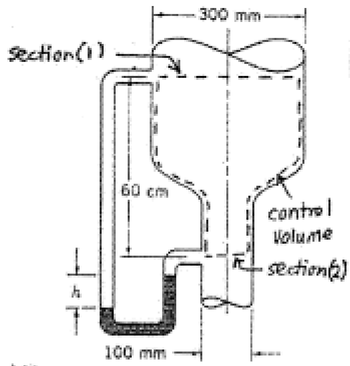


FIGURE P5.102

The volume flowrate may be obtained with

$$Q = A_1 V_1 = A_2 V_2 = \frac{\pi D_1^2}{4} V_1 = \frac{\pi D_2^2}{4} V_2 \qquad (1)$$

To determine either V_1 or V_2 we apply the energy equation (Eq. 5.82) to the flow between sections (1) and (2). Thus,

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + \underbrace{w_{\text{shaft net in}}}_{0} - \underbrace{\text{loss}}_{\text{neglect}} \qquad (2)$$

Combining Eqs. 1 and 2 we obtain

$$\frac{V_2^2}{2}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{p_1 - p_2}{\rho} + g(z_1 - z_2) \qquad (3)$$

To determine $\frac{p_1 - p_2}{\rho}$ we use the manometer equation from section 2.6 to obtain

$$\frac{p_1 - p_2}{\rho} = gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right) - g(z_1 - z_2) \qquad (4)$$

Combining Eqs. 3 and 4 we get

$$V_2 = \sqrt{\frac{2gh\left(\frac{SG_{Hg}}{SG_{oil}} - 1\right)}{1 - \left(\frac{D_2}{D_1}\right)^4}}$$

or

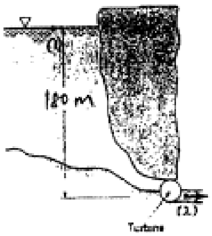
$$V_2 = \sqrt{\frac{(2)(9.81\,\frac{m}{s^2})(0.1\ m)\left(\frac{13.6}{0.9} - 1\right)}{1 - \left(\frac{100\ mm}{300\ mm}\right)^4}} = 5.29\ \frac{m}{s}$$

and from Eq. 1 we have

$$Q = \frac{\pi (0.1 m)^2}{4}\left(5.29\ \frac{m}{s}\right) = \underline{\underline{0.042\ \frac{m^3}{s}}}$$

Actual flowrate would be less than the frictionless value because the loss would be greater than the zero amount used above.

5.115 The hydroelectric turbine shown in Fig. P5.115 passes 500 kL/S across a head of 180 m. What is the maximum amount of power output possible? Why will the actual amount be less?



■ FIGURE P5.115

From the energy equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + h_s - h_L = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

where $p_1 = 0$, $p_2 = 0$, and $V_1 = 0$.

Thus,

$$h_s = (z_2 - z_1) + h_L + \frac{V_2^2}{2g}$$

And, the power is given by

$$\dot{W}_{turb} = \gamma Q h = \gamma Q \left[(z_2 - z_1) + h_L + \frac{V_2^2}{2g} \right]$$

The maximum power would occur if there were no losses ($h_L = 0$) and negligible kinetic energy at the exit ($V_2 \approx 0$; large diameter outlet).

Thus,

$$\dot{W}_{turb\,max} = \gamma Q\,(z_2 - z_1) = 9.8\,\frac{kN}{m^3}\left(500 \times 10^3\,\frac{L}{s}\right)\left(0.001\,\frac{m^3}{L}\right)(-180\,m)$$

$$= -88.2 \times 10^7\,\frac{N \cdot m}{s}$$

or $\dot{W}_{turb\,max} = \underline{\underline{-822\text{ MW}}}$

The minus sign is associated with power out.

The actual power will be less by amounts corresponding to loss and exit kinetic energy.

5.121 The turbine shown in Fig. P5.121 develops 74.5 kW when the flowrate of water is 0.6 m³/s. If all losses are negligible, determine (a) the elevation, h, (b) the pressure difference across the turbine, and (c) the flowrate expected if the turbine were removed.

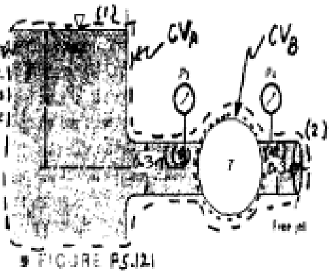


FIGURE P5.121

(a) Using control volume A and the energy equation (Eq. 5.84) we get:

$$\frac{\cancel{p_2}}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{\cancel{p_1}}{\gamma} + \frac{\cancel{V_1^2}}{2g} + z_1 + h_s - \cancel{h_L} \qquad (1)$$

For a turbine, $h_T = -h_s$ and from Eq. 5.85 we get:

$$h_T = \frac{\dot{W}_{\text{shaft net out}}}{\gamma Q} = \frac{(74.5\,kN)}{\left(9.80\,\frac{kN}{m^3}\right)(0.6\,m^3/s)} = 12.6\ m$$

Since $Q = AV$, we have

$$V_2 = \frac{Q}{A_2} = \frac{0.6\ m^3/s}{\frac{\pi}{4}\,d_2^2} = \frac{0.6\ m^3/s}{\frac{\pi}{4}(0.3\,m)^2} = 8.5\ m/s$$

Then from Eq. 1

$$z_1 - z_2 = h = \frac{V_2^2}{2g} - h_s = \frac{(8.5\ m/s)^2}{2(9.81\ m/s^2)} + 12.6\ m = \underline{\underline{16.3\ m}}$$

(b) For control volume B the energy equation yields

$$p_3 - p_4 = \gamma h_T = \left(9.8\,\frac{kN}{m^3}\right)(12.6\,m) = \underline{\underline{123.5}}\ \frac{kN}{m^2}$$

(c) Since $Q = VA = V_2 A_2$, if we know value of V_2 with the turbine removed, we could calculate Q with the turbine removed. Without the turbine, Eq. (1) reduces to

$$\frac{V_2^2}{2g} = z_1 - z_2 = h$$

and $V_2 = \sqrt{2gh} = \sqrt{2\left(9.81\,\frac{m}{s^2}\right)(16.3\,m)} = 18\ m/s$

Thus,

$$Q_{\text{w/o turbine}} = \frac{\pi d_2^2}{4} V_2 = \frac{\pi}{4}(0.3\,m)^2\,(18\ m/s) = \underline{1.27}\ \frac{m^3}{s}$$

5.129

5.129 Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.129. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m², respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. **(a)** Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. **(b)** Determine the force that the air puts on the object.

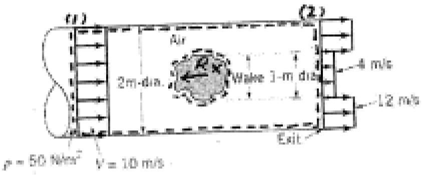


FIGURE P5.129

(a) To determine the loss suffered by a fluid particle as it flows from (1) to a location in the wake at (2) we apply the energy equation (Eq. 5.84) to that particle flow to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (1)$$

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

and

$$h_L = \frac{\left(50\ \frac{N}{m^2}\right)}{\left(12\ \frac{N}{m^3}\right)} + \frac{\left(10\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} - \frac{\left(4\ \frac{m}{s}\right)^2}{2\left(9.81\ \frac{m}{s^2}\right)} = \underline{\underline{8.45\ m}}$$

To determine the head loss associated with the entire flow across the object we use the non-uniform flow energy equation (Eq. 5.89) for flow from (1) to (2) through the control volume shown in the sketch to get:

$$\cancel{\frac{p_2}{\gamma}}^{\,0} + \frac{\alpha_2 \bar{V}_2^2}{2g} + \cancel{z_2} = \frac{p_1}{\gamma} + \frac{\alpha_1 \bar{V}_1^2}{2g} + \cancel{z_1} + \cancel{\frac{w_{\text{shaft net in}}}{g}}^{\,0} - h_L \qquad (2)$$

From Eq. 5.86 we get:

$$\frac{\alpha \bar{V}^2}{2g} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\dot{m}} = \frac{\int \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{\rho \bar{V} A}$$

Eq. (2) becomes

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{\int_{A_2} \frac{V^2}{2g} \rho \vec{V} \cdot \hat{n}\, dA}{(\rho V A)_{4\frac{m}{s}} + (\rho V A)_{12\frac{m}{s}}}$$

(con't)

5-143

5.129 (Con't)

or

$$h_L = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} - \frac{1}{2g}\left[\frac{V^3_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^3_{3\frac{m}{s}} A_{3\frac{m}{s}}}{V_{4\frac{m}{s}} A_{4\frac{m}{s}} + V_{12\frac{m}{s}} A_{12\frac{m}{s}}}\right]$$

and

$$h_L = \frac{\left(50\frac{N}{m^2}\right)}{\left(12\frac{N}{m^3}\right)} + \frac{\left(10\frac{m}{s}\right)^2}{2\left(9.81\frac{m}{s^2}\right)} - \frac{1}{2\left(9.81\frac{m}{s^2}\right)}\left\{\frac{\left(12\frac{m}{s}\right)^3 \pi\left[\frac{(2m)^2-(1m)^2}{4}\right] + \left(4\frac{m}{s}\right)^3 \frac{\pi(1m)^2}{4}}{\left(4\frac{m}{s}\right)\frac{\pi(1m)^2}{4} + \left(12\frac{m}{s}\right)\pi\left[\frac{(2m)^2-(1m)^2}{4}\right]}\right\}$$

$$h_L = \underline{\underline{2.58\ m}}$$

(b) To determine the force that the air puts on the object, R_x, we use the horizontal component of the linear momentum equation to get:

$$-\rho V_1^2 A_1 + \rho V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + \rho V^2_{4\frac{m}{s}} A_{4\frac{m}{s}} = p_1 A_1 - R_x$$

and thus

$$R_x = p_1 A_1 + \rho V_1^2 A_1 - \rho\left(V^2_{12\frac{m}{s}} A_{12\frac{m}{s}} + V^2_{4\frac{m}{s}} A_{4\frac{m}{s}}\right)$$

So

$$R_x = \left(50\frac{N}{m^2}\right)\frac{\pi(2m)^2}{4} + \left(1.23\frac{kg}{m^3}\right)\left(10\frac{m}{s}\right)^2 \frac{\pi(2m)^2}{4}\left(1\frac{N\cdot s^2}{m\cdot kg}\right)$$
$$-1.23\frac{kg}{m^3}\left\{\left(12\frac{m}{s}\right)^2 \frac{\pi\left[(2m)^2-(1m)^2\right]}{4} + \left(4\frac{m}{s}\right)^2\frac{\pi(1m)^2}{4}\right\}\left(\frac{1\ N\cdot s^2}{m\cdot kg}\right)$$

and

$$R_x = \underline{\underline{110\ N}}$$

5-144

5.133

5.133 When fluid flows through an abrupt expansion as indicated in Fig. P5.133, the loss in available energy across the expansion, loss_{ex}, is often expressed as

$$\text{loss}_{ex} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where A_1 = cross-sectional area upstream of expansion, A_2 = cross-sectional area downstream of expansion, and V_1 = velocity of flow upstream of expansion. Derive this relationship.

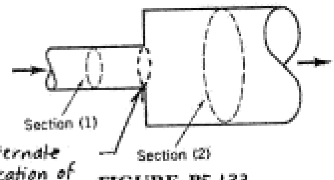


FIGURE P5.133

Applying the energy equation (Eq. 5.82) to the flow from section (1) to section (2) we obtain

$$\text{loss}_{ex} = \frac{p_1 - p_2}{\rho} + \frac{V_1^2 - V_2^2}{2} \quad (1)$$

Applying the axial direction component of the linear momentum equation (Eq. 5.22) to the fluid contained in the control volume from section (1) to section (2) we obtain

$$R_x + p_1 A_1 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \quad (2)$$

Now, if we consider section (1) as occurring at the end of the smaller diameter pipe (the beginning of the larger diameter pipe) as indicated in the sketch above, Eq. 1 still yields the expansion loss and Eq. 2 becomes

$$R_x + p_1 A_2 - p_2 A_2 = -V_1 \rho A_1 V_1 + V_2 \rho A_2 V_2 \quad (3)$$

Note that with section (1) positioned at the end of the smaller diameter pipe, p_1 acts over area A_2. Also, because of the jet flow from the smaller diameter pipe into the larger diameter pipe, the value of R_x will be small enough compared to the other terms in Eq. 3 that we can drop R_x. From Eq. 3

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1^2 \frac{A_1}{A_2} \quad (4)$$

Combining Eqs. 1 and 4 we obtain

$$\text{loss}_{ex} = V_2^2 - V_1^2 \frac{A_1}{A_2} + \frac{V_1^2 - V_2^2}{2}$$

(con't)

5-149

5.133 (con't)

From conservation of mass (Eq. 5.13) we have

$$V_2 = V_1 \frac{A_1}{A_2} \qquad (6)$$

Combining Eqs. 5 and 6 we get

$$loss_{ex} = V_1^2\left(\frac{A_1}{A_2}\right)^2 - V_1^2\left(\frac{A_1}{A_2}\right) + \frac{V_1^2 - V_1^2\left(\frac{A_1}{A_2}\right)^2}{2}$$

or

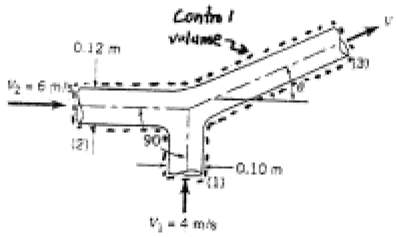
$$loss_{ex} = \frac{V_1^2}{2}\left[2\left(\frac{A_1}{A_2}\right)^2 - 2\frac{A_1}{A_2} + 1 - \left(\frac{A_1}{A_2}\right)^2\right]$$

and

$$loss_{ex} = \frac{V_1^2}{2}\left(1 - \frac{A_1}{A_2}\right)^2$$

5.134

5.134 Two water jets collide and form one homogeneous jet as shown in Fig. P5.134. (a) Determine the speed, V, and direction, θ, of the combined jet. (b) Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



■ FIGURE P5.134

For the water flowing through the control volume sketched above, the x- and y-direction components of the linear momentum equation are

$$-V_2 \rho V_2 A_2 + V_3 \cos\theta \, \rho V_3 A_3 = 0 \qquad (1)$$

and

$$-V_1 \rho V_1 A_1 + V_3 \sin\theta \, \rho V_3 A_3 = 0 \qquad (2)$$

From the conservation of mass principle we get

$$-\rho V_1 A_1 - \rho V_2 A_2 + \rho V_3 A_3 = 0 \qquad (3)$$

Combining Eqs. 1 and 2 we obtain

$$\tan\theta = \frac{V_1^2 A_1}{V_2^2 A_2} = \frac{V_1 \frac{\pi d_1^2}{4}}{V_2 \frac{\pi d_2^2}{4}} = \frac{\left(4 \frac{m}{s}\right)^2 \frac{\pi (0.1 m)^2}{4}}{\left(6 \frac{m}{s}\right)^2 \frac{\pi (0.12 m)^2}{4}} = 0.3086$$

So

$$\theta = \tan^{-1} 0.3086 = \underline{\underline{17.2^\circ}}$$

Now, combining Eqs. 1 and 3 we get

$$-V_2^2 \rho A_2 + V_3 \cos\theta \left(\rho V_1 A_1 + \rho V_2 A_2\right) = 0$$

or

$$V_3 = \frac{V_2^2 A_2}{\cos\theta \left(V_1 A_1 + V_2 A_2\right)} = \frac{V_2^2 d_2^2}{\cos\theta \left(V_1 d_1^2 + V_2 d_2^2\right)}$$

Thus

$$V_3 = \frac{\left(6 \frac{m}{s}\right)^2 (0.12 m)^2}{(\cos 17.2^\circ)\left[\left(4 \frac{m}{s}\right)(0.1 m)^2 + \left(6 \frac{m}{s}\right)(0.12 m)^2\right]}$$

and

$$V_3 = \underline{\underline{4.29 \frac{m}{s}}}$$

(con't)

5-151

5.134 (con't)

To determine the loss of available energy associated with the flow through this control volume we obtain by applying the energy equation (Eq. 5.64)

$$-\left(\check{u}_1 + \frac{V_1^2}{2}\right)\dot{m}_1 - \left(\check{u}_2 + \frac{V_2^2}{2}\right)\dot{m}_2 + \left(\check{u}_3 + \frac{V_3^2}{2}\right)\dot{m}_3 = 0 \qquad (4)$$

Also, the conservation of mass equation, Eq. 3, can also be written as

$$-\dot{m}_1 - \dot{m}_2 + \dot{m}_3 = 0 \qquad (5)$$

Combining Eqs. 4 and 5, we obtain

$$\dot{m}_1(\check{u}_3 - \check{u}_1) + \dot{m}_2(\check{u}_3 - \check{u}_2) = \dot{m}_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \dot{m}_2\left(\frac{V_2^2 - V_3^2}{2}\right) \qquad (6)$$

The left hand side of Eq. 6 represents the rate of available energy loss in this fluid flow. Thus rate of available energy loss is

$$\text{rate of loss} = \rho V_1 A_1\left(\frac{V_1^2 - V_3^2}{2}\right) + \rho V_2 A_2\left(\frac{V_2^2 - V_3^2}{2}\right)$$

or

$$\text{rate of loss} = \frac{\rho\pi}{4}\left[d_1^2 V_1\left(\frac{V_1^2 - V_3^2}{2}\right) + d_2^2 V_2\left(\frac{V_2^2 - V_3^2}{2}\right)\right]$$

Thus

$$\text{rate of loss} = \frac{\left(999\,\frac{kg}{m^3}\right)(3.14)\left(1\,\frac{N\cdot s^2}{kg\cdot m}\right)}{4}\left\{(0.10\,m)^2\left(4\,\frac{m}{s}\right)\left[\frac{\left(4\,\frac{m}{s}\right)^2 - \left(4.29\,\frac{m}{s}\right)^2}{2}\right] + (0.12\,m)^2\left(6\,\frac{m}{s}\right)\left[\frac{\left(6\,\frac{m}{s}\right)^2 - \left(4.29\,\frac{m}{s}\right)^2}{2}\right]\right\}$$

and

$$\text{rate of loss} = \underline{\underline{558\ \frac{N\cdot m}{s}}}$$

5-152